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Chaotic Complex-Valued Bidirectional Associative Memory — One-to-Many Association Ability —

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Abstract—In this paper, we propose a Chaotic Complex-valued Bidirectional Associative Memory (CCBAM) which can realize one-to-many associations of multi-valued patterns. The proposed model is based on the Bidirectional Associative Memory, and is composed of complex-valued neurons and chaotic complex-valued neurons. In the proposed model, associations of multi-valued patterns are realized by using complex-valued neurons, and one-to-many associations are realized by using chaotic complex-valued neurons. We carried out a series of computer experiments and confirmed that the proposed model can realize one-to-many associations of multi-valued patterns.

1. Introduction

Recently, neural networks are drawing much attention as a method to realize flexible information processing. And, the associative memories have been proposed such as the Associatron, the Hopfield network[1], the Bidirectional Associative Memory[2]. However, those models can not deal with multiple-valued patterns. The associative memory based on the Self-Organizing Map[3] which can deal with real-valued patterns has been proposed. However, it is based on the local representation, therefore it is not robust for damage of neurons.

As the model which can deal with multi-valued patterns, the complex-valued neuron model has been proposed[4]. In the complex-valued model, input, output and internal states of neurons have complex-value. The network is composed of complex-valued neurons can deal with multi-valued pattern[4].

On the other hand, chaos is drawing much attention as a method to realize flexible information processing as well as neural networks, fuzzy logic and genetic algorithms. Chaos is a phenomenon which is observed in deterministic nonlinear systems and the behavior is unpredictable. The chaotic behavior exists in many fields such as hydrodynamics, electric circuits and biological systems. In particular, it is considered that chaos plays an important role in the memory and learning of a human brain. In order to mimic the real neurons, a chaotic neuron model has been proposed by Aihara *et al.*[5]. In this model, chaos is introduced by considering the following properties of the real neurons: (1) spatio-temporal summation, (2) refractoriness and (3) continuous output function. It is known that the dynamic (chaotic) association is realized in the associative memories composed of the chaotic neurons[5]. And, the Chaotic Bidirectional Associative Memory (CBAM)[6] realizes one-to-many associations by introducing chaotic neurons to a part of the Bidirectional Associative Memory[2].

The chaotic complex-valued neuron model[7] which is based on the complex-valued neuron model[4] and the chaotic neuron model[5] has been proposed. The chaotic complex-valued associative memory[7] composed of chaotic complex-valued neuron models can realize dynamic associations of multi-valued patterns.

In this paper, we propose the Chaotic Complex-valued Bidirectional Associative Memory (CCBAM) which can realize one-to-many associations of multi-valued patterns. The proposed model is based on the Bidirectional Associative Memory, and is composed of complex-valued neurons and chaotic complex-valued neurons. In the proposed model, associations of multi-valued patterns are realized by using complex-valued neurons, and one-to-many associations are realized by using chaotic complex-valued neurons.

2. Chaotic Complex-Valued Neuron Model

Here, we explain the chaotic complex-valued neuron model[7] which is used in the proposed model. The chaotic complex-valued neuron model is the extended chaotic neuron model in order to deal with complex-value as internal states and output of neurons. It is known that the chaotic complex-valued associative memory composed of chaotic complex-valued neurons can realize dynamic associations of multi-valued patterns[7].

The dynamics of the chaotic complex-valued neuron is given by

$$x(t+1) = f\left(A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta\right) \quad (1)$$
$$(A(t), x(t), \theta \in \mathrm{C} \qquad k, \alpha \in \mathrm{R})$$

where $x(t)$ is the output of the neuron at the time t, $A(t)$ is the external input at the time t, α is the scaling factor of the refractoriness, k is the damping factor ($0 < k_r < 1$), and θ is the threshold of the neuron. $f(\cdot)$ is the output function is

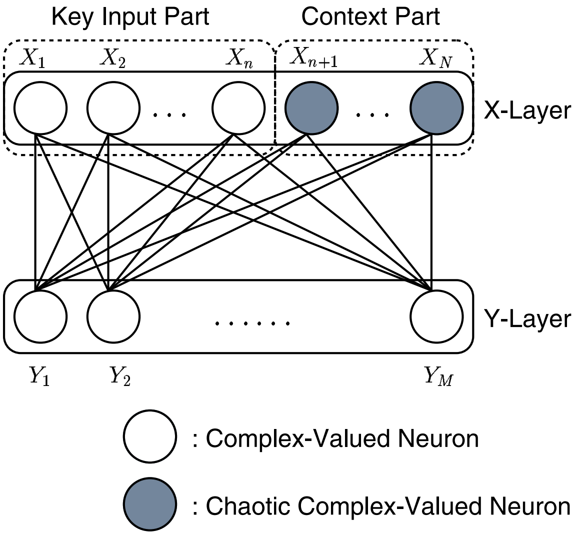


Figure 1: Structure of Proposed CCBAM.

given by

$$f(u) = \frac{\eta u}{\eta - 1.0 + |u|} \qquad (\eta \in \mathrm{R}) \tag{2}$$

where η is the constant ($\eta > 1$).

If the external input $A(t)$ is constant ($A(t) = A$), Eq.(1) can be described as

$$\begin{aligned} x(t+1) &= f(u(t+1)) \\ &= f(ku(t) - \alpha f(u(t)) + (A - \theta)(1-k)) \\ &= f(ku(t) - \alpha f(u(t)) + a) \end{aligned} \tag{3}$$

where $u(t)$ is the internal state of the neuron at the time t, $a(= (A - \theta)(1 - k))$ is the bifurcation parameter.

3. Chaotic Complex-Valued Bidirectional Associative Memory

Here, we explain the proposed Chaotic Complex-valued Bidirectional Associative Memory (CCBAM).

3.1. Structure

As shown in Fig.1, the proposed CCBAM has two layers (the X-Layer and the Y-Layer) as similar as the conventional Bidirectional Associative Memory[2]. The X-Layer has two parts; (1) Key Input Part composed of complex-valued neurons and (2) Context Part composed of chaotic complex-valued neurons.

3.2. Learning Process

Generally, the associative memory which is trained by a correlation matrix can not deal with one-to-many associations because the stored common data cause superimposed patterns. In the conventional Chaotic Bidirectional Associative Memory (CBAM)[6], each training pair is memorized together with its own contextual information in order to memorize the training set including one-to-many relations. In the proposed model, we use the same method to memorize the training set including one-to-many relations.

When the training set including one-to-many relations which is given by

$$\{(\boldsymbol{X}_1, \boldsymbol{Y}_1), (\boldsymbol{X}_1, \boldsymbol{Y}_2), (\boldsymbol{X}_2, \boldsymbol{Y}_3)\} \tag{4}$$

is memorized in the proposed model, each training pair is memorized together with its own contextual information. Namely, as for the training set shown in Eq.(4), it is modified as follows:

$$\{(\boldsymbol{X}_1\_\boldsymbol{C}_1, \boldsymbol{Y}_1), (\boldsymbol{X}_1\_\boldsymbol{C}_2, \boldsymbol{Y}_2), (\boldsymbol{X}_2\_\boldsymbol{C}_3, \boldsymbol{Y}_3)\} \tag{5}$$

In the proposed model, the patterns with its own contextual information are memorized by the orthogonal learning. The connection weights from the X-Layer to the Y-Layer $\boldsymbol{w}^{YX}$ and the connection weights from the Y-Layer to the X-Layer $\boldsymbol{w}^{XY}$ is given by

$$\boldsymbol{w}^{YX} = \boldsymbol{Y}(\boldsymbol{X}^*\boldsymbol{X})^{-1}\boldsymbol{X}^* \tag{6}$$

$$\boldsymbol{w}^{XY} = \boldsymbol{X}(\boldsymbol{Y}^*\boldsymbol{Y})^{-1}\boldsymbol{Y}^* \tag{7}$$

where * shows the conjugate transpose, and −1 shows the inverse. $\boldsymbol{X}$ and $\boldsymbol{Y}$ are the learning pattern matrixes, and are given by

$$\boldsymbol{X} = \{\boldsymbol{X}^{(1)}, \cdots, \boldsymbol{X}^{(p)}, \cdots, \boldsymbol{X}^{(P)}\} \tag{8}$$

$$\boldsymbol{Y} = \{\boldsymbol{Y}^{(1)}, \cdots, \boldsymbol{Y}^{(p)}, \cdots, \boldsymbol{Y}^{(P)}\} \tag{9}$$

where $\boldsymbol{X}^{(p)}$ is the pth pattern in the X-Layer, $\boldsymbol{Y}^{(p)}$ is the pth pattern in the Y-Layer, and P is the number of training pairs.

3.3. Recall Process

Since we assume that contextual information is usually unknown for users, in the recall process of the proposed model, only the Key Input Part receives input. For example, in the training pairs of Eq.(5), $\boldsymbol{X}_1$ is used as an input to the proposed CCBAM. In the proposed model, when $\boldsymbol{X}_1$ is given to the network as an initial input, since the chaotic complex-valued neurons in the Contextual Information Part change their states by chaos, we can expect that they can realize one-to-many associations as follows:

$$\begin{aligned} (\boldsymbol{X}_1\_\boldsymbol{0}, ?) \rightarrow \cdots \rightarrow (\boldsymbol{X}_1\_\boldsymbol{C}_1, \boldsymbol{Y}_1) \rightarrow \cdots \\ \rightarrow (\boldsymbol{X}_1\_\boldsymbol{C}_2, \boldsymbol{Y}_2) \rightarrow \cdots \end{aligned} \tag{10}$$

Since the proposed CCBAM is based on the BAM, the recall procedure of the CCBAM is almost the same as the BAM's.

Step 1 : Input to X-Layer

The input pattern is given to the X-Layer.

Step 2 : Calculation of Output of Neurons in Y-Layer

The output of the neuron k in the Y-Layer $x_k^Y(t)$ is given by

$$x_k^Y(t) = f\left(\sum_{j=1}^{N} w_{kj}^{YX} x_j^X(t)\right) \quad (11)$$

where N is the number of neurons of the X-Layer, w_{kj}^{YX} is the connection weight from the neuron j in the X-Layer to the neuron k in the Y-Layer, $x_j^X(t)$ is the output of the neuron j in the X-Layer at the time t, and $f(\cdot)$ is the output function given by Eq.(2).

Step 3 : Calculation of Output of Neurons in X-Layer

The output of the neuron j of the Key Input Part in the X-Layer at the time $t+1$ $x_j^X(t+1)$ is given by

$$x_j^X(t+1) = f\left(\sum_{k=1}^{M} w_{jk}^{XY} x_k^Y(t) + vA_j\right) \quad (12)$$

where M is the number of neurons of the Y-Layer, w_{jk}^{XY} is the connection weight from the neuron k in the Y-Layer to the neuron j in the X-Layer, v is the connection weight from the external input, A_j is the external input (See **3.4**) to the neuron j in the X-Layer.

The output of the neuron j of the Contextual Information Part in the X-Layer $x_j^X(t+1)$ is given by

$$x_j^X(t+1) = f\Bigg(\sum_{k=1}^{M} w_{jk}^{XY} \sum_{d=0}^{t} k_m^d x_k^Y(t-d) - \alpha \sum_{d=0}^{t} k_r^d x_j^X(t-d)\Bigg) \quad (13)$$

where k_m and k_r are damping factors, α is the scaling factor of the refractoriness.

Step 4 : Repeat

Steps 2 and **3** are repeated.

3.4. External Input

In the proposed model, the external input A_j is always given so that the key pattern does not change into other patterns.

If the initial input includes noise, we can use the initial input pattern $x_j^X(0)$ as the external pattern. However, the initial input pattern sometimes includes noise. So we use the following pattern $\hat{x}_j^X(t_{in})$ when the network becomes stable t_{in} as an external input.

$$t_{in} = \min\left\{ t \;\middle|\; \sum_{j=1}^{N_k} (\hat{x}_j^X(t) - \hat{x}_j^X(t-1)) = 0 \right\} \quad (14)$$

where N_k is the number of neurons in the Key Input Part. $\hat{x}_j^X(t)$ is the quantized output of the neuron j in the X-Layer at the time t, and is given by

$$\hat{x}_j^X(t) = \arg\min(\omega^s - x_j^X)^*(\omega^s - x_j^X) \quad (s = 1, 2, ..., S-1) \quad (15)$$

where S is the number of states and ω is given by

$$\omega = \exp(i2\pi/S) \quad (16)$$

where i is the imaginary unit.

4. Computer Experiment Results

Here, we show the computer experiment results to demonstrate the effectiveness of the proposed model.

4.1. Storage Capacity

Figure 2 shows the storage capacity of the proposed model which has 100/200 neurons in the Key Input Part of the X-Layer and the Y-Layer. As shown in this figure, the storage capacity of the proposed model depends on the number of neurons in the Key Input Part of the X-Layer and the Y-Layer, and the number of neurons in the Context Part in the X-Layer.

4.2. One-to-Many Association Ability

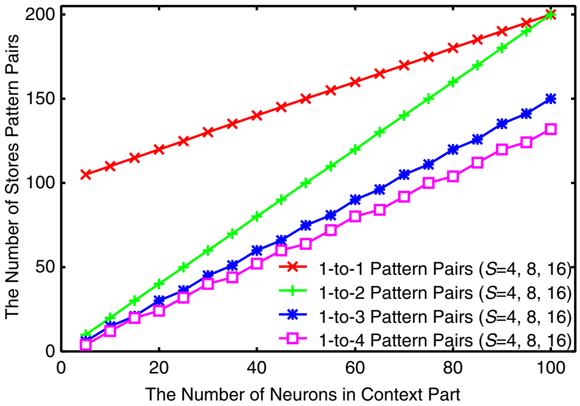
Figure 3 shows the relation between N (in 1-to-N pattern pairs) and the one-to-many association ability of the proposed model which has 100 neurons in the Key Input Part of the X-Layer and the Y-Layer and 50 neurons in the Context Part of the X-Layer. Figure 4 shows the relation between the number of states S and the one-to-many association ability of the proposed model. As shown in these figure, the proposed model can recall all patterns corresponding to the common term if the N or S is small.

4.3. Noise Reduction Effect

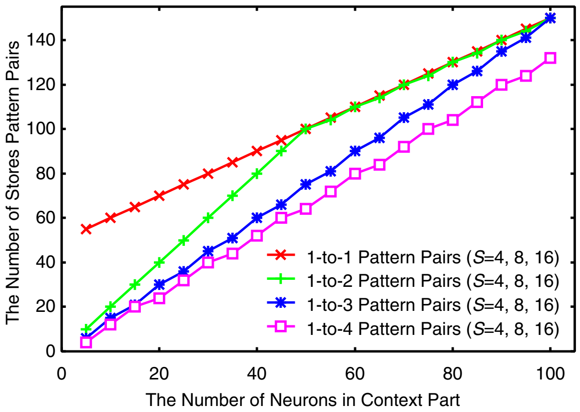
Figure 5 shows the noise reduction effect of the proposed model which has 100 neurons in the Key Input Part of the X-Layer and the Y-Layer and 50 neurons in the Context Part of the X-Layer.. As shown this figure, the proposed model can recall correct patterns even when the noisy pattern is given.

5. Conclusion

In this paper, we have proposed the Chaotic Complex-valued Bidirectional Associative Memory (CCBAM) which can realize one-to-many associations of multi-valued patterns. The proposed model is based on the Bidirectional Associative Memory, and is composed of complex-valued neurons and chaotic complex-valued neurons. In the proposed model, associations of multi-valued patterns are realized by using complex-valued neurons, and one-to-many associations are realized by using chaotic complex-valued neurons. We carried out a series of computer experiments and confirmed that the proposed model can realize one-to-many associations of multi-valued patterns.



(a) 100 neurons in the Key Input Part in X-Layer and Y-Layer



(b) 200 neurons in the Key Input Part in X-Layer and Y-Layer

Figure 2: Storage Capacity.

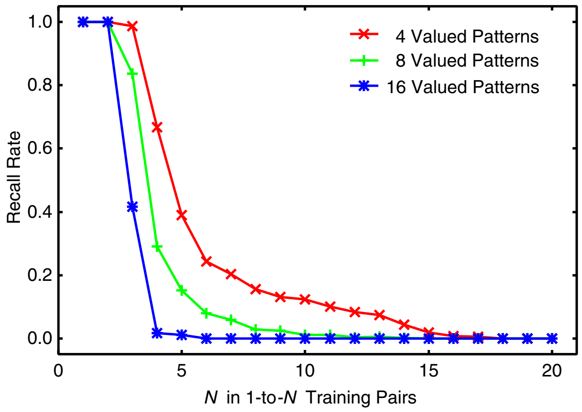


Figure 3: Relation between N and One-to-Many Association Ability.

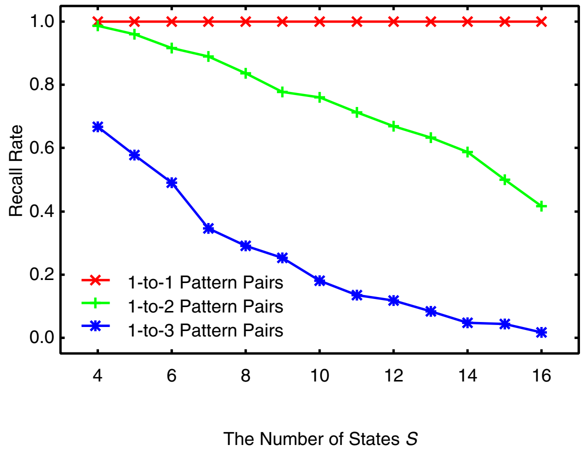


Figure 4: Relation between the Number of States and One-to-Many Association Ability.

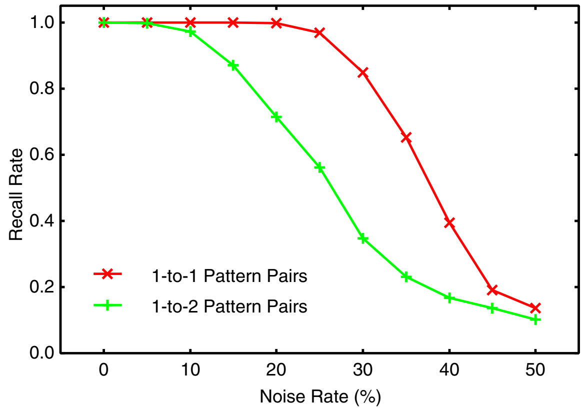


Figure 5: Noise Reduction Effect.

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