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Controlling Unstable Orbits via Varying Switching Time in a Simple Hybrid Dynamical Systems

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Abstract—In this paper, we discuss a controlling method of the switching time in the state- and period-dependent interrupted electric circuit. There are many state- and period-dependent interrupted electric circuits in the electrical engineering field. Therefore, the controlling method reported in this paper is able to apply the practical circuits such as the dc/dc converters and dc/ac inverters. First, we explain the algorithm. Next, we show an example of the application by demonstrating the proposed algorithm to a simple state- and period-dependent interrupted electric circuit with one dimensional topology. Finally we discuss validity of the method.

1. Introduction

It is known that the power conversion circuits exhibit rich nonlinear phenomena including the chaotic attractor [1, 2]. In many cases, the power conversion circuits are designed to prevent the unstable behavior, i.e. unstable orbits and chaotic attractors. Therefore, it is important to control the unstable orbits including the chaotic attractors from practical point of view.

There are many chaos controlling method applicable to the switching circuits. For example, Ref. [3] reports a duty ratio controlling method. Moreover, Refs. [4, 5] vary circuit parameters, such as the input voltage and reference values, for controlling the chaotic attractor. Using these controlling method, we can control the chaotic attractor to the periodic orbit.

On the other hand, a controlling method that focuses on the switching time may not be reported. The controlling method that focuses on the switching time has advantage in simplicity compared with the previous methods reported in Refs. [4, 5]. In addition, the simplicity will make possible to controlling the unstable orbits in the switching circuits with high-dimensional topology. Here, note that the duty ratio controlling method proposed in Ref. [3] has a simple algorithm, however it may not be able to apply the switching circuits with high-dimensional topology. In fact, there are many switching circuits that with two or more dimensional topology. For targeting these switching circuits, we have to propose a simple method for controlling unstable orbits.

This paper reports a controlling method that varying the switching time. The application target is switching circuits with one dimensional topology. First, we explain the algorithm of the controlling method. Next, we show the test circuit, which we apply the controlling method. Finally, we control the unstable orbits via varying the switching time in the test circuit. The test circuit has same switching rule with the current-controlled dc/dc converters. Therefore, it will be possible to control the unstable orbits in the current-controlled dc/dc converters by improving the proposed method.

2. Algorithm of the controlling method via varying the switching time

2.1. Hybrid dynamical system with one dimensional topology

We consider the following hybrid dynamical system with one dimensional topology.

$$\begin{aligned}\frac{dx}{dt} &= f(x,\lambda)\\ &= \begin{cases} f_1(t,x,\lambda,\lambda_1), & \text{subsystem-1}\\ f_2(t,x,\lambda,\lambda_2), & \text{subsystem-2}\end{cases}\end{aligned} \tag{1}$$

Let λ be a parameter included in both subsystem-1 and subsystem-2, whereas λ_1 and λ_2 are the parameters included only in subsystem-1 or subsystem-2. Moreover, we assume that the system has clock pulse at every period of T. Let x_n be a sampled data of the waveform at a time $t = nT$, where $n = 1, 2, 3 \ldots$. Therefore, the solution of Eq. (1) is described as follows:

$$x(t) = \begin{cases} \varphi_1(t, x_n, \lambda, \lambda_1), & \text{subsystem-1} \\ \varphi_2(t, x_n, \lambda, \lambda_2), & \text{subsystem-2} \end{cases} . \quad (2)$$

Figure 1 shows waveform behavior. If the waveform reaches the reference value x_{ref}, the subsystem switches from subsystem-1 to subsystem-2. If the clock pulse appears at every period of T, the subsystem returns to subsystem-1.

The periodic orbit satisfies the following equations:

$$x(nT) - x((n+m)T) = 0 \quad (3)$$

and

$$x(nT) - x((n+l)T) \neq 0, \quad (4)$$

where $1 \leq l < m$. Note that $x^{*m}(t)$ denotes period-m orbit that satisfies Eqs. (3) and (4).

We discuss stability of the period-one orbit as an example. Figure 2 shows the period-one orbit, i.e. $x^{*1}(t)$, and its perturbed orbit, i.e. $x(t)$, where x_n and x_n^{*1} denote an initial values at a time $t = nT$. Let a perturbation of the orbits at a time $t = nT$ be Δx_n. The perturbation Δx_n is expressed as follows:

$$\Delta x_n = x_n - x_n^{*1}. \quad (5)$$

Likewise, let the perturbation at a time $t = (n+1)T$ be Δx_{n+1}. The perturbation Δx_{n+1} is expressed as follows:

$$\Delta x_{n+1} = x_{n+1} - x_{n+1}^{*1}. \quad (6)$$

Therefore, it can be said that the orbit expressed as $x(t)$ is stable under $\frac{\Delta x_{n+1}}{\Delta x_n} < 1$, otherwise the orbit is unstable. In the following analysis, we control the unstable orbits via varying the switching time of the system.

2.2. Controlling method

Figure 3 shows a conceptual diagram of the controlling method. In the figure, the black waveform denotes $x^{*1}(t)$ and gray waveform denotes $x(t)$, respectively. Note that subsystem-1 is kept during t_{ON}, whereas subsystem-2 is kept during t_{OFF}. We vary the switching time of t_{ON} based on the controlling theory and getting a solution x'_{n+1} at a time $t = (n+1)T$.

The perturbation between x'_{n+1} and x_{n+1} is described as follows:

$$\Delta \hat{x}_{n+1} = x_{n+1} - x'_{n+1}. \quad (7)$$

Likewise, the perturbation between x'_{n+1} and x_{n+1}^{*1} is described as follows:

$$\Delta \bar{x}_{n+1} = x'_{n+1} - x_{n+1}^{*1}. \quad (8)$$

Let the controlling gain be k. Therefore, perturbation of the switching time, i.e. Δt_{ON}, can be derived as follows:

$$\Delta t_{\mathrm{ON}} = k \Delta x_n. \quad (9)$$

Here, Eq. (8) is rewritten as follows:

$$\Delta \bar{x}_{n+1} = \Delta x_{n+1} + \Delta \hat{x}_{n+1}. \quad (10)$$

Here, we assume that $\frac{\Delta \bar{x}_{n+1}}{\Delta x_n} = \mu$, where μ denotes the characteristic multiplier of Eq. (10). Note that we use the deadbeat control. Therefore, the control is completed when $\mu = 0$ is satisfied. We can get the controlling gain k based on Eq. (10). By substituting the controlling gain to Eq. (9), the perturbation of the switching time, i.e. Δt_{ON}, is defined. In this paper, we consider a simple hybrid dynamical system with one dimensional topology. Therefore, the

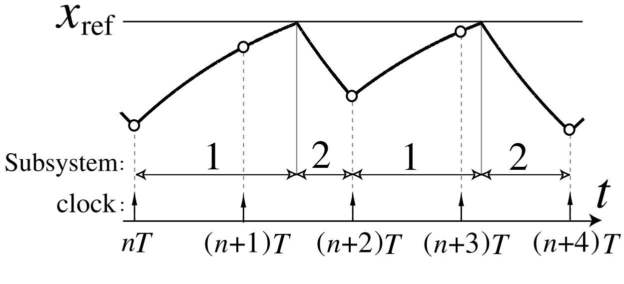
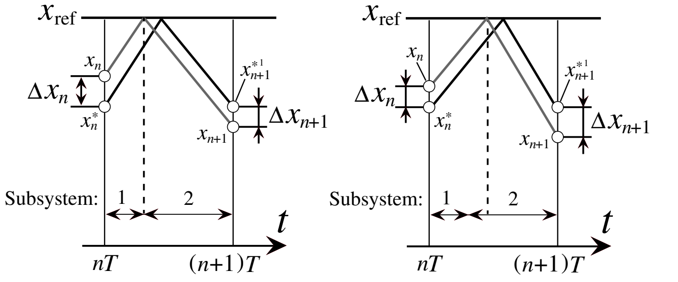


Figure 1: Waveform behavior.



(a) $\frac{\Delta x_{n+1}}{\Delta x_n} < 1$ (b) $\frac{\Delta x_{n+1}}{\Delta x_n} > 1$

Figure 2: Stability of the solution.

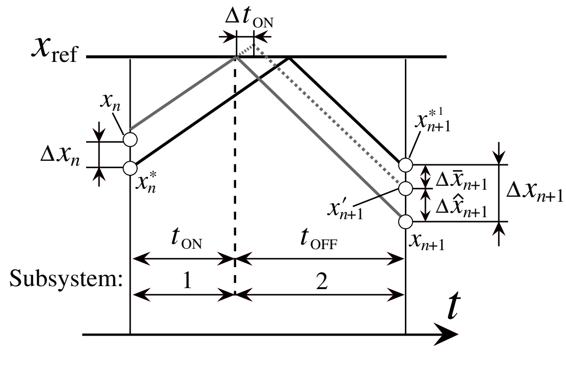


Figure 3: Controlling method of the switching time.

characteristic multiplier of Eq. (10), i.e. μ, denotes the slope of the return map. If the return map is defined with exact solution, we can get μ easily, otherwise we have to calculate the value of μ numerically based on the following equations:

$$\begin{aligned}\mu &= \frac{\Delta x_{n+1}}{\Delta x_n} \\ &= \frac{\partial \varphi_1(T, x_n^*, \lambda, \lambda_1)}{\partial x_n^*}.\end{aligned} \quad (11)$$

or

$$\begin{aligned}\mu &= \frac{\Delta x_{n+1}}{\Delta x_n} \\ &= \frac{\partial \varphi_2(T - t_{ON}, x^*(t_{ON}), \lambda, \lambda_2)}{\partial x^*(t_{ON})} \boldsymbol{S} \frac{\partial \varphi_1(t_{ON}, x_n^*, \lambda, \lambda_1)}{\partial x_n^*},\end{aligned} \quad (12)$$

where $\boldsymbol{S}$ denotes the saltation matrix. In this study, the system has one-dimensional topology. Therefore, the saltation matrix is to be a scalar value. The detail expression for getting the slope of the return map, i.e. stability calculation method applicable to the non-linear hybrid dynamical system, was reported in Ref. [7].

3. An example of the application

Figure 4 shows a simple state- and period-dependent interrupted electric circuit. This circuit model has been proposed in Ref. [6]. The position of the switch changes depending on the capacitor voltage and appearance of the clock pulse. If the capacitor voltage reaches the reference voltage, the position of the switch changes from A's side to B's side. On the other hand, if the clock pulse appears at every period of T, the position of the switch changes from B's side to A's side.

We fix the circuit parameters as follows:

$$R = 10[\mathrm{k}\Omega],\ C = 0.33[\mu\mathrm{F}],\ E = 3.0[\mathrm{V}],\ T = 2.0[\mathrm{ms}]. \quad (13)$$

The circuit equation is described as follows:

$$RC\frac{dv}{dt} = \begin{cases} -v + E, & \text{for switch A} \\ -v, & \text{for switch B} \end{cases}. \quad (14)$$

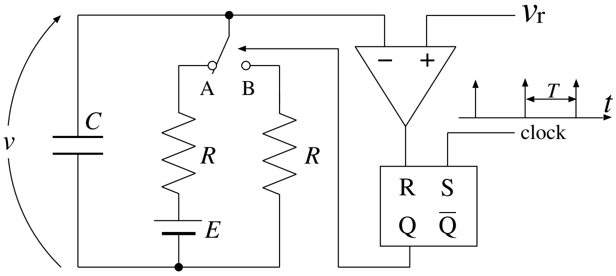


Figure 4: State- and period-dependent interrupted electric circuit.

The solution of Eq. (14) can be derived with exact solution as follows:

$$v(t) = \begin{cases} (v_k - E)e^{-\frac{1}{RC}(t-kT)} + E, & \text{for switch A} \\ v_k e^{-\frac{1}{RC}(t-kT)}, & \text{for switch B} \end{cases}, \quad (15)$$

where v_k is an initial capacitor voltage at a time $t = kT$. We use the following dimensionless values:

$$x = v,\ x_{\mathrm{ref}} = v_{\mathrm{ref}},\ B = E,\ t' = \frac{t}{RC},\ T' = \frac{T}{RC}, \quad (16)$$

where we assume $B = 1$ and we rewrite t' and T' as t and T, for the sake of the simplicity.

Figure 5 shows examples of the waveform behavior. By changing the reference voltage x_r, we observe many kinds of the periodic and non-periodic orbits. Based on Eqs. (3) and (4), we can understand that the period-two orbit is observed at $x_r = 0.68$, the period-four orbit is observed at $x_r = 0.78$, the period-three orbit is observed at $x_r = 0.84$ and the non-periodic orbit is observed at $x_r = 0.92$. It can be said that all of these periodic and non-periodic orbits are the unstable period-one orbit because these orbits never be the period-one orbit without controlling the switching time. Therefore, we apply the controlling method to these periodic and non-periodic orbits shown in Fig. 5. Figure 6 shows the application results. We start the controlling around at $t = 10$. It is clear that the period-two, period-four, period-three and non-periodic orbits are controlled to the unstable period-one orbit. The test circuit shown in Fig. 4 has the same switching rule with the current-controlled dc/dc converters. Therefore, the algorithm for controlling unstable orbits may be applicable to the current-controlled dc/dc converters.

4. Conclusion

We reported a controlling method of the unstable orbits via varying the switching time in a simple hybrid dynamical systems. First, we defined a hybrid dynamical system with one dimensional topology. Next, we explained the controlling method with focus on the switching time. Finally, we applied the proposed controlling method to a simple state- and period-dependent interrupted electric circuit. We confirmed validity of the controlling method.

In future, we improve the proposed method for controlling unstable orbits in two or more dimensional hybrid dynamical systems. Moreover, we have to improve the controlling method for applying the hybrid dynamical system with nonlinear characteristics.

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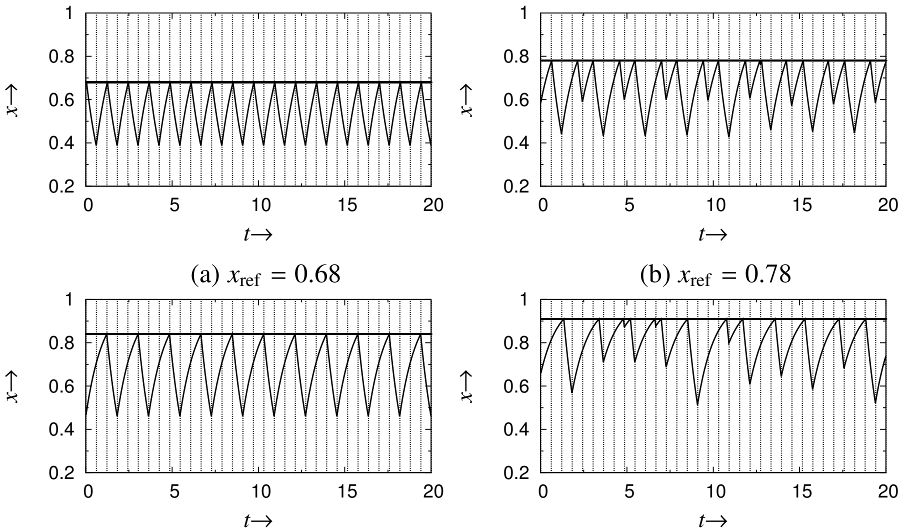
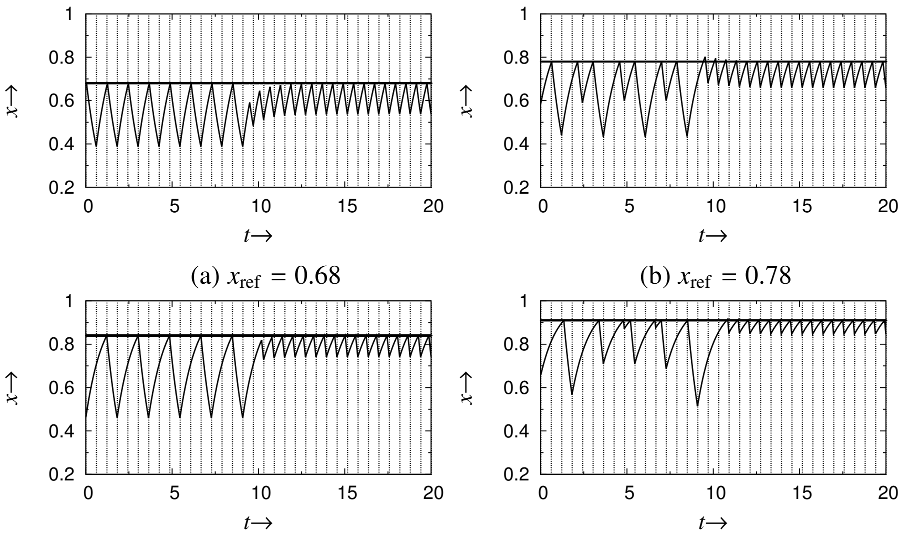


(a) $x_{\text{ref}} = 0.68$ (b) $x_{\text{ref}} = 0.78$

(c) $x_{\text{ref}} = 0.84$ (d) $x_{\text{ref}} = 0.92$

Figure 5: Capacitor voltage without controlling.



(a) $x_{\text{ref}} = 0.68$ (b) $x_{\text{ref}} = 0.78$

(c) $x_{\text{ref}} = 0.84$ (d) $x_{\text{ref}} = 0.92$

Figure 6: Capacitor voltage with controlling.

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