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Analyzing and Composing Music from Motifs

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Abstract—In this paper we apply the concept of musical motifs and network motifs to artificially compose music. We construct two different types of complex networks for music, namely, tonal network and rhythmic network, and identify tonal motifs and rhythmic motifs, respectively, from these networks. Then, from these two kinds of motifs, we define a set of composite motifs which represent basic groups of chronologically ordered notes. For example, t tonal motifs and r rhythmic motifs give a set of m composite motifs, where $m \geq \max(t, r)$. By combining these m motifs, a piece of music can be produced. We analyze a popular Japanese folk song, and identify its motifs. Music reconstructed from combining motifs has been found to "sound like" the original composition from which the motifs are constructed.

Keywords—Network, motif, music, composition.

1. Introduction

Network motifs are patterns of interconnecting nodes and links frequently found in a given complex network [1]. Most complex networks, such as social networks, gene regulation network and others, have been found to display characteristic motifs. Network motifs can thus be regarded as signature constituents of a network. Network motifs may appear in different forms for different kinds of networks. Network motifs of economic systems are different from those of biological ones. In Fig. 1, we show two simple network motifs. In music, we may conveniently define network motifs according to the way in which notes are connected chronologically as in a usual co-occurrence network.

In fact, the word "motif" (motive) appears equally in music and is the foundation of musical composition. It was created and first used by the German music critic Hans Paul von Wolzogen (1848-1938) [2]. A motif is connected with a particular character, place, or idea in a piece of music and may be heard whenever that character reoccurs, representing an important part of the plot. One famous motif in classical music is in Beethoven's Symphony No. 5. The four-note motif, as shown in the score of Fig. 2, forms a distinctive "short-short-short-long" pattern, and has been described by music critics as a representation of "fate knocking at the door" [3].

Motifs will appear repeatedly in a composition, either in their original forms or their variations. Almost every piece of musical work is composed of original motifs and their variations [4]. In this paper, we examine the connections between network motifs and music motifs, the purpose being to identify the structure of music and to propose a reconstruction method (artificial composition). We show that in terms of morphological characteristics, music motifs relate to network motifs when musical notes are connected as a co-occurrence network [5]. Furthermore, we also show that motifs are useful in the analysis of music in terms of complex network properties [6] as well as in computerized composition of music.

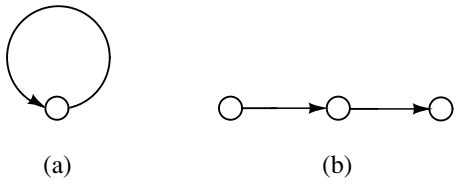


Figure 1: Examples of network motifs. (a) Self-regulating motif; (b) feed-forward motif.

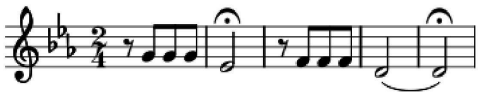
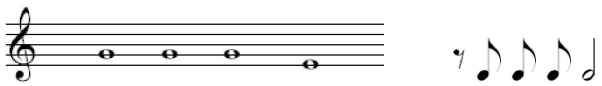


Figure 2: The two opening 4-note motifs in Beethoven's Symphony no. 5.

The rest of the paper is organized as follows. In Section 2, we define network motifs in music co-occurrence networks, and apply the concepts of network motifs in characterization of the structure of a given piece of music. Motifs for a selected Japanese folk song "Sato-no-aki" (里の秋) [7, 8] will be given. In Section 3 we propose a method for composing music using network motifs. Some computer generated music will be provided online and played in the presentation. Section 4 concludes the paper.

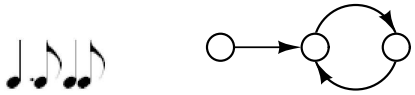
2. Analysis Based on Motifs

A music motif embeds two basic elements, namely, tone and rhythm. Specific motifs are defined by arranging tone and rhythm. With a given set of motifs, one may generate more motifs by re-arranging the tone and rhythm of the original motifs. Such process is called motif development. Typically, motif development would attempt to retain some



(a) (b)

Figure 3: (a) Tonal motif and (b) rhythmic motif of Beethoven's Symphony no. 5.



(a) (b)

Figure 4: Example of (a) rhythmic motif and (b) corresponding flow graph (network motif like notation).

characteristics of the original set of motifs. Two common approaches of motif developments are: (i) retaining the tonal sequence and changing the rhythmic sequence; (ii) retaining the rhythmic sequence and changing the tonal sequence [4]. Music with rich motif developments sounds dulcet and that without usually sounds dull and unappealing.

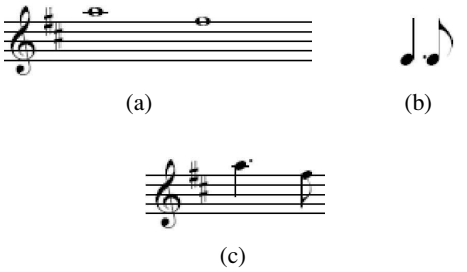
2.1. Tonal and Rhythmic Motifs

Based on the foregoing discussion, we define two types of motifs for music. First, we construct the co-occurence network for a given piece of music, using the procedure described in Tse *et al*. [5]. From the network we may identify short sequences of notes that repeat frequently throughout the piece. These sequences basically form the network motifs, and we decompose each sequence into its tonal sequence and rhythmic sequence, which define the *tonal motif* and the *rhythmic motif* for the given piece of music.

For example, for the opening 4-tone motif of Beethoven's Symphony no. 5 as shown earlier in Fig. 2, the tonal motif and rhythmic motif can be defined as shown in Fig. 3.

2.2. Example

We consider the popular Japanese folk song "Sato-no-aki" (里の秋) [7, 8], and construct its tonal network and rhythmic network. Basically the nodes in the tonal network are the tones used in the song, and the nodes in the rhythmic network are the time durations used. The tonal network is constructed by connecting the nodes (tones) that occur in a row, i.e., adopting the co-occurence network construction. Likewise, the rhythmic network can be constructed. From the co-occurence networks, we can readily find interconnections of two nodes, three nodes, four nodes and five nodes that repeatedly occur in the song. Thus, we can define tonal motifs and rhythmic motifs, respectively, from the tonal network and the rhythmic network. For instance, the rhythmic motif of four nodes and its corresponding network motif are shown in Figs. 4 (a) and (b), respectively. The complete list of motifs for "Sato-no-aki" is given in Tables 1 and 2.



(a) (b) (c)

Figure 5: Combining (a) tonal motif T_1 and (b) rhythmic motif R_1, giving (c) new motif.

3. Composition Using Motifs

In the foregoing section, we have illustrated the derivation of tonal and rhythmic motifs for any given piece of music. Our purpose here is to re-compose music based on the motifs obtained. The first step is to generate a full set of music motifs which contain both tonal and rhythmic information.

Suppose we have identified t tonal motifs and r rhythmic motifs from a given song. Denote the tonal motifs as T_1, T_2, ..., T_t, and the rhythmic motifs as R_1, R_2, ..., R_r. Then, each tonal motif can team up with a rhythmic motif to form one musical motif. Thus, m composite musical motifs can be generated, where $m \geq \max(t, r)$. For instance, T_1 and R_1 gives a new motif, as shown in Fig. 5. Moreover, the tonal and rhythmic motifs to be combined must have the same number of notes. Thus, the total number of motifs generated cannot be larger than $t \times r$, i.e.,

$$\max(t, r) \leq m \leq t \times r$$

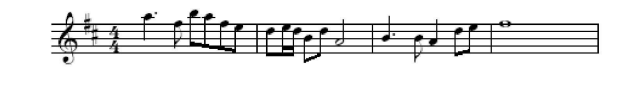
The composition of music can be proceeded by connecting the motifs using a biased random walk algorithm similar to the one proposed in Tse *et al*. [5]. Specifically, *the probability of a motif used is proportional to the relative frequency of the motif as it appears in the original composition*. Furthermore, correct rhythm has to be preserved in accordance with the original song by completing every bar with sufficient note duration. This further restricts the connection possibility. For instance, a duration equivalent to 4 crochets is needed for a 4/4 bar, etc.

3.1. Results and Demonstration

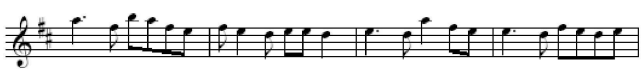
In the previous example of "Sato-no-aki" (里の秋) [7, 8], 18 motifs are identified, giving 13 tonal motifs and 11 rhythmic motifs. By re-combining the tonal and rhythmic motifs and taking into account the criterion of combining only those motif pairs having the same note number, 40

Table 1: Tonal motifs of "Sato-no-aki"

Name	Tonal motif	Network flow graph (motif)	Frequency
T_1			6
T_2			6
T_3			7
T_4			6
T_5			4
T_6			4
T_7			4
T_8			3
T_9			3
T_{10}			4
T_{11}			3



(a)



(b)

Figure 6: (a) Original score of "Sato-no-aki", (b) re-composed score using motifs.

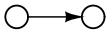
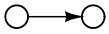
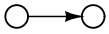
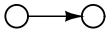
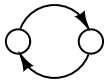
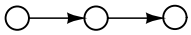
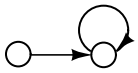
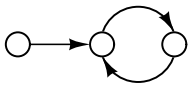
musical motifs can be generated. Among these, 16 are 2-tone motifs, 16 are 3-tone, 4 are 4-tone and 2 are 5-tone.

Applying the biased random walk algorithm mentioned above, motifs are combined to form bars which are linked to compose music. A part of the newly composed music is shown in Fig. 6 (b), which preserves the characteristic of the original song shown in Fig. 6 (a). Sound files can be downloaded from http://cktse.eie.polyu.edu.hk/MUSIC, and will be played in the presentation.

4. Conclusion

Motifs represent signatures of a piece of music. On hearing a motif, the audience knows what the music is, who composes it, and the subtle message being conveyed. In this work we have examined motifs in music and attempted to make a connection with network motifs. We have identified motifs from networks that are formed by connecting co-occurring notes in a given piece of music, defined tonal and rhythmic motifs for that music, generated new motifs that preserve the characteristic themes, and made an attempt to re-compose music from motifs. Future work will incorporate the network properties such as degree distribution to compose music, along with rudimentary constraints

Table 2: Rhythmic motifs of “Sato-no-aki”

Name	Rhythmic motif	Network flow graph (motif)	Frequency
R_1			7
R_2			25
R_3			7
R_4			6
R_5			6
R_6			12
R_7			4
R_8			6
R_9			7
R_{10}			4
R_{11}			6
R_{12}			5
R_{13}			4

that prevent irregularities and hence improve the “quality” of the recomposed music.

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