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Chaos in Chua's Oscillator with Chua's Diode and Memristor

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Abstract—This paper presents a chaotic oscillator obtained by modifying the canonical Chua's oscillator. Chua's diode and memristor are combined into a circuit and investigated.Simulation result shows a strange chaotic attractor confirmed by positive Lyapunov exponents.

1. Introduction

A two-terminal circuit element called – the memristor was first postulated by Leon O. Chua in September 1971 [1]. It is known as the forth basic circuit element after resistor (R), capacitor (C), and inductor (L). In April 2008, Stanley Williams and researchers in HP Information and Quantum Systems Laboratory announced the fabrication of a nano scale memristor [2]. From this milestone discovery, memristor has received sharply increasing attention in both research and industry.So far, many potential applications of memristor have been proposed, as in artificial biological systems, non-volatile RAM (NVRAM), application specific integrated circuits (ASICs) and field programmable gate arrays (FPGAs). For integrated circuit technology, a significant reduction in area with an unprecedented memory capacity and device density of memristors enables the maintaining of Moore's law. Many researchers around the world have been focusing on memristor applications in various areas of circuit design, alternative materials, spintronic memristors and memristor modeling.

With the nonlinear characteristic, memristor exhibits rich behaviors in dynamical system, especially in chaotic circuits. In this paper, we study the phenomena when adding memristor into the canonical Chua's oscillator. Simulation results and Lyapunov exponents calculation demonstrate that the modified Chua's circuit can generate chaos attractor.

1.1 Monotone-increasing piecewise-linear memristor

The memristor shown in Figure 1 is characterized by a non-linear constitutive relation between the voltage v and current i across the element as

$$v = M(q)i, \text{ or } i = W(\varphi)v, \qquad (1)$$

Where q , φ , $M(q)$ and $W(\varphi)$ are the charge, flux, memristance and memductance of the memristor, respectively. Two function $M(q)$ and $W(\varphi)$ are defined below:

$$M(q) = \frac{d\varphi(q)}{dq}, \qquad (2)$$

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi}, \qquad (3)$$

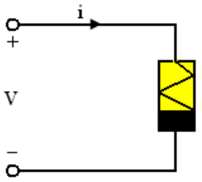


Figure 1. A two-terminal memristor

The charge-controlled memristor [3] has the "*monotone-increasing*" and "*piecewise-linear*" nonlinearity shown in Figure 2, with the relation between charge and flux demonstrated by the function $q(\varphi)$.

$$q(\varphi) = b\varphi + 0.5(a-b)(|\varphi+1| - |\varphi-1|), \qquad (4)$$

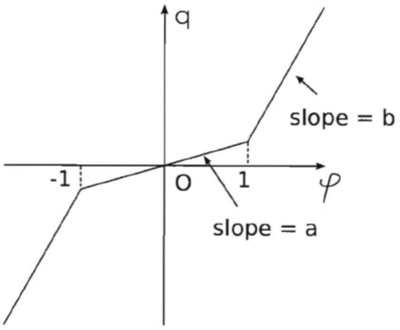


Figure 2. $q-\varphi$ characteristic of a monotone-increasing piecewise-linear memristor.

1.2. Canonical Chua's oscillator

The canonical Chua's oscillator depicted in Figure 3 consists of aninductor L, two capacitors C1, C2, a Chua's diode and a negative conductance –G.

The function $F(v)$ defined below represent the $i-v$ characteristic of the Chua's diode shown in Figure 4.

$$i = F(v) = G_b v + 0.5(G_a - G_b)(|v + B_p| - |v - B_p|) \qquad (5)$$

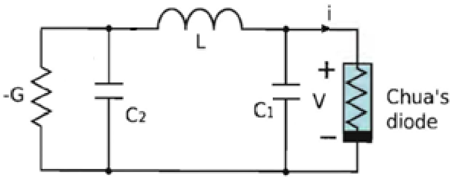


Figure 3. The canonical Chua's oscillator.

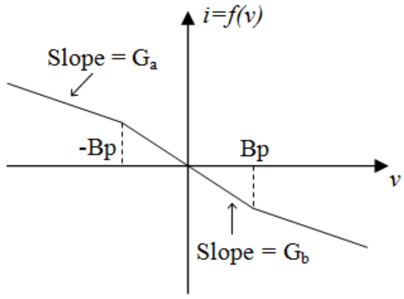


Figure 4. $i-v$ characteristic of Chua's diode.

2. Fourth-order chaotic oscillator

We now study the modified canonical Chua's oscillator shown in Figure 5.

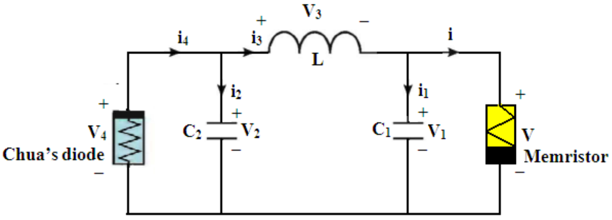


Figure 5. The modified canonical Chua's oscillator with memristor and Chua's diode

The Kirchoff equations of this circuit are here below:

$$\begin{cases} i_1 = i_3 - i, \\ v_3 = v_2 - v_1, \\ i_2 = -i_3 + i_4, \end{cases} \quad (6)$$

Since

$$i_1 = C_1 \frac{dv_1}{dt}, \ i = W(\varphi)v, \ v_3 = L\frac{di_3}{dt},$$

$$i_4 = -F(v_4), \ v_4 = v_2, \ v = v_1.$$

We have

$$\begin{cases} C_1 \dfrac{dv_1}{dt} = i_3 - W(\varphi)v_1, \\ L\dfrac{di_3}{dt} = v_2 - v_1, \\ C_2 \dfrac{dv_2}{dt} = -i_3 - F(v_2), \\ \dfrac{d\varphi}{dt} = v_1, \end{cases} \quad (7)$$

Let $x = v_1$, $y = i_3$, $z = v_2$, $w = \varphi$, $\alpha = 1/C_1$, $\beta = 1/C_2$, $\gamma = 1/L$.

Above equations become

$$\begin{cases} \dfrac{dx}{dt} = \alpha[y - W(w)x], \\ \dfrac{dy}{dt} = \gamma[z - x], \\ \dfrac{dz}{dt} = -\beta[y + F(z)], \\ \dfrac{dw}{dt} = x, \end{cases} \quad (8)$$

where

$$\begin{cases} q(w) = bw + 0.5(a-b)(|w+1| - |w-1|), \\ W(w) = \dfrac{dq(w)}{dw} = \begin{cases} a, & |w| < 1, \\ b, & |w| > 1, \end{cases} \end{cases} \quad (9)$$

and

$$F(z) = G_b z + 0.5(G_a - G_b)(|z + B_p| - |z - B_p|) \quad (10)$$

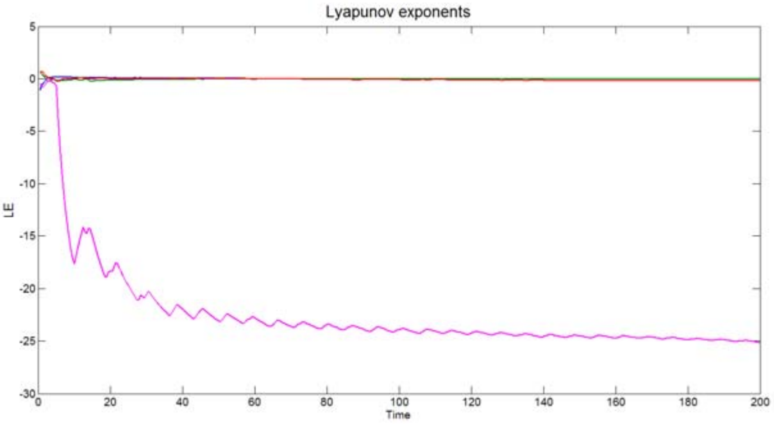
Simulation result for Eq.(8) with the parameter set as listed in TableI isdepicted in Figure 6, 7 and 8. Figure 6 illustrates the result of Lyapunov exponents using Lyapunov Exponents Tools [5] (LET). It is clear that two of four Lyapunov exponents are positive at around $\lambda_1 \approx 5.2\ 10^{-3}$ and $\lambda_2 \approx 2.2\ 10^{-3}$; therefore, the system exhibits chaotic behavior. Figure 7 and 8 depict the strange attractors of the system.

TABLE I. CIRCUIT PARAMETER SET FOR A CHAOTIC ATTRACTOR

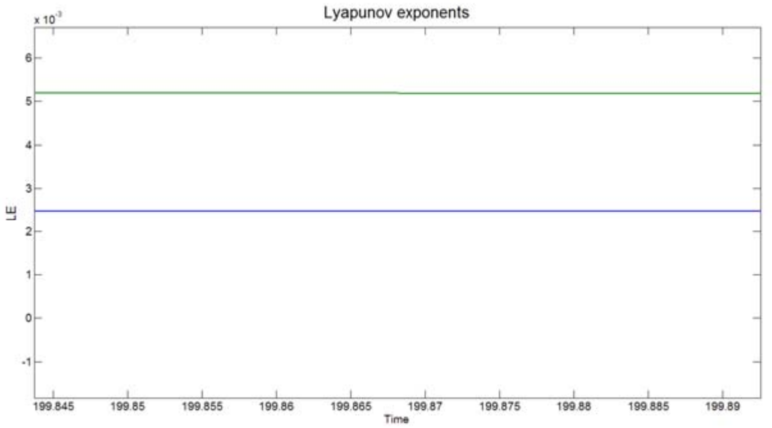
Parameter	a	b	G_a	G_b	B_p	α	β	γ
Value	0.2	9	–0.6	–0.4	0.5	4	1	1

3. Conclusion

There are some memristor-based chaotic circuits proposed in recent papers, these circuits obtained by replacing the Chua's diode in Chua's circuit by a memristor. This paper considers the case when the modified canonical Chua's oscillator contains both Chua's diode and memristor. With the results obtained, we conclude that this circuit can entrances the chaotic circuit family, extending the knowledge of memristor behaviors and chaotic circuits.

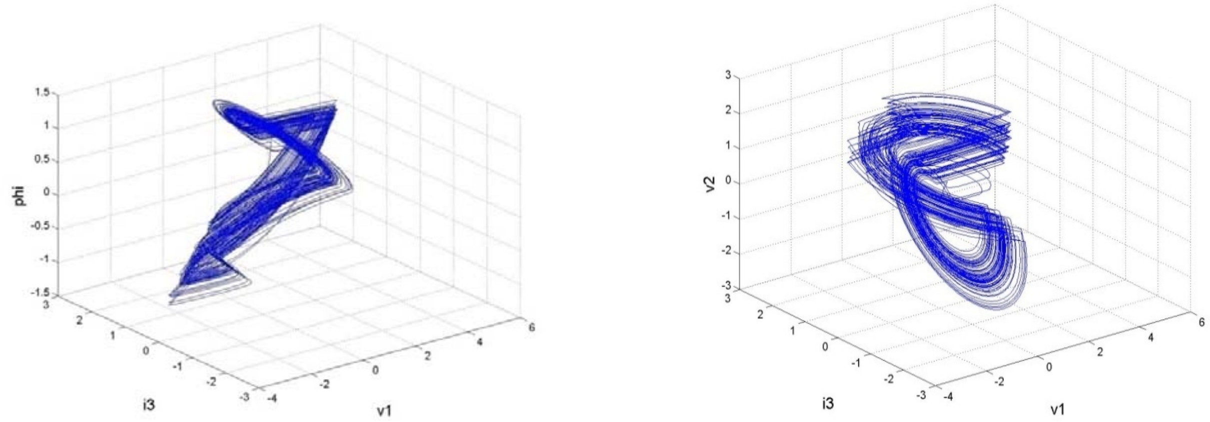


(a). Lyapunov Exponents of the system



(b). Two positive Lyapunov Exponents zoomed from (a)

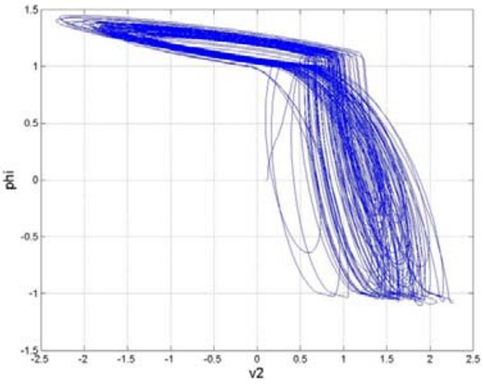
Figure 6. Lyapunov exponents calculation



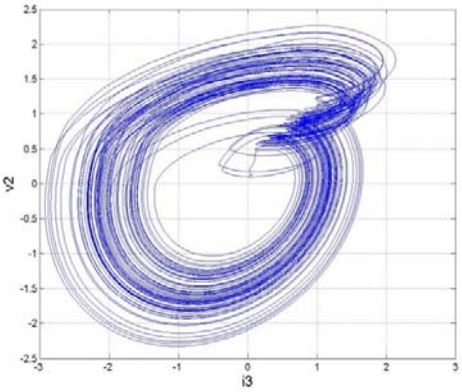
(a). 3D chaotic attractor, φ vs. i_3 vs. v_1

(b). 3D chaotic attractor, v_2 vs. i_3 vs. v_1

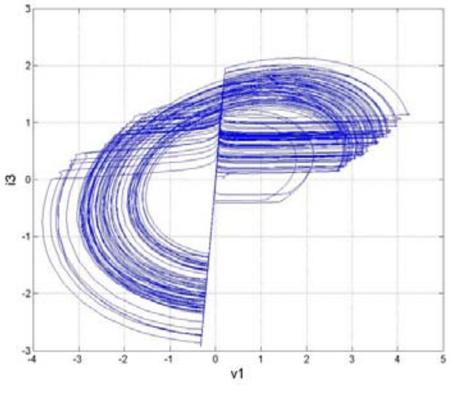
Figure 7. Trajectories in state space



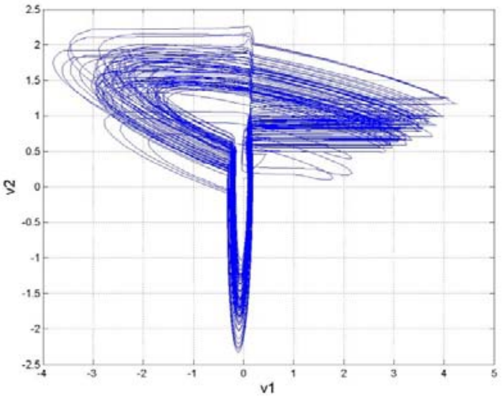
(a). 2D chaotic attractor, φ vs. v_2



(c). 2D chaotic attractor, v_2 vs. i_3



(b). 2D chaotic attractor, i_3 vs. v_1



(d). 2D chaotic attractor, v_2 vs. v_1

Figure 8. Chaotic attractors

Acknowledgments

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