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Chaos control method based on the Stability Transformation Method utilizing the Occasional Proportional Feedback control

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Abstract—In this paper, we consider a novel chaos control method based on the Stability Transformation Method for the continuous-time chaotic systems. Previously, we have proposed a stabilization method that can stabilize unknown unstable periodic orbits embedded in chaotic attractors in continuous-time dynamical systems detected via the approach of the Stability Transformation Method. However, the application range of this control method has been restricted since the method requires discontinuity of state variables. This paper presents a stabilization method of unknown unstable periodic orbits without state-jump dynamics by utilizing the Occasional Proportional Feedback technique. The stability of the control system is theoretically analyzed. Some results are verified by laboratory measurements.

1. Introduction

Chaos is an intrinsically complex phenomenon in deterministic nonlinear dynamical systems. It is often treated as a phenomenon to be suppressed in many engineering systems. In general, the procedure to stabilize unstable periodic orbits (UPOs) embedded in a chaotic attractor is called controlling chaos. Since the 1990's, the idea of stabilizing UPOs embedded in chaotic attractor through only small perturbations of system parameters introduced by Ott, Grebogi and Yorke[1] (OGY) has attracted great interest among physicists. Many researchers have applied OGY method to both discrete and continuous time chaotic systems, and have reported interesting results. However, this approach requires the knowledge of position of the desired periodic orbits, so that it is not able to stabilize the periodic orbits whose information of positions is unknown. In previous work[2], we have proposed a chaos control method based on Stability Transformation Method (STM) to stabilize unknown UPOs of a chaotic system. STM proposed by Pingel at al[3] can detect the location of UPOs by constructing a transformed system which has the stable periodic orbits with the same location of UPOs exhibited in the orginal chaotic system. The control method realized the transformed system as a real system. The control principle is to construct the transformed system which has the stable periodic orbits at the same location of UPOs of the target chaotic system. Then we used Instantaneous State Setting (ISS) method[4] in order to realize the transformed system for continuous-time dynamical system. However, the proposed method used instantaneously change of system state variables and makes them discontinuous. Therefore it may not be applicable for mechanical engineering systems. In other words, expanding scope of application of our proposed approach by making the state variables continuous has been expected. In this paper, we introduce a novel chaos control method based on STM utilizing Occasional Proportional Feedback (OPF) method to stabilize the unknown UPOs of chaotic systems whose state variables are continuous. We realize the controller to an autonomous continuous time system, 3-dimensional (3-D) hysteresis chaos generator, and present the stabilization of its unknown UPOs. We also confirm the effectiveness of the control system to the real system.

2. 3-dimensional hysteresis chaos generating system

2.1. 3-D hysteresis chaos generator

We consider an autonomous piecewise linear system which can generate chaos characteristic, 3-D hysteresis chaos generator, as our control target. The dynamics are described as follows:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & 2\delta\end{bmatrix}\left[\begin{bmatrix}x\\ y\end{bmatrix} - \begin{bmatrix}p\\ 0\end{bmatrix}h(x)\right], \tag{1}$$

where x and y are system state variables, " $\cdot$ " denotes the differentiation by normalized time τ, δ and p are 2 parameters of system which controlling a damping and equilibrium points, respectively. $h(x)$ is normalized hysteresis which switched from 1 to -1 if x reaches the threshold $x = -1$ and switched from -1 to 1 if x reaches the threshold $x = 1$. For simplicity, we focus on the following parameters range: $0 < \delta < 1, \quad p > 0$. In this range, Eq. (1) has the conjugate complex eigenvalue $\delta \pm j\omega$, where $\omega = \sqrt{1-\delta^2}$. We can confirm that the chaotic system exhibits chaos and bifurcation phenomena within this parameters range. [4] The piecewise solution of Eq. (1) is given by:

$$\begin{bmatrix}x(\tau)\mp p\\ y(\tau)\end{bmatrix} = \frac{e^{\delta\tau}}{\omega}\begin{bmatrix}\cos(\omega\tau+\phi) & \sin\omega\tau\\ -\sin\omega\tau & \cos(\omega\tau-\phi)\end{bmatrix} \times \begin{bmatrix}x(0)\mp p\\ y(0)\end{bmatrix} \tag{2}$$

where $\phi = \tan^{-1}(\delta/\omega)$.

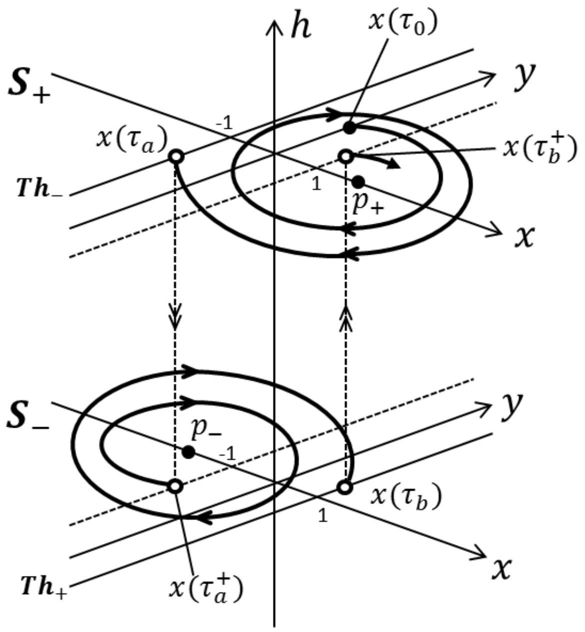

Figure 1: Behavior of 3-D Hysteresis Chaos Generator

To present the behavior of the system trajectory, we focus on the following 2 half-planes in a 3-dimensional phase space as shown in Fig. 1.

$$S_+ \equiv \{(x, y, h) | x \geq -1, h = 1\},$$
$$S_- \equiv \{(x, y, h) | x \leq 1, h = -1\}.$$

In order to derive a return map, we also define some objects shown in Fig. 1:
1) Equilibrium point on S_+: $\boldsymbol{p_+} = (p, 0, 1) \in S_+$,
2) Equilibrium point on S_-: $\boldsymbol{p_-} = (-p, 0, -1) \in S_-$,
3) Threshold on S_+: $Th_- = \{(x, y, h) | x = -1, h = 1\}$,
4) Threshold on S_-: $Th_+ = \{(x, y, h) | x = 1, h = -1\}$.
Note that the vector field on S_+ is symmetrical to that on S_-. We consider that the trajectory starts from $\boldsymbol{x}(\tau_0)$ on S_+ at $\tau = \tau_0$. It rotates around the equilibrium point $\boldsymbol{p_+}$ and expands as shown in Fig. 1. When the trajectory hits Th_- at τ_a, $h(x)$ is switched from 1 to -1 and the state jumps from $\boldsymbol{x}(\tau_a)$ on S_+ to the same coordinates of $\boldsymbol{x}(\tau_a)$ on S_-. The point on S_- is $\boldsymbol{x}(\tau_a^+)$. Then the trajectory starting from $\boldsymbol{x}(\tau_a^+)$ rotates divergently around $\boldsymbol{p_-}$ and must hit Th_+ at τ_b. At this moment, $h(x)$ is switched from -1 to 1 and the state jumps from $\boldsymbol{x}(\tau_b)$ on S_- to the same coordinates of $\boldsymbol{x}(\tau_b)$ on S_+. The point on S_- is $\boldsymbol{x}(\tau_b^+)$. The trajectory starting from $\boldsymbol{x}(\tau_b^+)$ rotates divergently around $\boldsymbol{p_+}$ similarly and it repeats such behavior. Here, the switching time τ_a^+ and τ_b^+ are defined by:

$$\tau_a^+ = \lim_{\Delta\tau \to 0} (\tau_a + \Delta\tau), \tau_b^+ = \lim_{\Delta\tau \to 0} (\tau_b + \Delta\tau). \qquad (3)$$

2.2. Return map

Since the trajectory starting from any point on $y = 0$ will definitely return to $y = 0$ within a finite time, we choose $J \equiv \{(x, y, h) | y = 0\}$ as the domain of our return map and define it as follows:

$$\begin{aligned} F : J \to J, \quad & \boldsymbol{R} \times \boldsymbol{B} \ \to \ \boldsymbol{R} \times \boldsymbol{B}, \\ \boldsymbol{\xi}_{n+1} &= (x_{n+1}, h_{n+1}) \\ &= F(x_n, h_n) = (f(x_n, h_n), g(x_n, h_n)). \end{aligned} \qquad (4)$$

where $x_n \in \boldsymbol{R}$ denotes coordinate of state $\boldsymbol{x}$ on $y = 0$ when the trajectory hits J at n-th time and $h_n \in \boldsymbol{B} \equiv \{-1, 1\}$ is the value of $h(x)$ at that moment. $f(x_n, h_n)$ and $g(x_n, h_n)$ in Eq. (4) are described by:

$$f(x_n, h_n) = \begin{cases} f_1(x_n) \text{ for } x_n > x_{Th}, \\ f_2(x_n) \text{ for } [(-1 \leq x_n \leq x_{Th}) \\ \qquad\qquad \text{and } (h_n = 1)], \\ f_3(x_n) \text{ for } [(-x_{Th} \leq x_n \leq 1) \\ \qquad\qquad \text{and } (h_n = -1)], \\ f_4(x_n) \text{ for } x_n < -x_{Th}, \end{cases} \qquad (5)$$

$$g(x_n, h_n) = \begin{cases} 1 \text{ for } \{(1 \leq x_n \leq x_{Th}) \\ \quad \text{or} [(-1 \leq x_n \leq 1) \cup (h_n = 1)]\}, \\ -1 \text{ otherwise}, \end{cases} \qquad (6)$$

where $x_{Th} = p + \frac{1}{\omega} \mathrm{e}^{-\delta\pi/\omega} \cos(-\pi + \phi)(-1 - p)$.

f_1, f_2, f_3, f_4 in Eq. (5) are represented as follows:

$$f_1(x_n) = -p + \frac{1}{\omega} \mathrm{e}^{\delta\tau_2} \Big[\frac{-1}{\omega} \mathrm{e}^{\delta\tau_1} \sin\omega\tau_1 \{(x - p) \sin\omega\tau_2 + (-1 + p) \cos(\omega\tau_2 + \phi)\} \Big], \qquad (7)$$

where

$$\tau_2 = \frac{1}{\omega} \Big[\pi + \tan^{-1} \frac{-e^{\delta\tau_1} \sin\omega\tau_1 (x - p)}{-1 + p + \frac{\delta}{\omega} e^{\delta\tau_1} \sin\omega\tau_1 (x - p)} \Big],$$

$$f_2(x_n) = p + \frac{e^{\delta\pi/\omega}}{\omega} \cos(\pi + \phi)(x - p), \qquad (8)$$

$$f_3(x_n) = -p + \frac{e^{\delta\pi/\omega}}{\omega} \cos(\pi + \phi)(x + p), \qquad (9)$$

$$f_4(x_n) = p + \frac{1}{\omega} \mathrm{e}^{\delta\tau_4} \Big[\frac{-1}{\omega} \mathrm{e}^{\delta\tau_3} \sin\omega\tau_3 \{(x + p) \sin\omega\tau_4 + (1 - p) \cos(\omega\tau_4 + \phi)\} \Big], \qquad (10)$$

where

$$\tau_4 = \frac{1}{\omega} \Big[\pi + \tan^{-1} \frac{-e^{\delta\tau_3} \sin\omega\tau_3 (x + p)}{-1 - p + \frac{\delta}{\omega} e^{\delta\tau_3} \sin\omega\tau_3 (x + p)} \Big].$$

τ_1 in Eq. (7) and τ_3 in Eq. (10) are given by solving the following equations with Newton method.

$$\begin{cases} \frac{e^{\delta\tau_1}}{\omega} \cos(\omega\tau_1 + \phi)(x - p) + 1 + p = 0, \\ \frac{e^{\delta\tau_3}}{\omega} \cos(\omega\tau_3 + \phi)(x + p) - 1 - p = 0. \end{cases} \qquad (11)$$

Here, we present the chaotic attractor exhibiting in the hysteresis chaos generating system and its corresponding 6 times composition mapping as shown in Fig. 2.

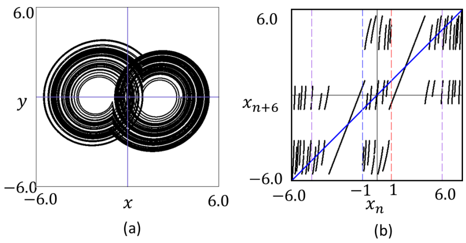

Figure 2: A chaotic attractor of system($\delta = 0.05, p = 2$): (a) $x - y$ phase space, (b) 6 times composition map

3. Chaos control method based on STM utilizing OPF

We propose a novel chaos control method based on STM utilizing OPF procedure to stabilize unknown UPOs of continuous-time chaotic systems. Block diagram of the controlled system is presented as shown in Fig. 3 and its dynamics are described as follows:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & 2\delta\end{bmatrix}\left[\begin{bmatrix}x\\ y\end{bmatrix} - \begin{bmatrix}p+\Delta p\\ 0\end{bmatrix} h(x)\right] \text{ for } SW = \text{on}. \tag{12}$$

Operation of controller switch SW is described by

$$SW = \begin{cases} \text{on} & \text{for } k\alpha\tau_n \le t \le k\alpha\tau_n + \dfrac{\pi}{\omega}, \\ & \qquad (\alpha = 0, 1, 2 ...) \\ \text{off} & \text{otherwise}, \end{cases} \tag{13}$$

where k denotes periodic number of the target trajectory.

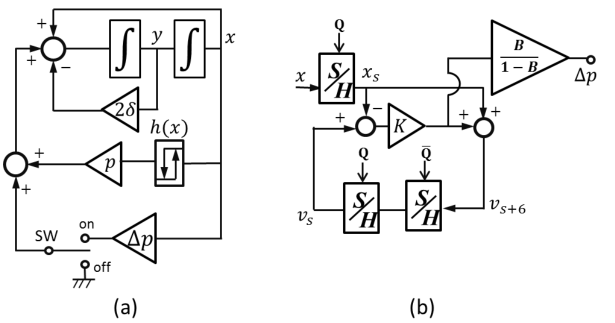

Figure 3: (a) Block diagram of controlled system, (b) Controlled variable Δp

Δp is the controlled variable applied to the equilibrium point p when SW is on.

$$\Delta p = \frac{1}{h(x)} K_{OPF}(x_n - v_{n+k}), \tag{14}$$

where K_{OPF} denotes the gain of controller utilizing OPF method. System will turn in super stable condition if K_{OPF} satisfies the following equation.

$$K_{OPF} = \frac{V}{V-1}, \tag{15}$$

where $V = \dfrac{1}{\omega} e^{\delta \frac{\pi}{\omega}} \cos(\pi + \phi)$.

v_{n+k} in Eq. (14) is determined as follows:

$$\begin{aligned} v_{n+k} &= q(\boldsymbol{v}_{n+k}), \\ \boldsymbol{v}_{n+k} &= (I - K_{STM})\boldsymbol{\xi}_n + K_{STM}\boldsymbol{v}_n, \end{aligned} \tag{16}$$

where $K_{STM} \in \boldsymbol{R}^2$ denotes the gain of controlled system based on STM. $\boldsymbol{v}_n = (v_n, h_n)$ and $\boldsymbol{\xi}_n = (x_n, h_n)$ represent the internal state of the controller and the state variable of system, respectively. $\boldsymbol{v}_n$ is updated whenever $y = 0$ as same as update of $\boldsymbol{\xi}_n$. Linear transformation q is defined by:

$$q : \boldsymbol{R}^2 \ \rightarrow \ \boldsymbol{R}, \quad (v, h) \ \mapsto v\ . \tag{17}$$

Here, we explain about the trajectory behavior of controlled system shown in Fig. 4. First, when $n = 0$ we consider that the trajectory hits J at $\tau = \tau_0$ on the half-plane S_+. The trajectory starting from this intersection point $\boldsymbol{x}(\tau_0)$ moves as well as the dotted line in case SW is off. When SW is on, at the moment the trajectory hits J, the controller's internal state is set to $\boldsymbol{v}_0$. Then v_6 is found by solving Eq. (16) with x_0 and v_0. We calculate Δp with Eq. (14) from the obtained v_6. We consider the hypothetical trajectory starting from v_6 is represented by the dashed line. Next, equilibrium point is shifted laterally by the controlled variable Δp between the moment from τ_0 to τ_1. In the result, the trajectory starting from $\boldsymbol{x}(\tau_0)$ is consistent with the hypothetical trajectory indicated by the dashed line which including $\boldsymbol{x}(\tau_1)$ on. Thereafter, shifted equilibrium point returns to its original position and the trajectory moves as well as behavior of 3-D hysteresis chaos generator as shown in Fig. 1. When it becomes $n = k\alpha$, the same control operation as above is repeated. We describe

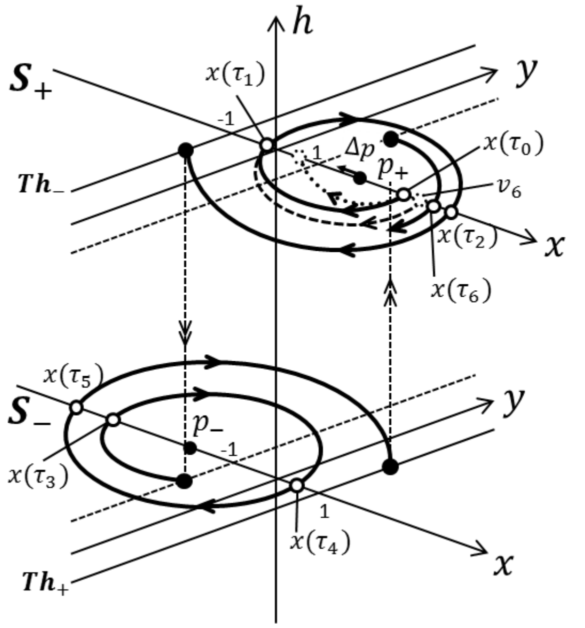

Figure 4: Typical trajectory of controlled system

dynamics of controlled system by this operation as follows:

$$
\begin{cases}
\boldsymbol{\xi}_{n+k} = F^k[(I - K_{STM})\boldsymbol{\xi}_n + K_{STM}\boldsymbol{v}_n] \\
\boldsymbol{v}_{n+k} = (I - K_{STM})\boldsymbol{\xi}_n + K_{STM}\boldsymbol{v}_n
\end{cases}
\text{for } n = k\alpha, \quad (\alpha = (0, 1, 2...))
$$

$$
\begin{cases}
\boldsymbol{\xi}_{n+l} = F^l(\boldsymbol{\xi}_n) \\
\boldsymbol{v}_{n+l} = \boldsymbol{v}_n
\end{cases}
\text{otherwise.} \quad (0 < l < k) \qquad (18)
$$

We consider the case of $K_{STM} = 0$, controlled system is equivalent to the hysteresis chaos generator in section 2.1. In the case of $K_{STM} \neq 0$, from the relation of $\boldsymbol{\xi}_n = F^k(\boldsymbol{v}_n)$, Eq. (18) can be abbreviated to the following system.

$$\boldsymbol{v}_{n+k} = (I - K_{STM})F^k(\boldsymbol{\xi}_n) + K_{STM}\boldsymbol{v}_n. \qquad (19)$$

Equation (19) is equivalent to transformed system. Regardless of the control gain K_{STM}, this transformed system (19) has fixed point(s) with the coordinate the same as fixed point(s) of the original chaos system. If we set control gain K_{STM} appropriately, transformed system (19) will exhibit a stable fixed point with the same position as $\boldsymbol{\xi}_f$. Our next step is to stabilize the fixed point $\boldsymbol{\xi}_f$ of the Transformed system (19) by suitable choices of K_{STM}. First, we consider the stability condition matrix of the controlled system as follows:

$$C_{ts} = \left| \left. \frac{\partial F^k(\boldsymbol{v}_n)}{\partial \boldsymbol{v}_n} \right|_{\boldsymbol{v}_n = \boldsymbol{\xi}_f} \right| = \left| \left. \frac{\partial \boldsymbol{v}_{n+k}}{\partial \boldsymbol{v}_n} \right|_{\boldsymbol{v}_n = \boldsymbol{\xi}_f} \right|. \qquad (20)$$

Since $\boldsymbol{\xi}_f$ is an unstable fixed point at least one of the eigenvalues of C_{ts} at $\boldsymbol{\xi}_f$ must possess an absolute value greater than 1. In order to stabilize $\boldsymbol{\xi}_f$ let us consider the stability matrix $M_{K_{stm}}$ which obey the following relation:

$$M_{K_{stm}} = (I - K_{STM})DF^k(\boldsymbol{\xi}_f) + K_{STM}. \qquad (21)$$

Then we adjust the control gain K_{STM} such that the eigenvalues of the stability matrix $M_{K_{stm}}$ at $\boldsymbol{\xi}_f$ must have absolute values less than 1. We also estimate the derivation of F^k at $\boldsymbol{\xi}_f$ as follows:

$$DF^k(\boldsymbol{\xi}_f) \simeq \frac{F^{k+1}(\boldsymbol{\xi}_f) - F^k(\boldsymbol{\xi}_f)}{F^k(\boldsymbol{\xi}_f) - \boldsymbol{\xi}_f}. \qquad (22)$$

Especially, if control gain K_{STM} is satisfied with the following equation, system will turn in super stable condition.

$$K_{STM} = \frac{-DF^k(\boldsymbol{\xi}_f)}{1 - DF^k(\boldsymbol{\xi}_f)}. \qquad (23)$$

Thus, if we set K_{STM} appropriately satisfied with the above stability condition, controlled system exhibits stable periodic orbit(s) with the same position as the unstable periodic orbit(s) of hysteresis chaos generating system. Since (18) does not include any information of the fixed point $\boldsymbol{\xi}_f$, it is possible to stabilize unknown UPOs of chaotic system if we can set control gain appropriately. Here, we verify the efficacy of the controlled system with our proposed method by numerical simulation. We choose 6-period stable periodic orbits as our goal and set $K_{STM} = 1.1$. We also confirm the stabilization of 3 kinds of different unstable periodic orbits depending on different initial values as shown in Fig. 5.

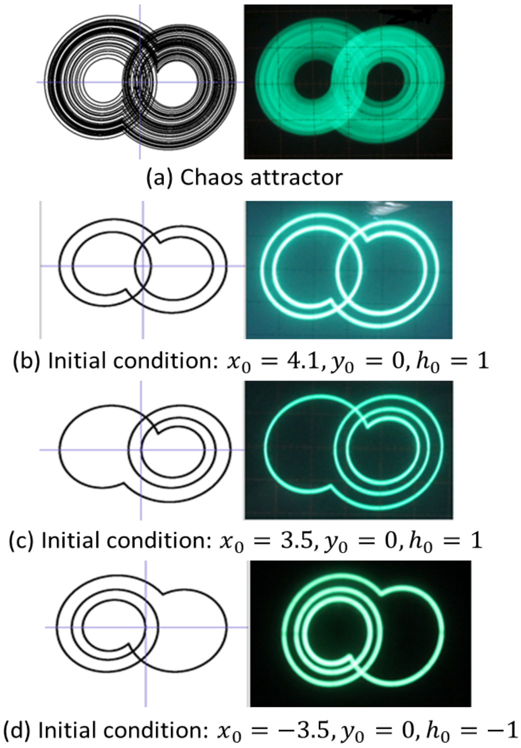

Figure 5: Stable periodic orbits exhibited by the controlled system ($\delta = 0.05, p = 2, k = 6, K_{STM} = 1.1$)

4. Conclusion

In this paper, we considered a basic approach to generalize techniques of controlling chaos. We proposed a novel chaos control method based on the STM utilizing OPF technique for the autonomous continuous-time chaotic systems, and described our approach for the case of stabilizing fixed points of 6 times composition map. We also presented the procedure to stabilize the unknown UPOs of a 3-D hysteresis chaos generator. Now we are trying to analyze the stability region of the control gain K_{OPF} and K_{STM} of our controlled system.

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