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Chaotic Rotor Associative Memory

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Abstract— Complex-valued Associative Memory (CAM) can deal with multi-valued patterns. Rotor Associative Memory (RAM) is an advanced model of the CAM. In this paper, we propose Chaotic Rotor Associative Memory (CRAM) which uses a chaotic neuron model. It is known that the CAM stores not only given learning patterns but also the rotated patterns. All of them appear in the recall process of the Chaotic CAM (CCAM). Our proposed model, the CRAM, can remove the rotated patterns. In addition, we found that most of the superimposed patterns vanished by computer simulations.

1. Introduction

Recently, Complex-valued Associative Memory(CAM) has been drawn attentions because the model can deal with multi-valued patterns like grayscale images[1][2].

When the complex-valued neurons take K states, it is known that the CAM stores $K-1$ spurious memories which are called rotated patterns, for a learning pattern. This phenomenon is one of the CAM's problems and corresponds to reversed patterns in the Hopfield Associative Memory(HAM). Nakata et al. proposed the Chaotic Complex-valued Associative Memory(CCAM)[6] and introduced chaos[3] to the CAM. By using chaotic neuron, the network can get out of stable states and search all learning patterns. The computer simulation shows that many rotated patterns and superimposed patterns appear in the association process. We regard the patterns as objectionable ones because they are different from the original learning patterns.

The Rotor neuron[7][8][9] is a model whose states are high-dimensional unit vectors. Especially, we are interested in the 2-dimensional rotor neuron related to the complex-valued neuron.

Rotor Associative Memory (RAM) is an associative memory model which the CAM is naturally extended to. The model does not store rotated patterns. Nakamura et al. showed that the capacity of the RAM is larger than that of the CAM[9].

In this paper, we proposed the Chaotic Rotor Associative Memory(CRAM) whose neurons are chaotic models for confirming effectiveness of the RAM. We can expect that the CRAM recalls less spurious memories than the CCAM does in their association process.

In computer simulation using grayscale patterns, we confirmed that the CRAM associated no rotated patterns and few superimposed patterns.

2. Complex-valued Associative Memory

2.1. Complex-valued neuron

At first, we define the complex-valued neuron whose input and state are complex number. K equally-divided points $c_k (k = 0, 1, \ldots, K-1)$ of the unit circle on complex plane are defined as Eq.(1).

$$c_k = \exp(2k\theta_K \sqrt{-1})(k = 0, 1, \ldots, K-1) \quad (1)$$
$$\theta_K = \frac{\pi}{K} \quad (2)$$

State of the neuron takes one of the set $\{c_k\}$. When we define the input, the output to a complex-valued neuron, the output function as $f(\cdot)$ and the neuron state as S, $f(\cdot)$ and z, updating is given by the following equation

$$z = f(S) \quad (3)$$
$$f(S) = \underset{c_k}{\operatorname{argmax}}\, Re(c_k \overline{S}). \quad (4)$$

2.2. Complex-valued Associative Memory

Next, we define the CAM which is an associative memory with complex-valued neuron.

The connection weights of CAM are complex-valued matrix. We denote the weight from the ith neuron to the jth neuron as w_{ji}, and then it must satisfy the relation

$$w_{ji} = \overline{w_{ij}}. \quad (5)$$

Let N and P be the number of neurons and learning patterns. Denote the pth learning pattern as $\boldsymbol{a}^{(p)} = (a_1^{(p)}, a_2^{(p)}, \ldots, a_N^{(p)})(p = 1, 2, \ldots, P)$. Then, w_{ji} is defined as

$$w_{ji} = \begin{cases} \sum_{p=1}^{P} a_j^{(p)} \overline{a_i^{(p)}} & (i \neq j) \\ 0 & (i = j). \end{cases} \quad (6)$$

We call this learning rule complex-valued hebb rule. It clearly meets Eq.(5).

The CAM can be regarded as the RAM as described in Sec.3.3.

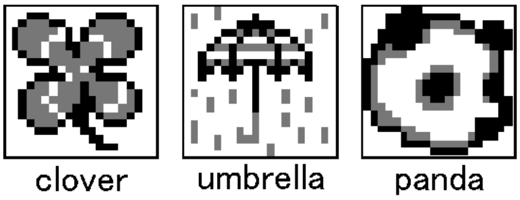

Figure 1: Learning patterns 1.

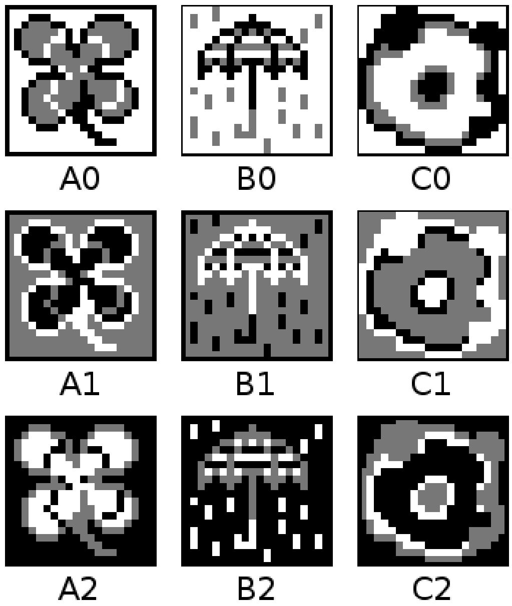

Figure 2: Rotated patterns.

2.3. Rotated patterns in CAM

When the CAM stores a pattern $\boldsymbol{a}$, it also stores the patterns $c_k\boldsymbol{a}(k = 1, 2, \ldots, K-1)$. The pattern $c_k\boldsymbol{a}$ is gained by rotating each neuron of the pattern $\boldsymbol{a}$ by $2k\theta_k$. The pattern $c_0\boldsymbol{a}$ is the learning pattern and in the case $k \neq 0$ the patterns $c_k\boldsymbol{a}$ is called the rotated patterns.

Suppose that the CAM stores three patterns of Fig.1. Each dot takes three states, white, gray, and black, and shows the state of a neuron. In this case, complex-valued neuron with $K = 3$ is used and the states c_0, c_1 and c_2 are assigned to white, gray and black, respectively.

The CAM will store nine patterns in Fig.2. The patterns A0, B0 and C0 are learning patterns, the patterns A1, B1 and C1 are patterns rotated by $2\theta_K$ and the patterns A2, B2 and C2 are patterns rotated by $4\theta_K$.

3. Rotor Associative Memory

3.1. Rotor neuron

At first, we define the rotor neuron whose input, output and state are 2-dimensional vectors. K equally-divided points $\boldsymbol{h}_k(k = 0, 1, \ldots, K-1)$ of the unit circle on x-y plane are defined as Eq.(7).

$$\boldsymbol{h}_k = \begin{pmatrix} \cos 2k\theta_K \\ \sin 2k\theta_K \end{pmatrix} \tag{7}$$

The state of neuron takes one of the set $\{\boldsymbol{h}_k\}$ and is expressed as

$$z = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{8}$$

We define the input to a rotor neuron, the output function and the neuron state as $\boldsymbol{S} = {}^t(x, y)$, $g(\cdot)$ and z , where ${}^t(x, y)$ means transposed (x, y). Updating is given by the following equation

$$z = g(\boldsymbol{S}) \tag{9}$$

$$g(\boldsymbol{S}) = \underset{\boldsymbol{h}_k}{\operatorname{argmax}} \, {}^t\boldsymbol{h}_k\boldsymbol{S}. \tag{10}$$

The complex-valued neurons behave as rotor neurons when complex numbers are regarded as 2-dimentional vectors.

3.2. Rotor Associative Memory

Next, we define the RAM which is an associative memory with rotor neurons.

The connection weights of RAM is 2-by-2 matrix. We denote the weight from the ith neuron to the jth neuron as $\boldsymbol{W}_{ji}$, and then it must satisfy the relation

$$\boldsymbol{W}_{ji} = {}^t\boldsymbol{W}_{ij}. \tag{11}$$

Denote the pth learning pattern as $\boldsymbol{A}^{(p)} = (\boldsymbol{a}_1^{(p)}, \boldsymbol{a}_2^{(p)}, \ldots, \boldsymbol{a}_N^{(p)})(p = 1, 2, \ldots, P)$. Then, $\boldsymbol{W}_{ji}$ is defined as

$$\boldsymbol{W}_{ji} = \begin{cases} \sum_{p=1}^{P} \boldsymbol{a}_j^{(p)t}\boldsymbol{a}_i^{(p)} & (i \neq j) \\ \boldsymbol{O} & (i = j). \end{cases} \tag{12}$$

We call this learning rule rotor hebb rule. It clearly meets Eq.(11).

3.3. Relationship with Complex-valued Associative Memory

We denote the state of the ith complex-valued neuron as $z_i = x_i + y_i\sqrt{-1}$. And we denote the weight from the ith neuron to the jth neuron as

$$w_{ji} = u_{ji} + v_{ji}\sqrt{-1}. \tag{13}$$

Then the following equation holds.

$$w_{ji}z_i = (u_{ji}x_i - v_{ji}y_i) + (v_{ji}x_i + u_{ji}y_i)\sqrt{-1} \tag{14}$$

If we denote

$$\boldsymbol{W}_{ji} = \begin{pmatrix} u_{ji} & -v_{ji} \\ v_{ji} & u_{ji} \end{pmatrix} \tag{15}$$

$$z_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}, \tag{16}$$

then we can regard the CAM as the special case of RAM. In fact, $\boldsymbol{W}_{ji}$ satisfies $\boldsymbol{W}_{ji} = {}^t\boldsymbol{W}_{ij}$ from $u_{ji} = u_{ij}$ and $v_{ji} = -v_{ij}$. However learning uses the complex-valued hebb rule not the rotor hebb rule. The RAM does not store rotated patterns by the rotor hebb rule, but stores the reversed patterns in the case that K is even.

3.4. Chaotic Rotor Associative Memory

We consider our proposed model CRAM. The network can get out of stable states and search all learning patterns by chaotic behavior. We construct the CRAM based on the Sato-Nagumo model[3]. The dynamics of chaotic rotor neurons are defined as

$$\boldsymbol{z}(t+1) = f\left(\boldsymbol{S}(t+1) - \alpha \sum_{d=0}^{t} k^d \boldsymbol{z}(t-d)\right). \tag{17}$$

The terms $\boldsymbol{z}(t)$ and $\boldsymbol{S}(t)$ are the neuron output and the neuron input at time t. The coefficients α and k are the scaling factor and damping factor. The term

$$-\alpha \sum_{d=0}^{t} k^d \boldsymbol{z}(t-d) \tag{18}$$

shows that the longer a stable state continues, the stronger the current state is moved toward the opposite direction. The CRAM can dynamically recall the stored patterns by this behavior.

3.5. Reason of memorizing no rotated patterns

The state of the ith neuron rotated by $2k\theta_K$ is denoted as $\boldsymbol{R}(k)\boldsymbol{z}_i^{(p)}$, where $\boldsymbol{R}(k)$ is defined as follows:

$$\boldsymbol{R}(k) = \begin{pmatrix} \cos 2k\theta_K & -\sin 2k\theta_K \\ \sin 2k\theta_K & \cos 2k\theta_K \end{pmatrix}. \tag{19}$$

When a pattern $\boldsymbol{R}(k)\boldsymbol{A}^{(p)}$ is given to the RAM, the input $\boldsymbol{S}_j$ to the jth neuron is defined as follows:

$$\begin{aligned} \boldsymbol{S}_j &= \sum_{i=1}^{N} \boldsymbol{W}_{ji}\boldsymbol{R}(k)\boldsymbol{a}_i^{(p)} \qquad (20) \\ &= \sum_{\substack{i=1 \\ i\neq j}}^{N} \boldsymbol{a}_j^{(p)t}\boldsymbol{a}_i^{(p)}\boldsymbol{R}(k)\boldsymbol{a}_i^{(p)} \\ &\quad + \sum_{\substack{i=1 \\ i\neq j}}^{N} \sum_{\substack{q=1 \\ q\neq p}}^{P} \boldsymbol{a}_j^{(q)t}\boldsymbol{a}_i^{(q)}\boldsymbol{R}(k)\boldsymbol{a}_i^{(p)} \qquad (21) \\ &= (N-1)\cos 2k\theta_K \boldsymbol{a}_j^{(p)} \\ &\quad + \sum_{\substack{i=1 \\ i\neq j}}^{N} \sum_{\substack{q=1 \\ q\neq p}}^{P} \boldsymbol{a}_j^{(q)t}\boldsymbol{a}_i^{(q)}\boldsymbol{R}(k)\boldsymbol{a}_i^{(p)}. \qquad (22) \end{aligned}$$

The first term of Eq.(22) move the state toward $\boldsymbol{A}^{(p)}$ in case $\cos 2k\theta_k > 0$ and $-\boldsymbol{A}^{(p)}$ in case $\cos 2k\theta_k < 0$. In addition, the larger $|\cos 2k\theta_k|$ is, the stronger the RAM recalls the learning pattern $\boldsymbol{A}^{(p)}$. Thus the RAM does not become stable because the model recalls $\pm\boldsymbol{A}^{(p)}$ when the rotated pattern is given to it. The second term is the noisy term.

4. Computer simulation

We experienced computer simulations for checking the behavior and confirming the effectiveness. The CCAM was regarded as a special case of CRAM. In this simulation, we regard continuously appearing patterns for more than 2 times as recalled patterns.

4.1. Chaotic Complex-valued Associative Memory

We confirmed the behavior of CCAM by computer simulation. The learning patterns were Fig.1. The parameters were $N = 400$, $k = 0.98$, $\alpha = 17$. Figure 3 shows the association result.

The initial state was set as clover. The followings are the times when the learning patterns were recalled.

clover	0 ~ 17, 125 ~ 130
umbrella	19 ~ 31, 141 ~ 154
panda	158 ~ 171

In addition, the followings were recalled rotated patterns and the time.

$2\theta_K$ rotated clover(A1)	65 ~ 79
$4\theta_K$ rotated clover(A2)	94 ~ 112
$2\theta_K$ rotated umbrella(B1)	46 ~ 59
$4\theta_K$ rotated umbrella(B2)	114 ~ 117
$2\theta_K$ rotated panda(C1)	202 ~ 222
$4\theta_K$ rotated panda(C2)	180 ~ 192

Furthermore, the superimposed patterns were recalled at $t = 35 \sim 43, 62 \sim 63, 133 \sim 138, 174 \sim 175, 198 \sim 199$.

From the results, it's confirmed that all learning patterns were recalled but all rotated patterns and many superimposed patterns were also recalled.

4.2. Chaotic Rotor Associative Memory

We confirmed the behavior of CRAM by computer simulation. The learning patterns were in Fig.1. The parameters were $N = 400$, $k = 0.97$, $\alpha = 14$. And Fig.4 shows the association result.

The initial state was set as clover. In addition, the followings are the times when the learning patterns were recalled.

clover	0 ~ 27, 119 ~ 122, 159 ~ 181
umbrella	29 ~ 67, 183 ~ 219
panda	69 ~ 117

From the results, it's confirmed that all learning patterns were recalled and the rotated and the superimposed patterns were not recalled.

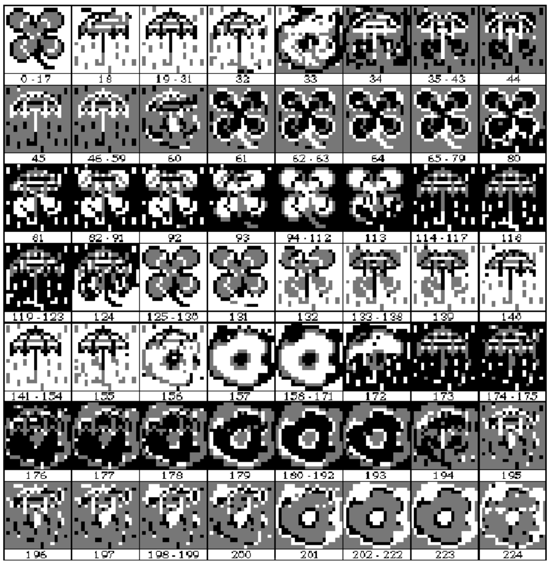

Figure 3: An association result of CCAM(N = 400, k = 0.98, α = 17).

5. Conclusion

In this paper, we proposed the CRAM which is RAM using chaotic neurons. We got the following results about our proposed model.

The CCAM cannot avoid recalling the rotated patterns. The CCAM recalls not only learning patterns but also rotated and superimposed patterns. However, the proposed CRAM can avoid them. In our experiment of the CRAM, rotated and superimposed patterns were not recalled.

When K is even, the CRAM recalls reversed patterns. However we can avoid them by adding one dummy state to the rotor neurons.

In this paper, we used grayscale patterns to visualize the chaotic behavior. To process image data, our simple method is not appropriate and we should take other approaches such as Nakata et al.[10].

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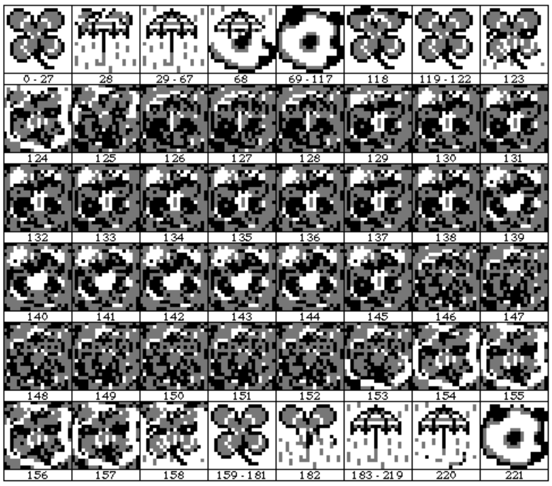

Figure 4: An association result of CRAM 1(N = 400, k = 0.97, α = 14).

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