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Self-organization of Motile Oscillators Inspired by Friendship Formation

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Abstract—Although dynamical order generated from local interactions among self-propelled entities, each of which has intrinsic property and internal state, is widely observed in nature and social systems, its mechanism is not yet clearly understood. To address this problem, we draw inspiration from the formation and collapse dynamics of friendship in human society. Although each individual has a different character, people interact with each other by properly balancing their self-assertion and compromise, consequently leading to the emergence of dynamical order. We express the characters of individuals using "phases" and propose a simple model in which the position and phase of each individual are adjusted on the basis of the phase difference and distance from the other individuals. Simulation results showed that various dynamic patterns emerge via changes to a small number of parameters.

1. Introduction

Self-organization of self-propelled entities are ubiquitous in nature and social systems such as traffic patterns [1], flocking of animals [2], and cell migration [3]. These intriguing phenomena have received considerable attention in recent years, and they are studied from both scientific and engineering viewpoints [1-7]. Mathematical modeling is a useful way for understanding the essential mechanism of these phenomena, and various models have been proposed thus far [1-4,6,7].

In general, each element constituting a system has its own internal state, which is influenced by its surroundings. Further, intrinsic properties of elements often differ from each other. The motion of each element should be determined on the basis of both information of other elements and its own information, *i.e.*, its intrinsic property and internal state. However, less attention has been paid to the effect of elements' intrinsic properties and internal states on macroscopic behavior of an entire system in the previous models, although it could play a crucial role in many systems.

To tackle this problem, we propose a mathematical model for self-organization of self-propelled entities inspired by the process of friendship formation. In human society, people having different characters interact with each other by properly balancing their self-assertion and compromise, consequently leading to the emergence of dynamical order. Hence, friendship formation is an attractive phenomenon for investigating the essential mechanism for self-organization of elements having intrinsic properties and internal states.

For constructing the model, we drew inspiration from the swarm oscillator model [7], in which dynamical order emerges from local interactions among motile elements with internal states. In our model, we add the effect of intrinsic properties of elements to the swarm oscillator model. Both intrinsic and superficial characters of individuals are expressed as "phases," and the position and superficial character of each individual are adjusted on the basis of the phase difference and distance from the other individuals. We demonstrate through simulations that various dynamic patterns such as the linear chain pattern, grid pattern, membrane pattern, and exocytosis-like pattern emerged via changes to a small number of parameters.

2. Model

We consider N particles on a two-dimensional plane. Each particle represents an individual, and its position is denoted by $\mathbf{r}_i$, where i $(1 \leq i \leq N)$ is the particle number. We assume that each individual has an "intrinsic character" and "superficial character." It is natural to consider that characters can be quantitatively described by points on a multi-dimensional plane comprising several paired concepts such as "patient and short-tempered" and "thorough and dissolute" (Fig. 1(a)). However, we simply express the characters in a two-dimensional plane by assuming that only two of the paired concepts are crucial for determining the compatibility between individuals (Fig. 1(b)). We further assume that the characters are on the unit circle and define the characters as deflection angles. Hereafter, we refer to these deflection angles as "phases." The phases for the intrinsic character and superficial character are denoted by $\tilde{\psi}_i$ and ψ_i, respectively (Fig. 1(b)). Here, $\tilde{\psi}_i$ is assumed to be time-invariant. We also assume that no individual can know the intrinsic characters of other individuals. Each individual changes its position and phase according to the position and superficial character of other individuals. Furthermore, we assume that interaction strength decreases as

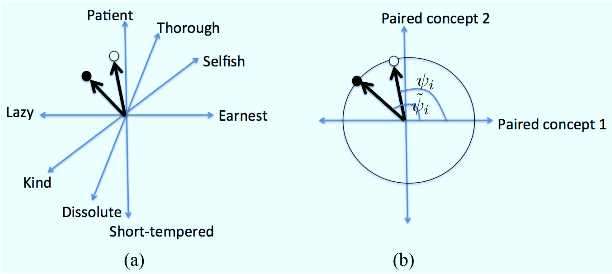

Figure 1: Definition of human character: (a) Human character defined in multi-dimensional phase space. (b) Simplified definition, *i.e.*, human character described by two-dimensional phase plane. The intrinsic character (filled circle) and superficial character (open circle) are described using phases $\tilde{\psi}_i$ and ψ_i, respectively.

the distance between individuals increases, although with the advent of the Internet, this assumption is not necessarily valid nowadays.

The time evolution of each particle is described by the following equations:

$$\dot{\psi}_i = \sum_{i \neq j} e^{-|\mathbf{R}_{ji}|} \frac{\partial F(\psi_j, \psi_i)}{\partial \psi_i} + k_i \sin(\tilde{\psi}_i - \psi_i), \quad (1)$$

$$\tau_i \dot{f}_i = k_i |\sin(\tilde{\psi}_i - \psi_i)| - f_i, \quad (2)$$

$$\dot{\mathbf{r}}_i = c_i \sum_{i \neq j} \hat{\mathbf{R}}_{ji} [e^{-|\mathbf{R}_{ji}|} (F(\tilde{\psi}_i, \psi_j) - \lambda_i f_i) - A|\mathbf{R}_{ji}|^{-\mu}], \quad (3)$$

where $\mathbf{R}_{ji} = \mathbf{r}_j - \mathbf{r}_i$ and $\hat{\mathbf{R}}_{ji} = \mathbf{R}_{ji}/|\mathbf{R}_{ji}|$. The function $F(\psi_j, \psi_i)$ represents the compatibility between j and i, which is large when "j prefers i." Although its functional form in real societies is unclear, here, we simply describe it as

$$F(\psi_j, \psi_i) = \cos(\psi_i - \psi_j + \beta), \quad (4)$$

where β is a constant that satisfies $-\pi \leq \beta < \pi$. Equation (4) implies that each individual prefers individuals whose phases differ from its own phase by β. Hence, individuals having an identical phase prefer each other if $\beta = 0$.

The first term in Eq. (1) implies that "each individual changes its superficial character so that others prefer it," while the second term implies that the superficial character is attracted to the intrinsic character. Here, k_i is a positive constant that represents the "stubbornness" of i. The frustration of i is denoted by f_i. Equation (2) implies that the frustration increases when the superficial character deviates from the intrinsic character. The parameter τ_i determines the time scale by which the frustration changes. The function $F(\tilde{\psi}_i, \psi_j)$ in Eq. (3) implies that i approaches or moves away from j when i prefers or hates j, respectively, whereas the term $\lambda_i f_i$ is used to express that i moves away from others when its frustration increases. The term $-A|\mathbf{R}_{ji}|^{-\mu}$ represents the exclusive volume effect. Here, c_i and λ_i are the parameters that express "interestedness in others" and "the tendency to get bored easily," respectively, and A and μ are positive constants. We note that k_i, τ_i, c_i, and λ_i are determined by the character of i, and these factors should actually constitute the orthogonal bases of the multi-dimensional space in Fig. 1(a). However, here, we assumed that these factors are irrelevant to the compatibility $F(\tilde{\psi}_i, \psi_j)$, for simplicity.

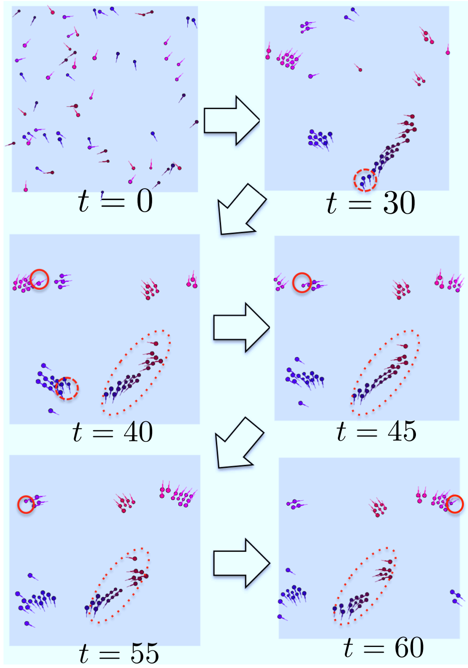

Figure 2: Simulation result. The condition and parameter values are described in the main text. The diameter and color of each particle denote frustration f_i and the intrinsic character $\tilde{\psi}_i$, respectively. The directions of thick short and thin long lines that originate from the center of the particle indicate the phases $\tilde{\psi}_i$ and ψ_i, respectively. These definitions also apply to Figs. 3-9.

3. Simulation

We performed simulations of the proposed model. We placed N particles in a square field of size $L \times L$. We set the values of k_i, τ_i, c_i, and λ_i independent of i (hereafter, we omit subscripts for these parameters). The values of A and μ were 8.86×10^{-7} and 11, respectively, and the time step was set to be 0.001.

Figure 2 shows the result under the following condition:

$N = 60$, $L = 10$, $k = 2.0$, $c = 1.0$, $\lambda = 40.0$, $\tau = 3.0$, $\beta = 0.0$
$\tilde{\psi}_i$ is uniformly distributed.
Periodic boundary condition.

We find that particles whose intrinsic characters are close to each other aggregated and formed clusters. However, several particles belonging to a cluster moved to another cluster (dashed circles in Fig. 2). A particle that oscillates between two clusters (solid circles in Fig. 2) and clusters that contact and detach periodically (dotted circles in Fig. 2) were also found.

Figure 3 shows the result in the case where k was changed with the other conditions unchanged. When $k = 0.0$, half of the particles formed a cluster while the other

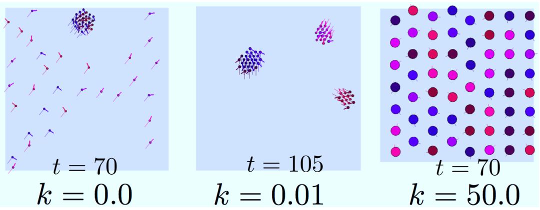

Figure 3: Simulation result when k is changed. The condition and other parameter values are described in the main text.

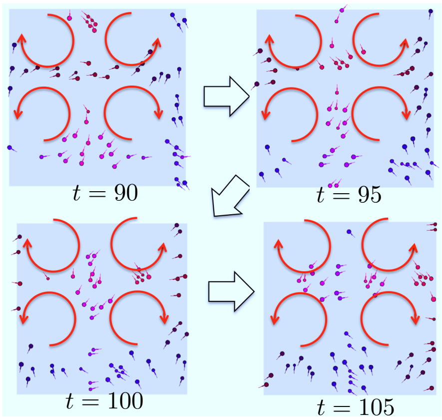

Figure 4: Simulation result (Convection pattern). The condition and parameter values are described in the main text.

half was distributed in the field. When $k = 0.01$, three clusters are formed. Further, when $k = 50.0$, a crystal structure is formed, wherein the frustration of each particle is extremely large. Figure 4 shows the result when β was changed from 0 to -0.15π with the other conditions being the same as those in Fig. 2. In this case, convection occurred because each particle chases other particles whose phases differ slightly from its own phase.

Figure 5 shows the result when β is changed under the following condition:

$N = 60, L = 10, k = 2.0, c = 1.0, \lambda = 40.0, \tau = 3.0$
$\tilde{\psi}_i = 0\ (i \leq 20),\ 2\pi/3\ (20 < i \leq 40)$, and $4\pi/3\ (40 < i \leq 60)$.
Periodic boundary condition.

When $\beta = -0.9\pi$, three particles with differing intrinsic characters formed a cluster and the clusters were aligned almost equidistantly. Particles within clusters rotated clockwise or counterclockwise. When $\beta = -0.4\pi$, particles having the same intrinsic character were aligned vertically and moved horizontally. When $\beta = -0.1\pi$, several membrane structures were formed, with several particles existing within the membranes. The particles that constitute the membranes and those within the membranes switched dynamically. When $\beta = -0.055\pi$, several clusters were formed and convection occurred within each cluster. Further, for β between -0.4π and -0.1π, convection similar to that shown in Fig. 4 was found (data not shown).

Figure 6(a) shows the result under the following condi-

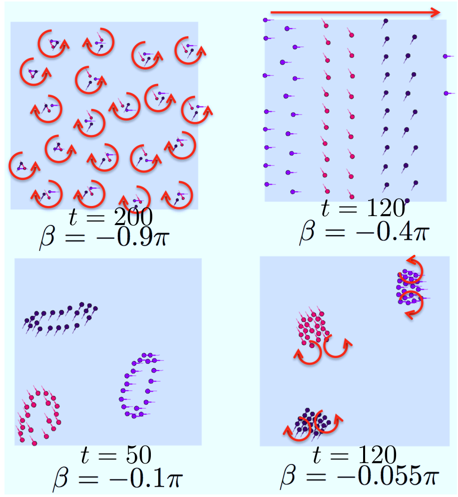

Figure 5: Simulation result when β is changed. The condition and other parameter values are described in the main text.

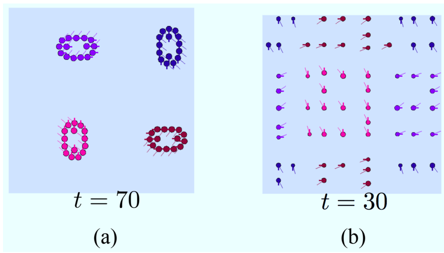

Figure 6: Simulation results: (a) stable membrane pattern and (b) grid pattern. The condition and parameter values are described in the main text.

tion:

$N = 60, L = 10, k = 0.065, c = 1.0, \lambda = 20.0, \tau = 3.0, \beta = -0.18\pi$
$\tilde{\psi}_i = 0\ (i \leq 15),\ \pi/2\ (15 < i \leq 30),\ \pi\ (30 < i \leq 45)$, and $3\pi/2\ (45 < i \leq 60)$.
Periodic boundary condition.

We find that ellipsoidal membranes were formed. A couple of particles existed within each membrane along the long axis of the ellipsoid. In contrast to the membrane pattern shown in Fig. 5, the membrane structures were symmetric and stable.

Figure 6(b) shows the result under the following condition:

$N = 60, L = 5, k = 0.065, c = 1.0, \lambda = 500.0, \tau = 3.0, \beta = -0.18\pi$
$\tilde{\psi}_i = 0\ (i \leq 15),\ \pi/2\ (15 < i \leq 30),\ \pi\ (30 < i \leq 45)$, and $3\pi/2\ (45 < i \leq 60)$.
Periodic boundary condition.

In this case, a grid pattern in which particles were aligned both vertically and horizontally was observed. Particles having the same $\tilde{\psi}_i$ were close to each other. The overall structure moved at a constant speed.

Figure 7 shows the result under the following condition:

$N = 100, L = 14, k = 0.02, c = 2.5, \lambda = 20.0, \tau = 3.0, \beta = -0.18\pi$

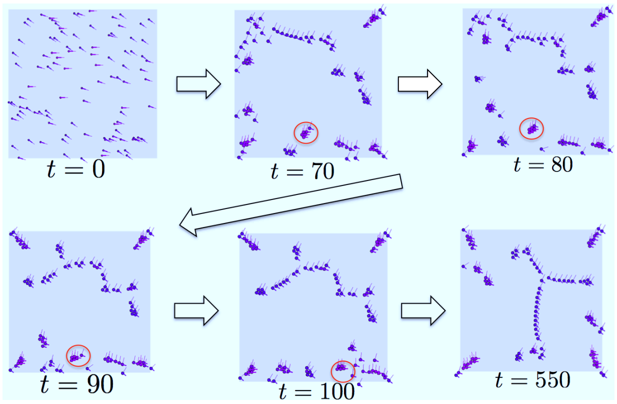

Figure 7: Simulation result (Linear chain pattern generated through chasing). The condition and parameter values are described in the main text.

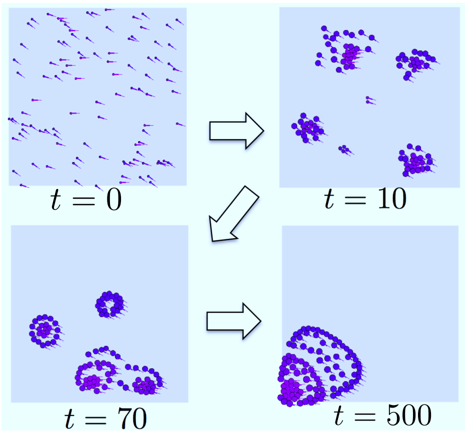

Figure 8: Simulation result (Exocytosis-like pattern). The condition and parameter values are described in the main text.

$\tilde{\psi}_i = 0$ $(i \leq 16)$, -0.1π $(16 < i \leq 50)$, and -0.2π $(50 < i \leq 100)$.
Periodic boundary condition was not used.

We find that particles having a large $\tilde{\psi}_i$ chased those having a small $\tilde{\psi}_i$. In such a case, a linear chain was gradually formed. Several particles were pushed toward the corner of the field.

Figure 8 shows the result under the following condition:

$N = 100$, $L = 14$, $k = 0.65$, $c = 0.5$, $\lambda = 20.0$, $\tau = 3.0$, $\beta = -0.18\pi$
$\tilde{\psi}_i = 0$ $(i \leq 16)$, -0.1π $(16 < i \leq 50)$, and -0.2π $(50 < i \leq 100)$.
Periodic boundary condition was not used.

In this case, several particles formed clusters and were surrounded by membranes. After a while, the clusters moved out from the membranes and the membranes fused again. This behavior was similar to exocytosis in biological systems.

4. Conclusion and future works

We proposed a mathematical model for self-organization of elements having internal states and different intrinsic properties, inspired by friendship formation in human society. Individuals were modeled by particles having intrinsic and superficial characters, which were expressed as phases. The position and phase of the particles were adjusted based on those of their neighboring particles. The simulation results showed that various dynamic patterns emerge via changes to a small number of parameters. Although it is still unclear whether these patterns can be observed in real human society, our model is interesting in that it shows various non-trivial collective behaviors emerging through simple local rules. Therefore, we expect that our model will enable us to understand the essential mechanism underlying collective behaviors of self-propelled entities. Further, this study could help develop swarm robots that can exhibit versatile collective behaviors in response to the circumstances.

Although we have shown various patterns qualitatively, the obtained results are not yet evaluated quantitatively. Detailed mechanism of the emergence of dynamical order is not yet clear. Further, the model is still somewhat complicated and there are many free parameters. In future, we will investigate the proposed model in more detail, and try to further simplify the model without losing its essence.

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