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# Synchrony sensitivity of a single neuron for various synaptic weight distributions

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**Abstract**—In this study, I investigated sensitivity of a single neuron to synchronous presynaptic firings for log-normal synaptic weight distribution from a viewpoint of spike transmission. For the comparison, I also tested various types of synaptic weight distribution.

As a result, the synchrony of presynaptic neurons was less efficient on the spike information transmission at the strongest synapse in systems with the log-normal synaptic weight distribution than in systems with the other types of synaptic weight distribution. My result suggest that the synchrony plays a role of efficiently improving the spike transmission of populations of weak synapses.

## 1. Introduction

The brain is a huge network consisting of a number of neurons. It is believed that neurons in the brain primarily communicate with spikes or action potentials. There are two main theories of neural computation in the field of neuroscience. The one is the spike-rate based theory. In this theory, neurons are thought to temporally integrate noisy inputs. The other is the spike-timing based theory, in which neurons can detect synchrony of inputs.

Many experimental observations suggest that the temporal spike coordination plays a role in sensory processing [1, 2]. Some theoretical studies support such a role of the correlated inputs into a single neuron [3, 4]. These studies showed that a few synchronous presynaptic spikes result in many extra postsynaptic spikes. Neurons are, then, sensitive to and can detect synchronous presynaptic inputs.

Neurons in the cortex or the hippocampus exhibit sparse, irregular, and asynchronous firings when external inputs are absent. Recent theoretical studies suggest that log-normal synaptic weight distribution makes these firings possible [5, 6]. Further, the network activity was sustainable without external inputs like real neural systems. These studies showed that the sustainable network activity was implicated in the stochastic resonance.

In a network with log-normal synaptic weight distribution, internal states of neurons might be much different from in a network with the other types of synaptic distribution, such as constant distribution and uniform distribution, that were typically used in many previous studies [3, 4]. Under such states, effects of synchronous inputs are also thought to be much different.

Then, in this study, I investigated synchrony sensitivity of a single neuron in a system with log-normal synaptic weight distribution. Since synaptic weight distribution gives an impact on postsynaptic responses, I also tested the other types of synaptic weight distribution for the comparison with log-normal distribution.

## 2. Materials and Methods

I modeled a neuron with a leaky integrate-and-fire (LIF) model. Model dynamics of the LIF was given by

$$C\frac{\mathrm{d}v}{\mathrm{d}t} = g_{\mathrm{leak}}(v_{\mathrm{rest}} - v) + g_{\mathrm{ex}}(v_{\mathrm{ex}} - v) + g_{\mathrm{inh}}(v_{\mathrm{inh}} - v), \quad (1)$$

where the variable $v$ was membrane potential and the variables $g_{\mathrm{ex}}$ and $g_{\mathrm{inh}}$ were respectively excitatory- and inhibitory-synaptic conductance. When $v$ reached a firing threshold $v_{\mathrm{thre}}$, it was reset to $v_{\mathrm{reset}}$. The neuron had the absolute refractory period $\tau_{\mathrm{ref}}$. All the parameters of Eq. (1) were same as Ref. [5]. An exponential function governed kinetics of $g_{\mathrm{ex}}$ and $g_{\mathrm{inh}}$ such as $\tau\dot{g} = -g + \sum_i \overline{g}_i$, where $\overline{g}_i$ was unitary synaptic conductance. The parameter $\tau$ determined the time scale of synaptic behaviors.

In my neural system, the neuron had $N_{\mathrm{ex}}$ excitatory and $N_{\mathrm{inh}}$ inhibitory presynaptic neurons. These presynaptic neurons organized synapses on the postsynaptic neuron. All the inhibitory synapses had the same value of conductance $\overline{g}_{\mathrm{inh}}$, whereas the excitatory synapses were heterogeneous. Five types of distribution realized the heterogeneity among synapses: log-normal, Gaussian, uniform, bimodal, and constant distribution. In all the types of distribution, unitary excitatory synaptic conductance was bounded for $\overline{g}_{\mathrm{thre}}$. The synaptic weight distribution except for the log-normal distribution was generated after the generation of the log-normal distribution. The mean value of the other types of synaptic distribution was inherited from the log-normal distribution.

**Log-normal distribution:** We generated log-normal distribution with the parameters of $\mu = 10^{-0.31}$ nS and $\sigma = 10^{-0.3}$ nS [5].

**Gaussian distribution:** In Gaussian distribution, negative numbers could appear, but values of conductance should be non-negative. Then, the lower bound was set to zero in the generation of the Gaussian distribution. In the limited range, the Gaussian distribution was generated as the mean value of the log-normal distribution was preserved. The standard deviation in the Gaussian distribution was also identical to that in the log-normal distribution.

**Uniform distribution:** In uniform distribution, the mean value located at the center of the distribution. The maximum value was twice of the mean value. Therefore, for the

Table 1: Model parameters in the system

| Implication | Parameter | Value | |
|---|---|---|---|
| Membrane capacitance | $C$ | 100 | pF |
| Leak conductance | $g_{\mathrm{leak}}$ | 4.5 | nS |
| Resting potential | $v_{\mathrm{rest}}$ | −70 | mV |
| Excitatory reversal potential | $v_{\mathrm{ex}}$ | 0 | mV |
| Inhibitory reversal potential | $v_{\mathrm{inh}}$ | −80 | mV |
| Spike threshold | $v_{\mathrm{thre}}$ | −55 | mV |
| Resetting potential | $v_{\mathrm{reset}}$ | −70 | mV |
| Inhibitory unitary synaptic conductance | $\overline{g}_{\mathrm{inh}}$ | 1 | nS |
| Excitatory unitary synaptic conductance threshold | $\overline{g}_{\mathrm{thre}}$ | 2.5 | nS |
| Synaptic time constant | $\tau$ | 2 | ms |
| Number of excitatory neurons | $N_{\mathrm{ex}}$ | 5,000 | |
| Number of inhibitory neurons | $N_{\mathrm{inh}}$ | 1,250 | |

uniform distribution, uniform random numbers between zero and the double of the mean value were generated.

**Bimodal distribution:** In the generation of bimodal distribution, the distribution was realized by two sets of uniform distribution. The one was in $[0, 0.1]$ and the other was $[M - 0.1, M]$, where $M$ was the maximum value and was the same value as that of the uniform distribution. The half of excitatory synapses were laid in the former range and the others were in the later range.

**Constant distribution:** In constant distribution, all excitatory synapses on the neuron were homogeneous. Their synaptic conductance was the mean value of the log-normal distribution.

Neurotransmitter release probability and its fluctuation of each synapse were functions of its synaptic conductance. Amounts of the neurotransmitter release probability and the fluctuation were given by $1 - e^{-12.3\overline{g}_i}$ and $0.22\overline{g}_i^{-0.5}$ [5].

Firings of both excitatory- and inhibitory-presynaptic terminals were in the manner of the Poisson process, which was characterized by the parameter of the mean firing rate $f$. All inhibitory presynaptic terminals fired at $f$. Excitatory presynaptic terminals had the mean firing rate proportional to their unitary synaptic conductance such as $f\overline{g}_i/\overline{g}_{\mathrm{thre}}$ [7]. Note that, among all types of synaptic weight distribution, the mean firing rate of a population of presynaptic terminals was the same, because the mean synaptic conductance was sustained among all the types of distribution. All the parameters in Eq. (1) are summarized in Tab. 1.

In my experiments to investigate synchrony input effects, synchronous events were generated as the following procedure.

1. Focus on a spike of the strongest synapse as an anchor
2. Randomly selected $n$ excitatory neurons from the other $N_{\mathrm{ex}} - 1$ excitatory neurons
3. Move one spike in individual spike trains of the $n$ selected neurons to the time of the anchor spike of the strongest synapse
4. 2. and 3. were repeated until all spikes of the strongest synapse were synchronized with $n$ spikes of the other synapses

In this operation, a spike was not allowed to be moved to the time of the other spikes in each spike train or to across neurons. Then, the total number of spikes was kept before and after the generation of synchrony events. All simulations were conducted for 100 s with the temporal resolution of 0.1 ms.

To evaluate spike information transmission of individual synapses in the system, the mutual information (MI) was employed. The MI was given by

$$I(X, Y) = \sum_{x,y} P(x)P(y|x) \log_2 \frac{P(y|x)}{P(y)}, \qquad (2)$$

where $P(x)$ and $P(y)$ were probability of input and output and $P(y|x)$ was conditional probability. For the evaluation of the MI, a spike train was regarded as a binary sequence with 10 ms bin width. In a binary sequence, each bin was represented with 1 or 0 whether the bin included spikes or not. Multiple spikes in a bin were permitted and such a bin was also represented with 1.

In addition, extra spikes per synchronous event was also evaluated. Because synchronous events were generated by just moving spikes within individual spike trains with referring an anchor neuron, the total number of presynaptic spikes were maintained before and after the synchronous event generation. Then, the increase or the decrease of postsynaptic spikes must be caused only by the generation of synchronous events. The extra spikes per synchronous event were defined by $(N_{\mathrm{sync}_n} - N)/M$, where $N_{\mathrm{sync}_n}$ was the number of postsynaptic spikes for $n > 0$ and $N$ was that for $n = 0$ in a simulation. $M$ was the occurrence of synchronous events in a simulation.

## 3. Results

### 3.1. Stochastic resonance phenomenon in system with log-normal distribution

First of all, an experiment analogous to Ref. [6] was conducted (Fig. 1). An apparent peak was observed in the function of MI against $f$ only in the case of the log-normal distribution (Fig. 1(A)). This was a typical stochastic resonance phenomenon. The peak located at $f = 1.7$ Hz, indicating that presynaptic spikes were optimally transmitted to the postsynaptic neuron with this parameter. Even though various types of synaptic weight distribution were tested, there was no peak in the other functions. Then, it was considered that the stochastic resonance could occur only in the system with the log-normal distribution.

When $f = 1.7$ Hz, in the case of the log-normal distribution, the mean membrane potential of the postsynaptic neuron was around −60 mV (Fig. 1(B)). Because the log-normal distribution had a heavy tail, the conductance of the strongest synapse in the system had a value close to $\overline{g}_{\mathrm{thre}}$. A spike arrival at a synapse with $\overline{g}_{\mathrm{thre}}$ was able to evoke

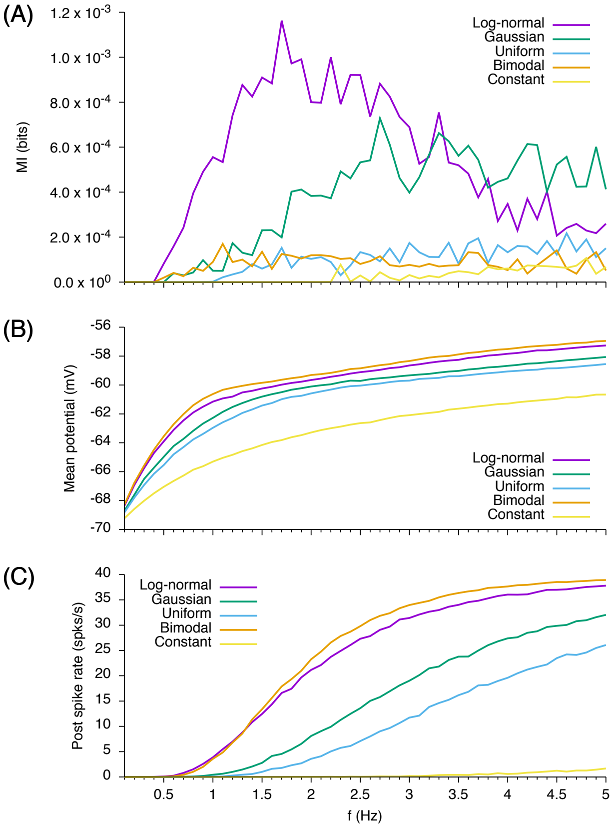


Figure 1: Stochastic resonance in the system with the log-normal synaptic weight distribution. **(A)** MI between spike trains of the presynaptic neuron with the strongest synapse and the postsynaptic neuron. Each value of MI was averaged over 20 simulations. The mean value was estimated by the bootstrap method. **(B)** The mean value of postsynaptic membrane potential in a simulation. **(C)** The mean postsynaptic spike rate per second. The number of spikes for each second was averaged over 100 seconds in a simulation.

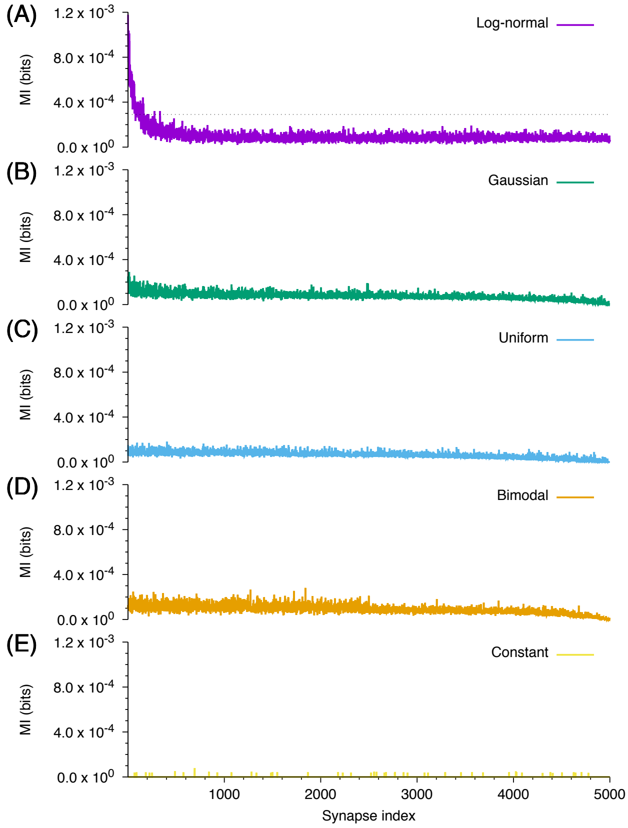


Figure 2: MI of each excitatory synapse at $f = 1.7$ Hz. The stronger synapse was, the younger its index was. In **(A)**, the black dashed horizontal line represented the maximum MI in all the other types of distributions.

about 10 mV excitatory postsynaptic potential when the postsynaptic neuron was in the resting state. In contrast to the log-normal case, the mean potential of the postsynaptic neuron in the case of the bimodal distribution was always higher. However, the MI values in the bimodal case were much smaller than that in the log-normal case (Fig. 1(A)). This was because the largest synaptic conductance in the bimodal distribution was far from $\overline{g}_{\mathrm{thre}}$, and was not enough to activate the postsynaptic neuron. This was the common mechanism for smaller values of MI in the four cases except for the log-normal case. However, even though $f$ increased and the mean membrane potential grew up, the MI increased but the stochastic resonance was not observed in the four cases. Then, the log-normal distribution seemed to be essential for the stochastic resonance in the system.

When the system exhibited the stochastic resonance, the postsynaptic neuron never realized log-frequency firings as observed in the cortex or the hippocampus (Fig. 1(C)). Then, a certain level of a high firing rate of a postsynaptic neuron in the brain might be needed for the stochastic resonance.

### 3.2. A few excitatory synapses work as spike transmitters in log-normal distribution

Next, MI values of individual synaptic terminals was weighed (Fig. 2). In the case of the log-normal distribution, a value of MI decreased as a neuron index increased (Fig. 2(A)). the MI values of about the 100 strongest synapses, corresponding to 2 % of the total number of synapses in the system, were larger than the maximum MI of any other types of synaptic distribution. Then, these synapses might not generate noise, but rather might transmit spike information to the postsynaptic neuron.

In the synaptic distribution except for the log-normal distribution, the MI function was almost flat (Fig. 2(B)–(E)). Therefore, it was considered that there was no distinction of a role among synapses.

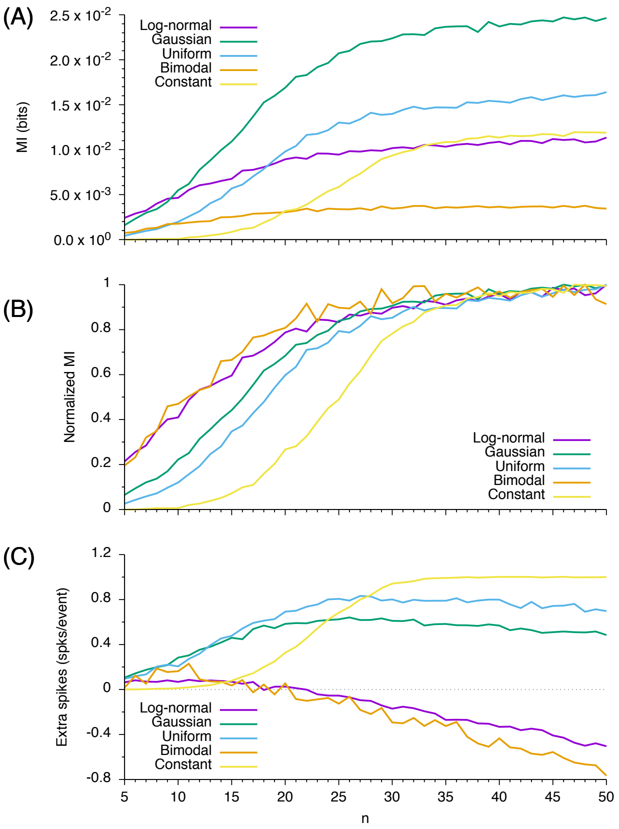


Figure 3: Synchrony sensitivity of the postsynaptic neuron. **(A)** MI between spike trains of the strongest synapse and the postsynaptic neuron for various *n*. Each value of MI was the mean value of 20 simulations. The mean value was estimated by the bootstrap method. **(B)** Same as **(A)** but in each type of synaptic distribution, the values of MI were normalized by the maximum value. **(C)** Extra spikes caused by synchronous events. The plotted values of extra spikes per synchronous event were the mean value of 20 simulations. The mean value was estimated by the bootstrap method. The black dashed horizontal line was a zero-base line, indicating that synchronous events had no effects on the postsynaptic neural activity.

### 3.3. Influence of synchronous events on spike information transmission

Finally, synchronous event effects were evaluated (Fig. 3). In the three types of distribution (the Gaussian, the uniform, and the constant), the spike information transmission of the strongest synapse drastically improved in $10 \le n \le 30$ (Fig. 3(A)). In particular, synchronous events enhanced the spike transmission of the strongest synapse in the system with the Gaussian distribution. Amazingly, the MI value of the strongest synapse in the Gaussian case reached 2.5 times as large as that in the log-normal case for $n = 50$. Synchronous events had smaller effects on the spike information transmission of the strongest synapse in the log-normal and the bimodal distribution than the other three types of distribution. Then, the postsynaptic neuron in the system with the log-normal distribution was less sensitive to synchronous events than in the system with the other types of distribution. This trend was able to be confirmed easier when the MI values were normalized (Fig. 3(B)).

The trend might be related to changes of postsynaptic neural activity caused by the difference of synaptic distribution. For this reason, I also evaluated extra spikes of the postsynaptic neuron as influences of synchronous events (Fig. 3(C)). In the Gaussian, the uniform, and the constant distribution, the extra spikes were always positive and increased as larger synchrony. Contrarily, although the extra spikes were positive until *n* was 20, the extra spikes decreased and were negative for larger *n* in the log-normal and the bimodal distribution. This negative effect of synchronous events might prevent the strongest synapse from enhancing the spike information transmission as observed in the other three types of synaptic distribution.

## 4. Summery and discussions

In this study, I investigated synchrony sensitivity of a single neuron for various types of synaptic weight distribution from a viewpoint of spike information transmission. In my experiments, the stochastic resonance was observed only in the system with the log-normal weight distribution. When synapses log-normally distributed and firing rates of excitatory presynaptic neurons were optimized, synchronous events had smaller effect on the enhancement of spike information transmission at the strongest synapse. This related with the decrease of postsynaptic firings by synchronous presynaptic inputs.

In my experiment without synchronous events, excitatory presynaptic inputs were optimized for the spike transmission at the strongest synapse. Then, the optimality of presynaptic inputs for the strongest synapse might be lost by the generation of synchronous events. My result suggests that the synchrony is efficient on the spike information transmission in populations of weak synapses. However, this is still unclear and further analyses are needed.

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