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Bifurcation Phenomena of Coupled Piecewise Affine System with Hysteresis Properties

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Abstract—We study bifurcation phenomena of the discrete time piecewise Affine systems with hysteresis. Although these systems are categorized as a hybrid systems, not much bifurcation analyses have been done. We firstly investigate the Lozi map with hysteresis and compute its bifurcation sets. And, we research the bifurcations of diffusively-coupled models. As a result, a lot of border-collision bifurcations and corresponding multi-stable periodic areas are found. And the size of chaotic area that generated by hysteresis characteristics can be controlled by coupling.

1. Introduction

The hybrid system is an important theme in a control engineering field and a dynamical system field, and it provoked a great deal of controversy. The normal definition of a hybrid system is a dynamical system whose continuous state is switched by other discrete events. The hybrid system is usually treated as a continuous time system[1]. However, in control engineering field, if a state space has some non-smooth characteristics, the system is called a hybrid system. Here, the discrete-time piecewise affine system is included in hybrid systems[2, 3]. The piecewise affine system is actively researched as a control problem of a mixed logical dynamical system(MLD).

Up to now, the border parameter of piecewise affine system is most constant. Here, a hysteresis characteristic is given to the border parameter of piecewise affine system, that bifurcation problem has never been examined.

In this paper, we study bifurcation phenomena of piecewise affine system with hysteresis properties. In addition, we examine the system that is coupled.

2. System

The Lozi equation[4, 5, 6] is a piecewise linear difference equation obtained from the Heńon map by approximating a square term by an absolute function:

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k) = \begin{pmatrix} 1 + y_k - a|x_k| \\ bx_k \end{pmatrix} \tag{1}$$

We can describe Lozi system(Eq. (1)) like:

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k) = \begin{cases} \boldsymbol{f}_1(\boldsymbol{x}_k) = \begin{pmatrix} 1 + y_k + ax_k \\ bx_k \end{pmatrix} & (\boldsymbol{x}_k \in S_1) \\ \boldsymbol{f}_2(\boldsymbol{x}_k) = \begin{pmatrix} 1 + y_k - ax_k \\ bx_k \end{pmatrix} & (\boldsymbol{x}_k \in S_2) \end{cases} \tag{2}$$

$$S_1 = \left\{(x, y) \in \boldsymbol{R}^2 | x < 0\right\}$$
$$S_2 = \left\{(x, y) \in \boldsymbol{R}^2 | x \geq 0\right\}$$

It can be considered the model that two linear changing systems. In Eq. (2), $x = 0$ is fixed border. We define the model that Lozi map with hysteresis,

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}) = \begin{cases} \boldsymbol{f}_1(\boldsymbol{x}_k) = \begin{pmatrix} 1 + y_k + ax_k \\ bx_k \end{pmatrix} & (\boldsymbol{x}_k \in S_1) \\ \boldsymbol{f}_2(\boldsymbol{x}_k) = \begin{pmatrix} 1 + y_k - ax_k \\ bx_k \end{pmatrix} & (\boldsymbol{x}_k \in S_2) \end{cases} \tag{3}$$

$$S_1 = \left\{(x, y) \in \boldsymbol{R}^2 | x \leq H_1\right\}$$
$$S_2 = \left\{(x, y) \in \boldsymbol{R}^2 | x \geq H_2\right\}$$

if $\boldsymbol{x} \notin S_1$ then $\boldsymbol{x} \in S_2$, if $\boldsymbol{x} \notin S_2$ then $\boldsymbol{x} \in S_1$.

In this model, H_1 and H_2 are parameters of the hysteresis characteristic, and $H_1 > H_2$.

Figure 1 is the state space of hysteresis system, and Fig. 2 is functions and regions of state.

We couple two systems that Eq. (3) by differential coupling, as follows:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{f}(\boldsymbol{x}_k) + \phi(\boldsymbol{x}_k - \boldsymbol{y}_k) \\ \boldsymbol{y}_{k+1} &= \boldsymbol{f}(\boldsymbol{y}_k) + \phi(\boldsymbol{y}_k - \boldsymbol{x}_k) \end{aligned} \tag{4}$$

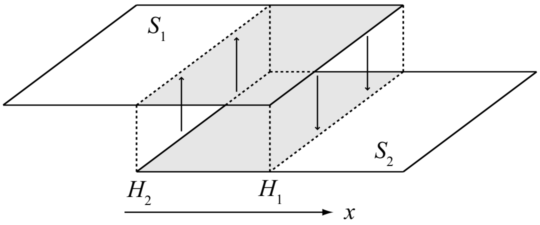


Figure 1: State space

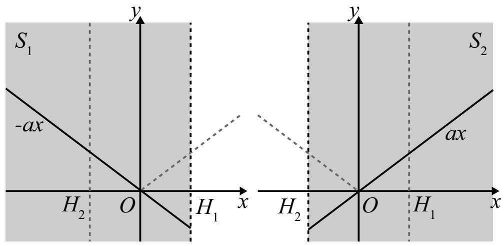


Figure 2: Region and function

3. Bifurcation

In this section, We shows the bifurcation phenomena of Lozi system with hysteresis. First, Fig. 3 show the bifurcation diagram and chaotic attractor of Lozi map. In the bifurcation diagram,The classification of stable periodic region by colors of Tb. 1. Bifurcation of Lozi map has already been analyzed[7].

Table 1: Color chart of the stable periodic region of bifurcation diagrams

Period-1	Period-2	Period-3	Period-4	Period-5
Period-6	Period-7	Period-8	Period-9	Period-10
Period-11	Period-12	Period-13	Chaotic	Divergence

Fig. 4 show the bifurcation diagram of hysteresis model(Eq. (3)). We define the parameter $H = H_1 = -H_2$.

In Fig. 3, three kinds of areas of a fixed point region, 2-periodic point region, and chaos region are seen, however the multi-periodic region more than period-3 and the chaotic region have been generated between the fixed point area and two cycle area in fig Eq. (3). This is because the boundary becomes two (H_1 and H_2) by hysteresis characteristics, and border-collision bifurcation was generated for the hysteresis border.

Fig. 5 show the bifurcation diagram of hysteresis model(Eq. (4)). In this figure, coupling factor $\phi = 0.1$, initial points $\boldsymbol{x}_0 = (0.3, 0.1)$ and $\boldsymbol{y}_0 = (-0.5, -0.3)$.

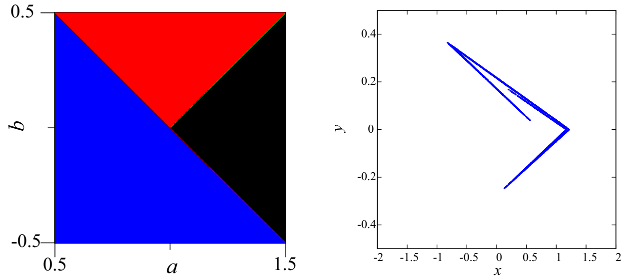


Figure 3: Bifurcation diagram and chaotic attractor of the Lozi map

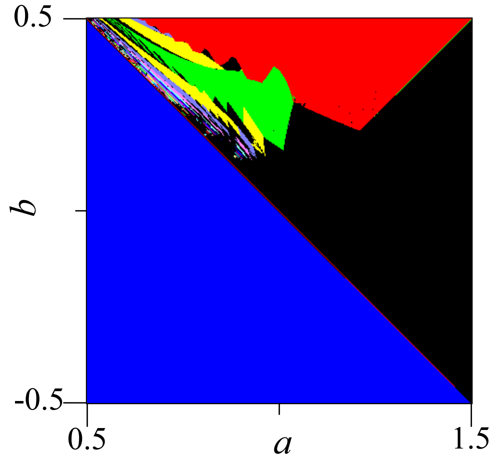


Figure 4: Bifurcation diagram of the Lozi map with hysteresis($H = 0.2$)

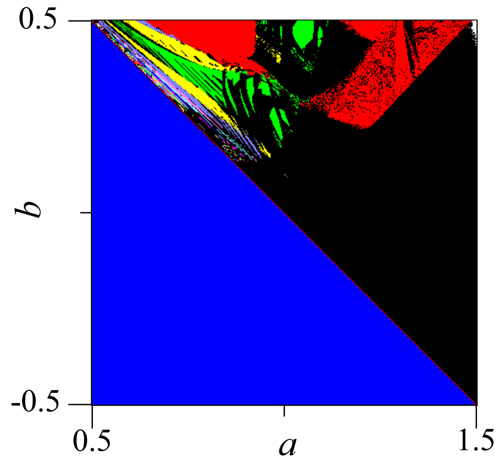


Figure 5: Bifurcation diagram of the coupled hysteresis system($H = 0.2$)

In Fig. 5, a chaotic region has been generated in the 2-periodic region. Fig. 6 is the one dimension bifurcation diagram of no-coupled model, and Fig. 7 is the one dimension bifurcation diagram of coupled model. In Fig. 7, chaos can be seen in the near of $a = 1.1$. Chaos is occurred by the coupling.

Fig. 8 shows the chaotic attractor, the attractor (a) of no-coupled model is composed of the combination of straight lines, however it has been blurred in (b).

Fig. 9 shows phase portraits of the coupled hysteresis system, there is the chaotic region(c) between the periodic point regions(a,b,d).

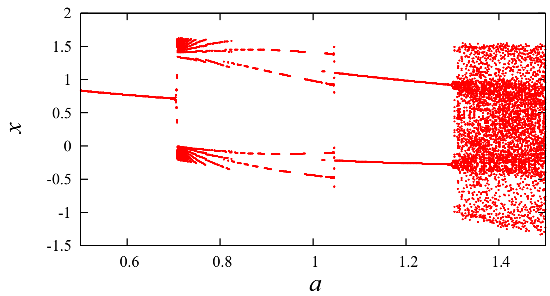


Figure 6: One dimension bifurcation diagram of the no-coupled model($b = 0.3$, $H = 0.2$)

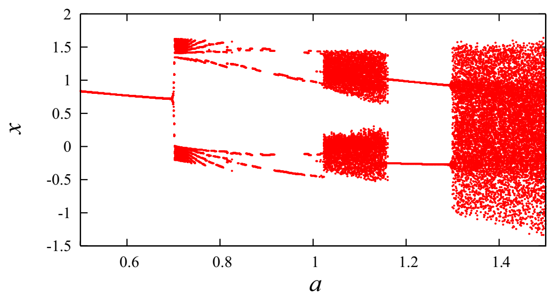
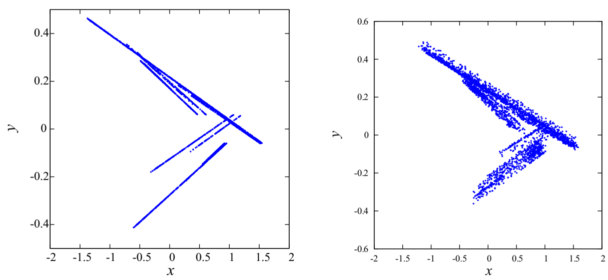


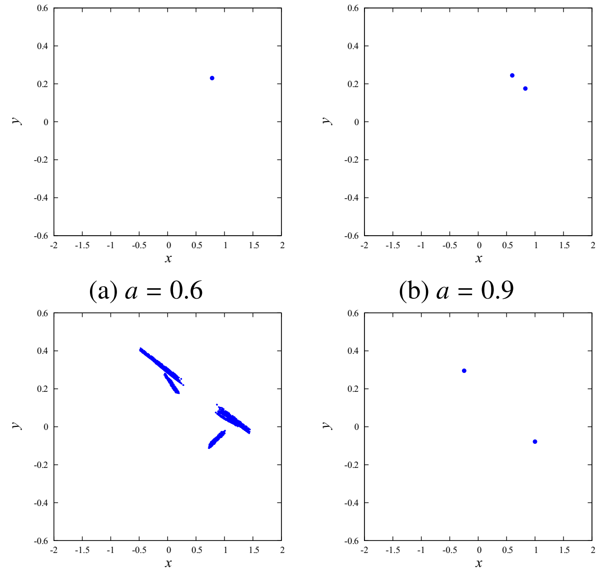
Figure 7: One dimension bifurcation diagram of the coupled model($b = 0.3$, $H = 0.2$, $\phi = 0.1$)



(a) (b)

Figure 8: Chaotic attractor, (a)no-coupled model, (b)coupled model

Fig. 10 is bifurcation diagrams of coupled hysteresis system that coupling parameter ϕ is changed, and Fig. 11



(a) $a = 0.6$ (b) $a = 0.9$

(c) $a = 1.1$ (d) $a = 1.2$

Figure 9: Phase portrait of the coupled hysteresis system ($b = 0.3$, $H = 0.2$, $\phi = 0.1$)

is a closeup for the characteristic region of Fig. 10. When coupling parameter ϕ is small, the chaotic region grows, however ϕ is increased, the stable periodic region covers over the chaotic region. The chaotic region can be changed into the stable periodic region by coupling.

Fig. 12 is bifurcation diagrams of coupled hysteresis system that hysteresis parameter H is changed, and Fig. 13 is a closeup for the characteristic region of Fig. 12.. When the width of hysteresis is enlarged, we can see that the multi-periodic region (chaotic region) between the fixed point region and the 2-periodic point region expend by border-collision bifurcations.

4. Conclusion

We study bifurcation phenomena of the discrete time piecewise Affine systems with hysteresis. In this study, we define the Lozi map with hysteresis characteristics, and we couple it system. As a result, the border has increased by the hysteresis characteristic, more than 2-periodic region and chaotic region appeared by border-collision bifurcation. The chaotic region can be enlarged or reduced by coupling.

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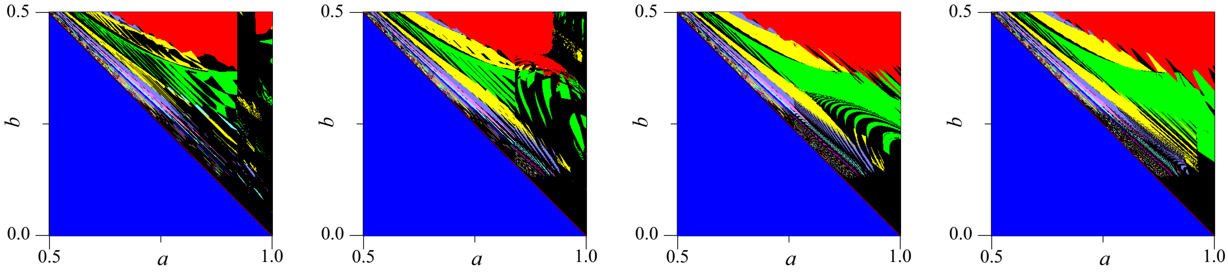
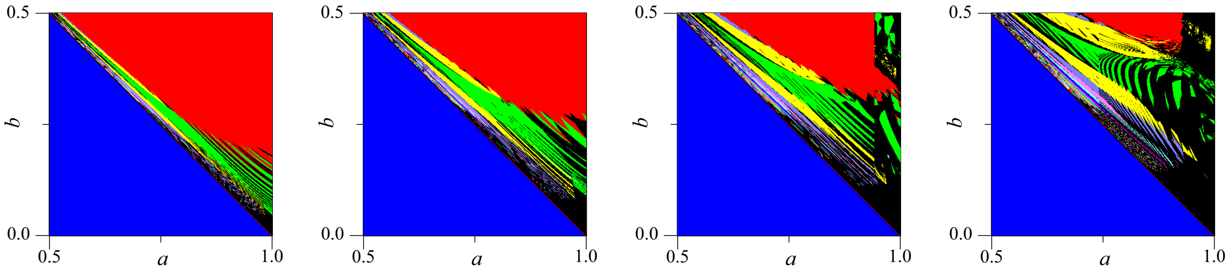


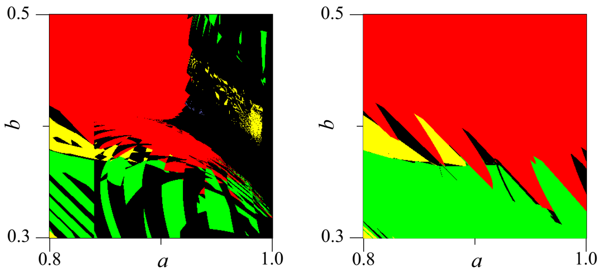
(a) ϕ = 0.05 (b) ϕ = 0.1 (c) ϕ = 0.15 (d) ϕ = 0.2

Figure 10: Bifurcation diagram of the coupled hysteresis system ϕ is changed



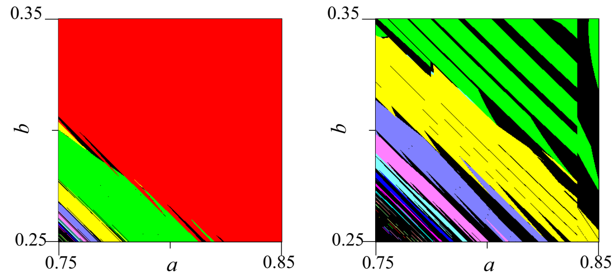
(a) H = 0.05 (b) H = 0.1 (c) H = 0.15 (d) H = 0.2

Figure 12: Bifurcation diagram of the coupled hysteresis system H is changed



(a) (b)

Figure 11: Closeup of Fig. 10



(a) (b)

Figure 13: Closeup of Fig. 12

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