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Control of intermittent chaos in semiconductor laser with short external cavity

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Abstract– We predict the occurrence of intermittent chaos in a semiconductor laser with a short external cavity. We observe the trajectory of intermittent chaos in the phase space to analyze the characteristics of chaotic bursts. We succeed in preventing the occurrence of intermittent chaos by perturbing the optical feedback phase in the semiconductor laser.

1 Introduction

Strong deviations from normal behaviors can occur in natural and engineered systems, which is called extreme events [1]. Extreme events include many phenomena that give great impacts on our societies, such as earthquakes, typhoons, and infectious diseases. These phenomena can be reproduced using a numerical model, such as plate movement and volcanic activity for earthquakes. Infectious diseases can also be described by a numerical model and the spread of infectious diseases can be predicted. In numerical simulations, an addition of a small perturbation may prevent the extreme events [1]. In addition, extreme events can be predicted using machine learning techniques [2-4].

Intermittent chaos is a similar phenomenon to extreme events that can occur in a semiconductor laser with optical feedback [5]. When intermittent chaos occurs, the laser amplitude changes with a low amplitude (called laminar), and a high amplitude (called burst). Intermittent chaos is observed in a certain range of parameter values, and mode hopping dynamics of the external cavity modes results in intermittent chaos. It is difficult to predict the occurrence of intermittent chaos owing to the unpredictability of chaotic temporal waveforms. If intermittent chaos can be predicted and prevented, it may have a large impact on the application of fast physical random number generation using chaotic lasers.

Intermittent chaos is a good model for analyzing and clarifying the mechanism of extreme events. The prediction of extreme events has been reported in chaotic systems [6-8]. However, the prediction and prevention of intermittent chaos in semiconductor lasers have not been reported yet.

In this study, we numerically investigate intermittent chaos in a semiconductor laser with optical feedback. We find the conditions for the occurrence of burst oscillations of intermittent chaos. We also prevent the occurrence of burst oscillations by adding a perturbation to the optical feedback phase.

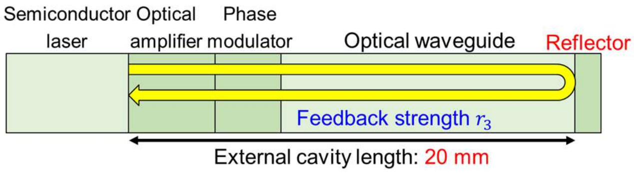

Fig. 1 Numerical model of a semiconductor laser with a short external cavity.

2 Numerical model

Figure 1 shows a numerical model of a photonic integrated circuit, consisting of a semiconductor laser, optical amplifier, phase modulator, and short external cavity. The external cavity length is set to 20 mm to observe intermittent chaos. The corresponding delay time is 133 ps.

The Lang-Kobayashi equations [5,9] for the dynamics of semiconductor lasers can be expressed as follows.

$$\frac{dE(t)}{dt} = \frac{1+ia}{2}\left\{\frac{G_N(N(t)-N_0)}{1+\epsilon|E(t)|^2} - \frac{1}{\tau_p}\right\}E(t) + \kappa E(t-\tau)\,exp[i(-\omega t+\phi_0)] \quad (1)$$

$$\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - \frac{G_N(N(t)-N_0)}{1+\epsilon|E(t)|^2}|E(t)|^2 \quad (2)$$

$$\kappa = \frac{(1-{r_2}^2)r_3}{\tau_{in} r_2} \quad (3)$$

Where $E(t)$ is the slowly varying complex electric-field amplitude and $N(t)$ is the carrier density. The term $\kappa E(t-\tau)\,exp[i(-\omega t+\phi_0)]$ in Eq. (1) indicates the feedback light and includes the term of the phase shift ϕ_0.

Intermittent chaos occurs when the parameter values are changed, and the reflectivity r_3 of the feedback light is a crucial parameter to observe intermittent chaos.

3 Intermittent chaos

Figure 2(a) shows the temporal waveforms of the laser intensity and optical frequency shift when the parameter is set to $r_3 = 0.0894$. The temporal waveforms show the occurrence of intermittent chaos. Laminar and burst oscillations are observed clearly in these temporal waveforms. It is worth noting that the optical frequency shift is clearly separated between the laminar and the burst oscillations.

Figure 2(b) shows the attractor of the intermittent chaos in the phase space of the optical frequency shift and carrier density. When the temporal waveform shows the laminar, the trajectory of the attractor is located in the left part. This is because the trajectory is attracted to the steady state solutions (modes) indicated by the black dots. When the temporal waveform shows the bursts, the trajectory moves to the right part of the attractor. In fact, mode hopping occurs and the trajectory moves to the neighboring steady state solutions via unstable steady state solutions (anti-mode), indicated by the white dots. Therefore, the attractors of the laminar and burst dynamics can be distinguished in the phase space.

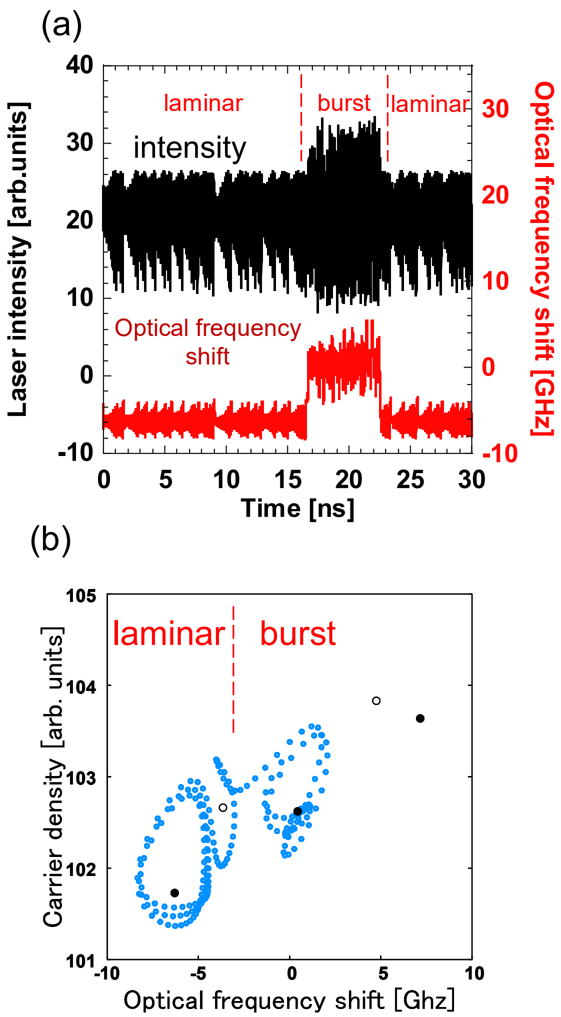

Fig. 2 (a) Temporal waveforms of laser intensity (black) and optical frequency shift (red). (b) Attractor of intermittent chaos in the phase space. Black and white dots indicate the external cavity modes.

4 Prevention method

We consider that the bursts can be prevented by adding a perturbation when the trajectory moves from left to right on the chaotic attractor. Figure 3 show the trajectory and the steady state solutions. The trajectory is located near one of the steady state solutions (black dots) and moves to another steady state solution (black dots) via unstable steady state solutions (white dots).

We introduce the conditions to observe burst oscillations as follows. First, the carrier density, which is indicated by the vertical axis of the attractor, shows a local minimum value. Then, the trajectory of the local minimum is located within the red dotted lines in Fig. 3. The trajectory moves to the right near the anti-mode (white dot). These conditions can be used for the prevention of the bursts.

We apply a perturbation to the optical feedback phase of the semiconductor laser when the above-mentioned phenomenon is observed. To prevent the occurrence of the bursts, a phase modulation of $-\pi/6$ is applied for 40 ps. By perturbing the optical feedback phase, the steady state solutions (black dots) are shifted to the right direction in the phase space, and the trajectory can move back to the steady state solutions to prevent the occurrence of the bursts.

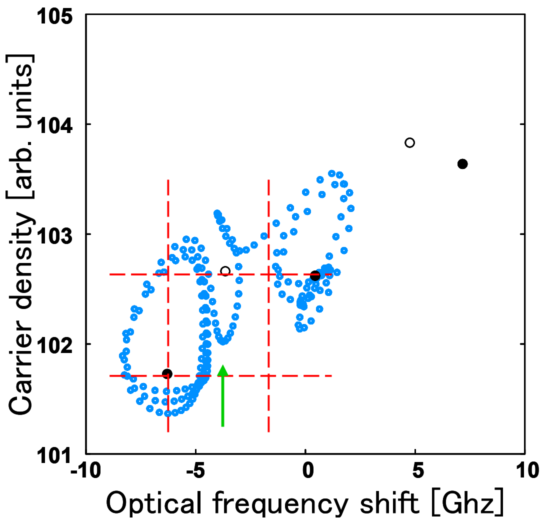

Fig. 3 Prevention method of the burst oscillation. The green arrow indicates the position for adding the perturbation.

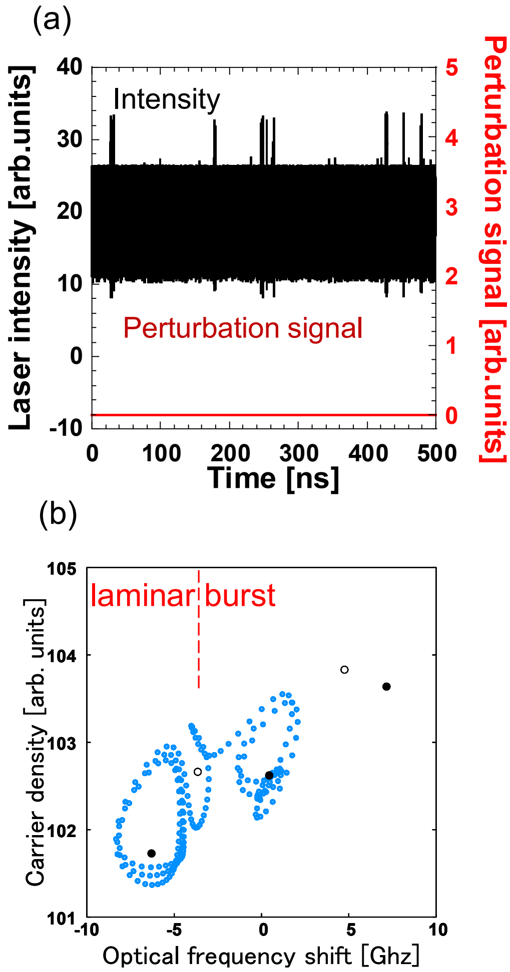

Fig. 4 Example of (a) temporal waveform of the laser intensity and perturbation signal and (b) attractor in the phase space without perturbation for prevention.

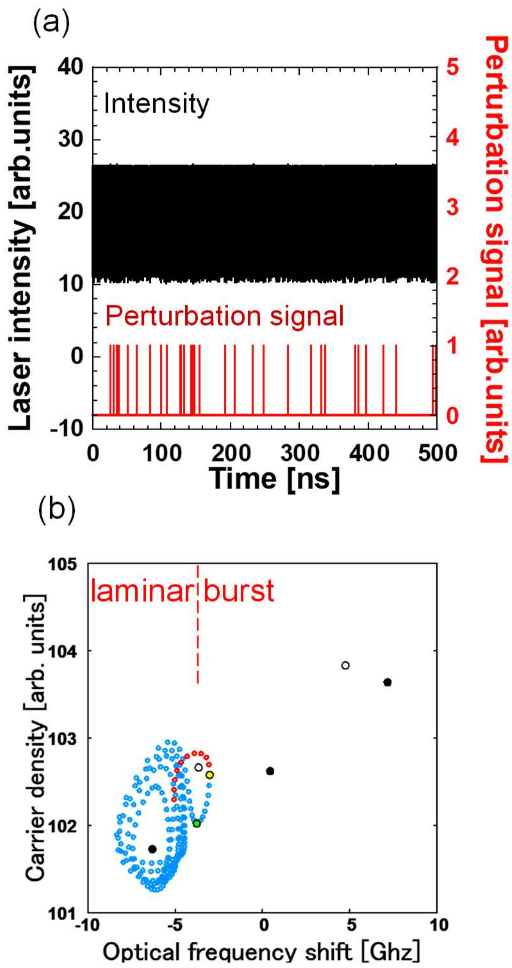

Fig. 5 Example of (a) temporal waveform of the laser intensity and perturbation signal and (b) attractor in the phase space with perturbation for prevention.

5 Results of prevention

Figure 4(a) shows an example of the temporal waveforms without perturbation for prevention. Both laminar and burst oscillations are observed without perturbation. Figure 4(b) shows the corresponding attractor. The trajectory is located in the wide area of the attractor.

We introduce the perturbation to the phase shift of the feedback light before the burst occurs. Figure 5(a) shows an example of the temporal waveform and the attractor with perturbation for prevention. Burst oscillations disappear in the temporal waveform in the presence of the perturbations. Figure 5(b) shows the corresponding attractor. After the perturbation, the steady state solution moves to the right, and the trajectory goes back to the original laminar attractor, as shown in Fig. 5(b). Therefore, we succeed in preventing burst oscillations by perturbing the optical feedback phase.

6 Prevention of bursts for different feedback strength

Next, we investigate the prevention of the bursts at different parameter values. Figure 6(a) shows the bifurcation diagram of the laser intensity as the feedback strength is changed. Intermittent chaos is observed in the parameter region indicated by the red arrow.

We apply the perturbation to the feedback phase at the conditions of different feedback strengths. Figure 6(b) shows the number of the bursts as the feedback strength is changed without and with prevention. Without perturbation (the black line in Fig. 6(b)), the number of bursts increases as the feedback strength is increased. However, when the perturbation is applied to the feedback phase (the red line in Fig. 6(b)), the number of bursts becomes zero. Therefore, we succeed in preventing the burst oscillations of intermittent chaos at different feedback strengths.

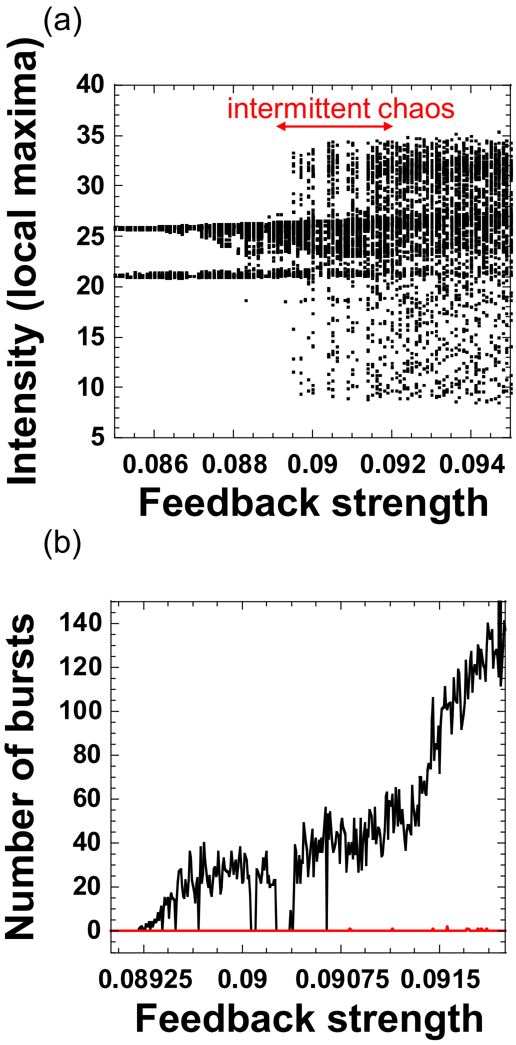

Fig. 6 (a) Bifurcation diagram of laser intensity as the feedback strength is changed. (b) The number of the bursts as the feedback strength is changed without (black) and with(red) prevention.

7 Conclusions

We investigated intermittent chaos in a semiconductor laser with a short external cavity. We identified the conditions for applying the perturbation to prevent the occurrence of burst oscillations. We succeeded in preventing burst oscillations by perturbing the optical feedback phase.

8 Acknowledgment

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