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Response of the *Physarum* Solver to a Sinusoidal Stimulation in a Path Finding Stage for a Transport Network

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Abstract—"*Physarum* solver" is the shortest path finding algorithm for a transport network mimicking an adaptation process in true slime mold developed by Tero *et al.* (2006) Physica A, 363. We introduce a periodic forcing term to the solver, and investigate a response to the stimulation. With the forcing term, there exists such a property that the required time to reach steady state becomes short. Also there exists trade-off between the convergence time and accuracy for the shortest path finding process with respect to the amplitude of the forcing term.

1. Introduction

Information processing in biological systems has attracted many researchers' attentions. From the engineering point of view, living organisms often behave effectively. As an example, they alter their behavior in accordance with varying environmental conditions. Novel methods which imitate such ability are proposed in various fields.

Nakagaki *et al.* found that the true slime mold *Physarum Polycephalum* is capable for finding shortest path in a transport network [1]. The organism does not utilize a central organ such as brain for making a decision on an issue, because it is an unicellulate. Instead, repeatedly contracting and relaxing oscillation of their body thickness (which is supposed to be induced by metabolism) has a crucial role [2]. In this sense, the organism is an attractive research subject of information processing of oscillatory media [3].

From an observation of the organism, a novel mathematical model mimicking their smart behavior has been derived by Tero *et al.* [4, 5]. An electric circuit analogue of the method is also proposed [6]. We are interested in how dynamics for path finding process is influenced by a forcing term. Since the organism utilize a oscillation pattern, the forcing term may generate an action for the finding process.

In this paper, we investigate an effect of a periodic forcing term to the solver [4]. We introduce a sinusoidal noise to the dynamics, and employ an amplitude of the forcing term as a key parameter. When there exist competing multiple paths in a transport network, the dynamics represents a steady state at an early stage of path finding process compared to the case where no periodic perturbation is applied. Also there exists trade-off between the convergence time and accuracy for the shortest path finding process with respect to the amplitude of the forcing term.

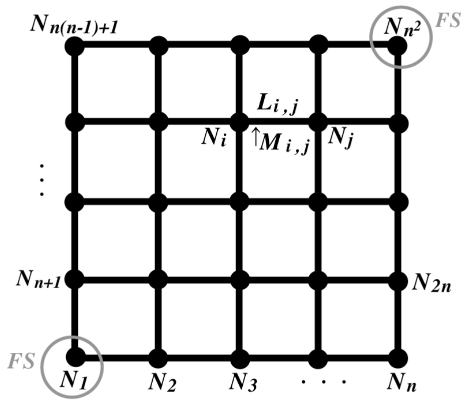


Figure 1: Lattice structure for a transport network.

2. Methods

Let us consider a 2-dimensional lattice structure for a transport network, as shown in Fig.1. We define nodes N_i at branching points, and the nodes N_i and N_j are connected by segments ($M_{i,j}$). The length of $M_{i,j}$ is denoted by $L_{i,j}$. If we assume $(n \times n)$ square lattice structure, the total number of nodes($\equiv N$) and segments($\equiv M$) are n^2 and $2n(n-1)$, respectively. We assume that two food sources are set at the nodes which are indicated with the label "*FS* " in Fig.1, namely at N_1 and N_{n^2}.

When the plasmodium of *Physarum Polycephalum* is put inside the structure, it forms a network with branch of tubes where the fluid like nutrients and oxygen are transported. Eventually, the organism connects the food sources with shortest path. In the following, we review the model in [5] for finding shortest path in the transport network, and introduce a periodic perturbation to this dynamics.

The flux in the tube through $M_{i,j}$ from N_i to N_j is expressed by the variable $Q_{i,j}$. Assuming Poiseuille flow in the tube, the flux $Q_{i,j}$ is written as

$$Q_{i,j} = \frac{D_{i,j}}{L_{i,j}}(p_i - p_j), \tag{1}$$

where $D_{i,j}$ represents the conductivity of the edge $M_{i,j}$, and p_i and p_j are the pressure at N_i and N_j, respectively. Regarding the $Q_{i,j}$, the following Poisson equation is derived because the total amount of liquid is conserved at each node,

$$\sum_i Q_{i,j} = \begin{cases} -1 & \text{for } j = 1, \\ +1 & \text{for } j = n^2, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Note that N_1 and N_{n^2} act as a source and a sink, respectively in this case.

To model the adaptation of tubular thickness to $Q_{i,j}$, the following dynamics called as "adaptation equation" is derived as follows,

$$\frac{dD_{i,j}}{dt} = f(|Q_{i,j}|) - D_{i,j}, \tag{3}$$

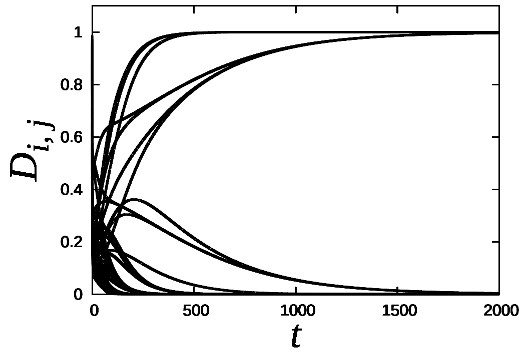
where the function $f(|Q_{i,j}|)$ is a monotonically increasing continuous function with $f(0) = 0$. In [5], two types of function for $f(|Q_{i,j}|)$ are employed. One is $f(Q) = Q^\mu$, and the other is $f(Q) = (1+a)Q^\mu/(1+aQ^\mu)$, where $\mu(> 0)$ and a are control parameters of the feedback regulation between the thickness of a tube and the flux. In this study, we assume the former function for $\mu = 1$. In this case, the method is called as "*Physarum* solver" because the shortest path always survives in any initial state.

In this study, we introduce a periodic forcing term to Eq.(3) as follows,

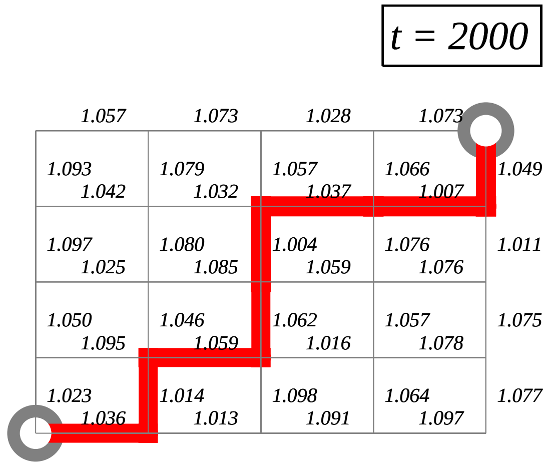
$$\frac{dD_{i,j}}{dt} = |Q_{i,j} + E \sin \omega t| - D_{i,j}, \tag{4}$$

where E and ω are the amplitude and angular frequency, respectively, of the periodic forcing term. We focus on the dynamics when the periodic forcing term is applied.

In the following results, we solve Eq.(2) (which yields a linear equation system with sparse symmetric matrix) by the conjugate gradient method [7], and numerical integrations of Eq.(4) are conducted using the fourth-order Runge-Kutta method with a step size 0.01. The initial states of $D_{i,j}$ are chosen by uniform random numbers within the interval $[0.5, 1.0]$, and we adopt the same values for $D_{i,j}$ throughout this paper. We assume the $n = 5$ case ($N = 25$ and $M = 40$). The values of $L_{i,j}$ are chosen by uniform random numbers within the interval [1.0, 1.1]. Note that multiple competing paths often appear in the situation despite of a small-scale lattice.



(a)



(b)

Figure 2: Example of (a) timeseries plot of $D_{i,j}$ and (b) steady state with the actual values of $L_{i,j}$, when no forcing term is applied ($E = 0$).

3. Results

First we review the result for $E = 0$. Figure 2 shows an example of timeseries plot of $D_{i,j}$, and the steady state in the path finding process. The actual values of $L_{i,j}$ are presented in the neighborhood of the corresponding $M_{i,j}$ in Fig.2 (b). In Fig.2 (b), the thicknesses of the red lines represent the conductivities $D_{i,j}$ of the corresponding segments of tube. From the figure, it is clear that only a shortest path remains eventually. In the case, a total length of the shortest path ($\equiv L_s$) is 8.268. The $D_{i,j}$ is closely related to the flux $Q_{i,j}$. Figure 3 presents the values of the pair $(|Q_{i,j}|, D_{i,j})$ of the corresponding segments $M_{i,j}$. In this case, all the pairs are separated into two groups at the steady state. One is the origin, and the other is (1, 1) in the $(|Q_{i,j}| - D_{i,j})$ plane.

Next we apply the periodic forcing term to this dy-

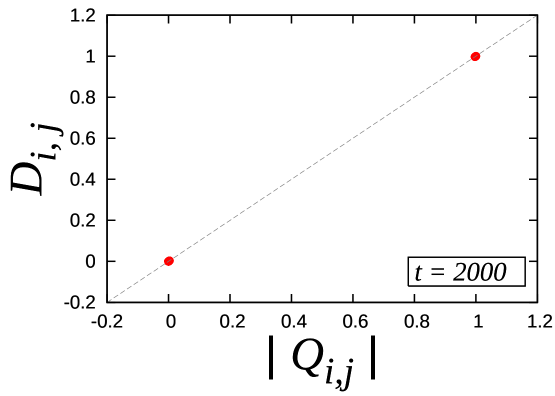


Figure 3: The values of the pair $(|Q_{i,j}|, D_{i,j})$ at the same time in Fig.2(b). The dotted line indicates the line where $|Q_{i,j}| = D_{i,j}$

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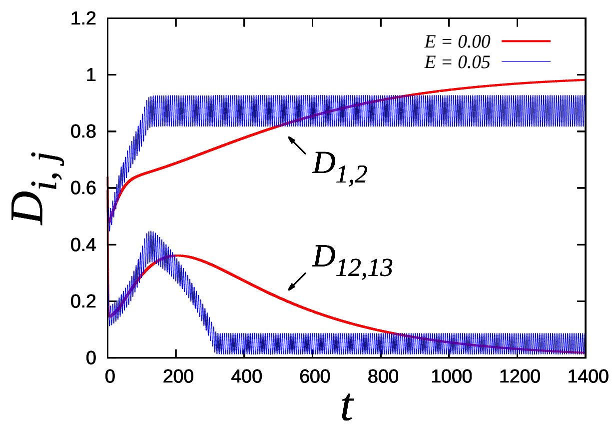
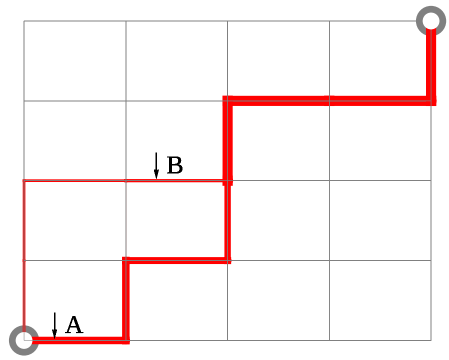


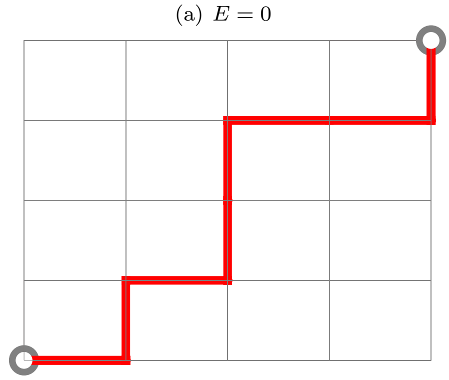
Figure 4: Timeseries of $D_{1,2}$ and $D_{12,13}$. The red and blue line for $E = 0$ and $E = 0.05$, respectively.

namics. Figure 4 represents the comparison between $E = 0$ and $E = 0.05$ case with $\omega = 1.0$. In the figure, the timeseries of $D_{1,2}$ and $D_{12,13}$ are shown, where the red and blue line correspond to the result for $E = 0$ and 0.05, respectively. Comparing with both cases, the following properties can be seen. (I) With the periodic forcing term, the conductivity settles down to a steady state at earlier stage than the case of $E = 0$. (II) The steady state becomes a periodic one by the forcing term, whereas it is horizontal when no periodic forcing term is applied.

Figure 5 shows the state of $D_{i,j}$ at $t = 327$ in both cases. When no forcing term is applied, there exist two competing paths, because the shortest path finding process is underway as shown in Fig.2(a). On the other hand, for $E = 0.05$ only the shortest path seems to be remained at the time. The $D_{1,2}$ in Fig.4 cor-



(a) $E = 0$



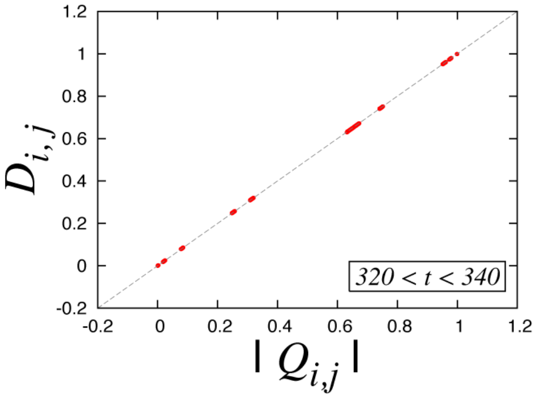
(b) $E = 0.05$

Figure 5: Comparison between the conductivities for $E = 0$ and $E = 0.05$ at $t = 327$ in Fig.4.

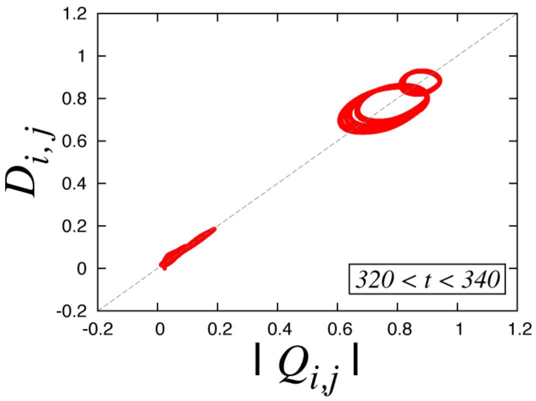
responds to the conductivity at the segment which is indicated with the label “A” in Fig.5(a). Similarly, the segment for $D_{12,13}$ is indicated as “B”. Paying attention to the variation of $D_{12,13}$ in Fig.4, the conductivity at B decays at an early stage when the periodic perturbation is applied with $E = 0.05$. Meanwhile, $D_{12,13}$ for $E = 0$ relates to some extent. That is why there exists only the shortest path for $E = 0.05$ at $t = 327$ as shown in Fig.5, which explains the property (I).

We should pay attention to the property (II), too. Figure 6 presents the dots of the pair $(|Q_{i,j}|, D_{i,j})$ for $320 < t < 340$. For $E = 0$, the dots are distributed along the dotted line where $|Q_{i,j}| = D_{i,j}$ as shown in Fig.6(a). For $E = 0.05$, the dots form into some arches as shown in Fig.6(b) because $D_{i,j}$ is perturbed by the periodic forcing term. Therefore, the tubular thicknesses in Fig.5(b) change periodically even at the steady state. In the case, some conductivities other than that for the shortest path develop to an extent.

Increasing the value of E gradually, interesting property can be observed. Figures 7(a) and (b) present the state of $D_{i,j}$ at the same time in Fig.5, for $E = 0.06$ and $E = 0.08$, respectively. In the case of $E = 0.06$, certain route appears, and a total length of the route (L) is 8.278. Compared with the total length of the



(a) $E = 0$



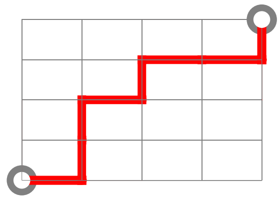
(b) $E = 0.05$

Figure 6: The dots of the pair $(|Q_{i,j}|, D_{i,j})$ for $320 < t < 340$.

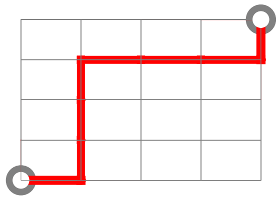
shortest route L_s, this route has slightly long length (the difference from L_s ($\equiv \Delta L, L - L_s$) is 0.010). It should be noted that the route contains the segment B. That is, the dynamics modifies the found path as the competing path. For $E = 0.08$, an additional change of the found path is observed as shown in Fig.7(b). In this case, ΔL is 0.033. As far as our numerical result is concerned, a required time to reach steady state is shorter for larger E. However, there is trade-off between convergence time and accuracy for the shortest path finding process. In addition, for comparatively large E, the dynamics is not useful due to the property (II) for the path finding scheme.

4. Concluding remarks

We investigated the property of a path finding process in the *Physarum* solver when the periodic perturbation is applied. One of the interesting characteristics is that a convergence time to reach steady state becomes shorter compared to the case with no periodic perturbation. Such a characteristic is observed



(a) $E = 0.06$



(b) $E = 0.08$

Figure 7: Alternation of the found path regarding the values of E.

for various combinations of $L_{i,j}$ and the initial states $D_{i,j}$, especially such a case that there exist multiple competing paths in a transport network. As a future problem, we will investigate a frequency dependence of the forcing term related to the path finding process.

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