2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Synchronization Phenomena in Complex Networks by Using Parametrically Excited Oscillators with Involvement of Small Mismatch

Kosuke Oi†, Yoko Uwate† and Yoshifumi Nishio†

†Department of Electrical and Electronic Engineering, Tokushima University
2-1 Minamijosanjima, Tokushima, 770-8506, Japan
Email: {ooi, uwate, nishio}@ee.tokushima-u.ac.jp

Abstract— Investigation of synchronization phenomena in chaotic oscillator is expected to application to physics, biology, medical science, and engineering equivalent. In this study, we focus on chaotic oscillation by applying parametrically excite to van der Pol oscillator, and apply small mismatch. We investigate synchronization phenomena in random network with hub by using computer simulation. In addition, we investigate more detailed effects of small mismatch added in hub. As a result, the small mismatch to hub has effect on synchronization phenomena in network.

1. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. We consider that it is important to investigate the synchronization phenomena of coupled oscillators for the future engineering application. The coupled van der Pol oscillator is one of coupled oscillators, and synchronization generated in the system can model certain synchronization of natural rhythm phenomena. The van der Pol oscillator is studied well because it is expressed in simple circuit. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In our research group, we have investigated synchronization of parametrically excited van der Pol oscillators [6]. By carrying out computer calculations for two or three subcircuits case, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed.

However, we have investigated the only simple network models. It is important to investigate more complex network for the broad-ranging future engineering applications. In our previous study, we investigated synchronization phenomena in random network by using parametrically excited van der Pol oscillators with small mismatch [7]. First, two oscillators are combined by resisters in one-dimensional coordinate system. We have investigated synchronization between two oscillators by changing the value of coupling strength. We also investigated synchronization phenomena of complex network by applying this circuit model to ten coupled oscillators as random network model with hub which include small mismatch in amplitude of parametrically excitation to each of node. We focus on effect of small mismatch of hub in random network through this previous study by reason of we expect this mismatch make an effect to synchronization phenomena in random network.

In this study, we investigate full synchronization in random network by adding several types of small mismatch to hub of random network. We confirm that the full synchronous state depends on the small mismatch in hub.

2. Circuit Model

Figure 1 shows the parametrically excited van der Pol oscillator.

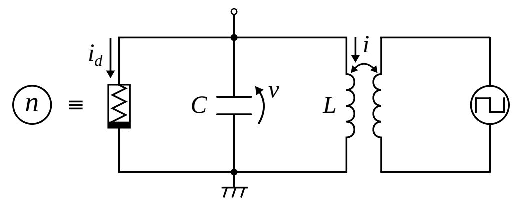


Figure 1: Parametrically excited van der Pol oscillator.

This circuit includes a time-varying inductor L whose characteristic is given as the following equation.

$$L = L_0\gamma(\tau). \tag{1}$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 2, and its amplitude and angular frequency are termed α and ω, respectively. The α expresses the amplitude of function

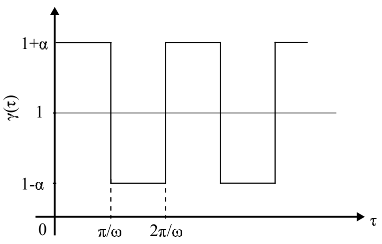


Figure 2: Function relating to parametrically excitation.

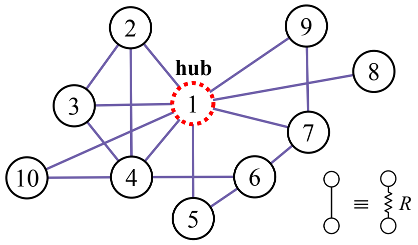


Figure 3: A random network model with a hub.

relating to parametrically excitation. The $v-i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

The normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} &= \varepsilon(x_n - x_n^3) - y_n + \delta \displaystyle\sum_{k\in S_n}(x_k - x_n) \\ \dfrac{dy_n}{d\tau} &= \dfrac{1}{\gamma(\tau)} x_n \quad (n = 1, 2, 3, \cdots, 10)) \end{cases} \tag{3}$$

S_n is the set of nodes which are directly connected to the node n.

3. Random Network with Hub

We apply the circuit model to ten coupled oscillators as random network model with hub. The ten coupled oscillator model is shown in Fig. 3. In this circuit system, there is a hub (1st oscillator) which is connected to many oscillators. Here, the small mismatch is added to α which is corresponding to the amplitude of the function relating to parametrically excitation. We investigate effect of small mismatch of hub in random network by considering three cases (Mismatch of hub: zero, random and constant). The small mismatch is generated by random and the range of the mismatch is set to [-0.01:0.01]. Each table (Table 1, Table 2 and Table 3) shows small mismatch pattern which is used in this computer simulations. In these Tables, w expresses the pattern of small mismatch. In Table 1, the value of small mismatch in hub (α_1) is zero. In Table 2, the value of α_1 is random. In Table 3, the value of α_1 is fixed with 0.01.

Figure 4 shows attractors and lissajous of each node and internode when w=1, $\alpha = 0.958$ and $\delta = 1.3$. This figure expresses a part of example of full synchronization. Figure 5 shows the simulation results of are mechanism of synchronization. The horizontal axis denotes the coupling strength δ, and the vertical axis denotes the parameter of α expresses the threshold of full synchronization in network. In this graph, the lower area of each line denotes full synchronous area and the upper area of each line denotes unsynchronous area. By increasing the value of the coupling strength, the full synchronous area becomes large. Namely, several types of full synchronization states can be observed from Fig. 5. We can see that the spread of full synchronous area is determined by the small mismatch pattern.

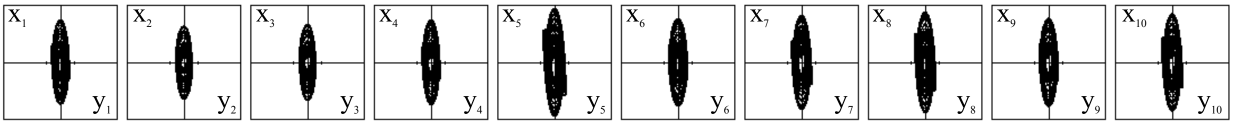
Next, we compare process of full synchronization by contriving ways to add small mismatch. We add small mismatch to all nodes include hub or except hub. Figure 6 shows an average of mechanism spreading full synchronization of three types. In this graph, the lower area of each line denotes full synchronous area and the upper area of each line denotes unsynchronous area in common with Fig. 5. In Fig. 6. (b) and Fig. 6. (c), become full synchronization in smaller value of α than Fig. 6. (a), and we confirm the difference of process of spreading full synchronization. In these results, we have confirmed that the full synchronous state depends on the pattern of small mismatch.

Table 1: Mismatch of hub: zero.

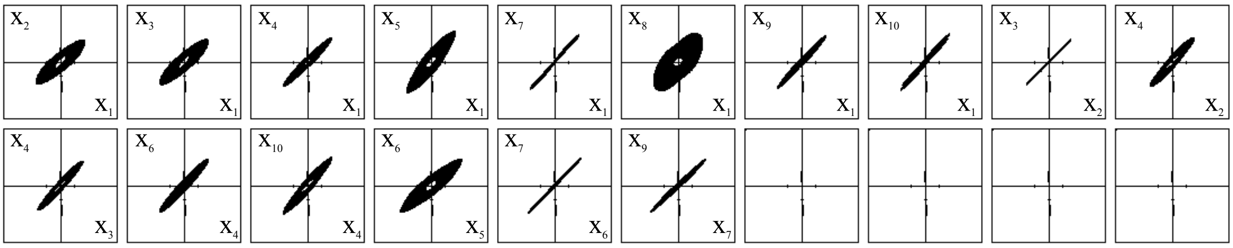
small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	0	0	0	0	0
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0.003	-0.001	-0.003	0.007	0.009
α_8	0.007	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001

Table 2: Mismatch of hub: random.

small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	-0.005	-0.001	0.009	-0.003	-0.006
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0.003	-0.001	-0.003	0.007	0.009
α_8	0.007	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001



(a) Attractors of each node.



(b) Lissajous of each internode.

Figure 4: Attractors and lissajous of each node and internode when w=1.

Table 3: Mismatch of hub: constant.

small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	0.01	0.01	0.01	0.01	0.01
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0.003	-0.001	-0.003	0.007	0.009
α_8	0.007	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001

4. Conclusions

In this study, we have investigated synchronization phenomena of parametrically excited van der Pol oscillators in random network with hub by adding several types small mismatch to hub of random network in order to investigate more detailed effects of small mismatch of hub in random network. In this result, we have confirmed that the full synchronous state depends on the small mismatch in hub. For the future work, we would like to consider the influence of the small mismatch for synchronization state of another type and more large scale network structure.

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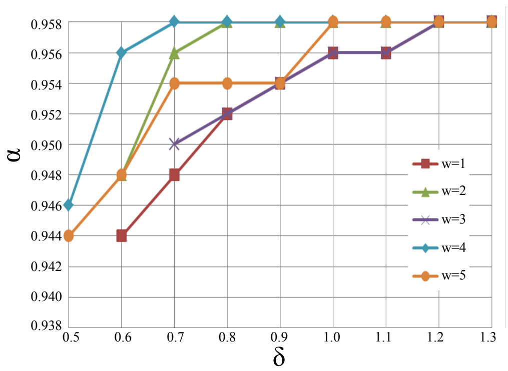
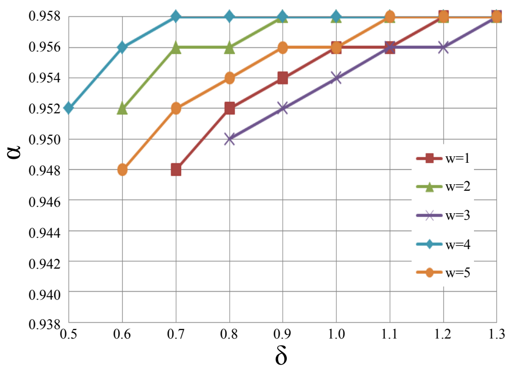
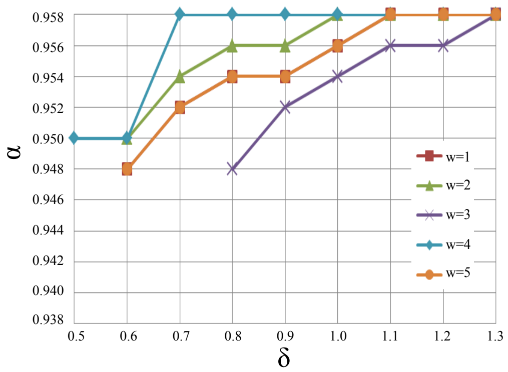


(a) Mismatch of hub: zero.

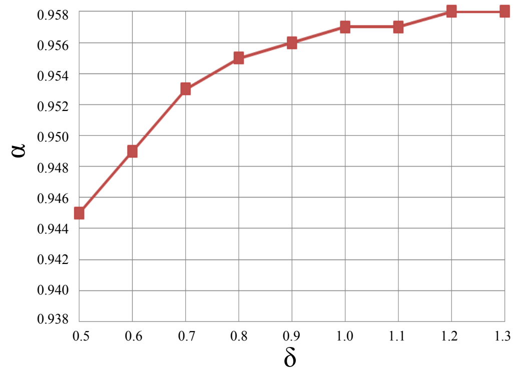


(b) Mismatch of hub: random.

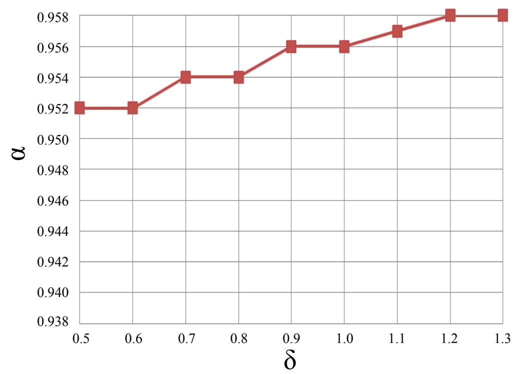


(c) Mismatch of hub: constant.

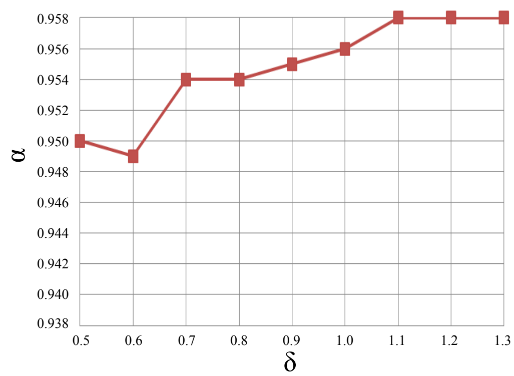
Figure 5: Mechanism of synchronization when $\alpha_1 = 0$.



(a) Mismatch of hub: zero.



(b) Mismatch of hub: random.



(c) Mismatch of hub: constant.

Figure 6: Average of mechanism of synchronization.