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Effects of Complex Chaotic Dynamics for Combinatorial Optimization

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Abstract—Chaotic dynamics has been shown effective for combinatorial optimization. In one of such approaches to optimization problems, the chaotic dynamics is utilized to efficiently drive a heuristic algorithm, which can be applied to large-scale problems. In this paper, the searching dynamics of such a chaotic search driving heuristic algorithm, which is a kind of hybrid dynamical systems, is analyzed. First, the performance of the chaotic search is compared with the stochastic search and the tabu search, and effectiveness of the chaotic dynamics is confirmed. Second, the searching dynamics of those algorithms are quantified by estimating expansion of a small difference added to their search, which corresponds to the largest Lyapunov exponent. By such an analysis, it has been clarified that the chaotic search expands a small difference slower but gradually makes it larger than the other compared heuristic approaches. It is also shown that the chaotic dynamics, which expands the difference the most slowly, corresponding to smaller Lyapunov exponent, has the best performance.

1. Introduction

Various heuristic methods have been proposed for NP-hard combinatorial optimization problems. Effectiveness of the chaotic dynamics to avoid being trapped at the state of a local minimum has been firstly found in the optimization algorithm using the mutually connected neural networks [1]–[4]. Such a method has been realized by extending or replacing the conventional Hopfield neural network to the chaotic neural network [5]. In the analyses of the effectiveness of such chaotic neurodynamics, it has been shown that the chaotic dynamics close to the edge of chaos has high performance [2, 4]. As the second approach using chaos for optimization method based on the Hopfield neural network, effectiveness of the chaotic noise has been also investigated and shown that the chaotic noise is more effective than the stochastic noise [6, 7]. Effects of the chaotic noise was analyzed using the method of surrogate data and it has been shown that the specific autocorrelation function of the chaotic noise is effective when it is added to each neuron [7, 8]. Different from these two approaches based on the Hopfield neural network, the third approach utilizes the chaotic neurodynamics to drive other simple heuristic algorithms [9]. In the third approach, it becomes possible to apply effective chaotic dynamical search to much larger combinatorial optimization problems, whose size is on the order of 10^4 for the Traveling Salesman Problems (TSPs), because it does not have any drawbacks on problem conding as the Hopfield neural network approaches have. Furthermore, the third chaotic approach can include effects of the tabu search by relate the tabu effect to refractoriness of the chaotic neurons. It has been shown that such chaotic methods including tabu effects are more effective than the conventional tabu searches or stochastic annealing methods, by comparing their performances on large TSPs and Quadratic Assignment Problems (QAPs) [10, 11].

Because the first and second approaches are based on the Hopfield neural network and they can be formulated as an autonomous dynamical system, its searching dynamics could be analyzed from various aspects of the chaotic dynamics as shown in Refs. [2, 4, 7, 8]. In the first approach, it has been clearly shown that the complex chaotic dynamics, whose Lyapunov exponents are close to 0, is the most effective. For the second approach, temporal structure of the chaotic sequence has been shown effective. The results of those analyses in the first and the second approaches are quite clear and necessity of the chaotic dynamics for realizing high performance in the neural network approaches have been shown evidently.

On the other hand, the method of the third approach cannot be defined only by the equations because the chaotic dynamics includes the gain inputs which are feedback from the heuristic algorithm updating the solutions. It is a kind of the hybrid dynamical systems connecting the heuristic algorithms and the chaotic dynamics. Therefore, simple analysis methods for autonomous chaotic dynamical systems could not be applied to such a hybrid dynamical system, and detail analysis has not been done yet. In the previous analysis in Ref. [12], periodicity and mutual information on variation of the solution, which is updated by a heuristics driven by the chaotic dynamics, are evaluated. By comparing such variation of the solution on several tabu searches and the chaotic search, and it has been clarified that the forbidding of the previous moves becomes gradually strict in the chaotic search and deeper search in a small area is much better that the conventional tabu searches. However, we still have a question how the chaotic dynamics in such a hybrid dynamical system effects combinatorial optimization.

In this paper, we evaluate the orbital instability of the searching dynamics of the chaotic methods combined with the heuristic algorithm. Since it is not possible to calculate the Lyapunov spectrum of such a hybrid dynamical system straight-forwardly, we observe expansions of a slight difference added to the searching dynamics, which corresponds to the maximum Lyapunov exponent. Such a feature quantity is calculated on several heuristic methods, the stochastic search, the tabu search and the chaotic search, and relation between their performances and orbital instability is investigated.

2. Chaotic Search Which Drives a Heuristic Algorithm

In the approach using chaotic dynamics to drive a heuristic algorithm, the chaotic neural network [5] has been utilized to generate the chaotic dynamics. Each firing of the chaotic neurons actuates the heuristic algorithm and one update of the solution to a neighboring state is performed. In the case of applying to the TSP, the 2-opt method or the Lin-Kerneighan method have been selected as a heuristics to perform a move [9, 11, 13]. This approach can also include the tabu effect of the tabu searches by defining a neuron for each move which will be memorized in the tabu list. In this paper, we apply such a method for the TSP proposed in Ref. [11] and analyze its searching dynamics.

The neurons in the chaotic neural network to drive a heuristic algorithm consist of three internal states, $\xi_i(t)$, $\eta_i(t)$ and ζ_i, which corresponds to the gain effects to minimize objective function, the connection effects to keep appropriate firing rate and the tabu effects including the chaotic dynamics, respectively. When the summation of these three internal state becomes large, the combined heuristic algorithm is actuated and a corresponding heuristic move is performed.

In this paper, we introduce the method which use N neurons for N city TSP [11], which can be realized by the following equations,

$$\xi_i(t+1) = \max_j \{\zeta_j(t+1) + \beta\Delta_{ij}(t)\}, \qquad (1)$$

$$\eta_i(t+1) = -W \sum_{k=1}^{N} x_k(t) + W, \qquad (2)$$

$$\zeta_i(t+1) = -\alpha \sum_{d=0}^{s-1} k_r^d x_i(t-d) + \theta, \qquad (3)$$

$$x_i(t+1) = f\{\xi_i(t+1) + \eta_i(t+1) + \zeta_i(t+1)\}. \qquad (4)$$

where, $x_i(t)$ is the output of the ith neuron at time t, $\Delta_{ij}(t)$ is the gain effect (difference of the tour length) made by a 2-opt exchange which connects the cities i and j, β is the strength of the gain effect, W is the weight of the mutual connections, k_r is the decay parameter of the tabu effect, α is the strength of the tabu effect, s is the length of the tabu tenure, and f is a sigmoidal function, $f(y) = 1/(1 + \exp(-y/\epsilon))$, respectively. This neural network is updated asynchronously. When $x_i(t+1) > \frac{1}{2}$, the city i is connected with the city j corresponding to the maximum of $\{\zeta_j(t+1) + \beta\Delta_{ij}(t)\}$ in Eq. (1), using the 2-opt exchange.

Eq. (3) can be modified to the following form suitable for numerical calculation, when $s = t$,

$$\zeta_i(t+1) = k_r\zeta_i(t) - \alpha x_i(t) + R, \qquad (5)$$

where R is a positive bias. In this chaotic neural network which drives the 2-opt algorithm, the internal state $\xi_i(t)$ includes not only the gain effect, but also the tabu effect of the city j which will be connected by the 2-opt exchange.

3. Analysis on the Searching Dynamics of the Chaotic Search which Drives a Heuristic Algorithm

In this section, performance and searching dynamics of the chaotic searching method described in Sec. 2 is compared with the conventional tabu search and an algorithm using the stochastic dynamics.

For a fair comparison, the tabu search is realized by same equations as the chaotic searches in Eqs. (1)–(3), with the following settings, $W = 0, k_r = 1, \alpha = \infty$ and s to the tabu tenure. When the $\xi_i(t+1) + \zeta_i(t+1)$ is the maximum in all neurons, the city i is connected to the city j corresponding to the maximum of $\{\zeta_j(t+1) + \beta\Delta_{ij}(t)\}$, using the 2-opt exchange. The stochastic search for comparison is also realized by the same equations, with introducing Gaussian random numbers for $\zeta_i(t+1)$.

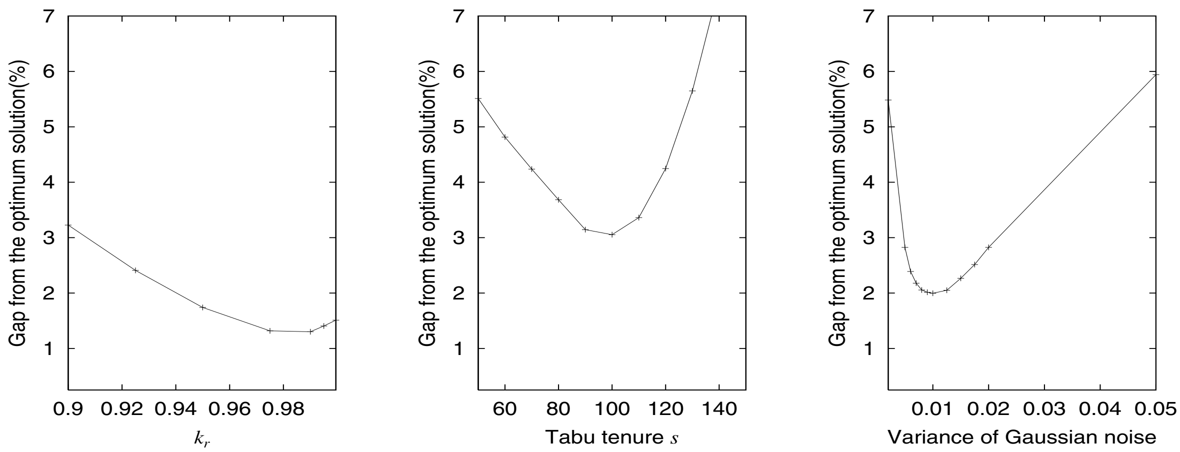
3.1. Comparison on the Solving Performance

The results of these three methods, the chaotic search, the tabu search and the stochastic search, are compared in Fig. 1. Those are the results on a 200-city TSP, KroA200. Each plot in Fig. 1 is an average solution obtained by 10000 runs with different initial conditions. The obtained average solutions are evaluated as gaps from the exact optimum solution. Cutoff time for each run is 1000 iterations, for every algorithm. By this simple and fair comparison, we can see that the chaotic dynamics has the best performance. The chaotic search achieves almost 1 % from the optimum solution only by 1000 iteration runs.

3.2. Analysis on the Searching Dynamics

In the conventional research on the approach that utilizes chaotic dynamics to drive a heuristic algorithm, periodicity and mutual information of the dynamics has been evaluated and compared with the conventional tabu searches [12]. In such analysis, it has been clearly shown that the chaotic search can have longer memory effect which gradually decreases, while the tabu search has limited memory effect corresponding to the tabu tenure length. It may be one of the factor for the chaotic search to realize an efficient search.

In this paper, we evaluate an orbital instability of the searching dynamics, as one of the nonlinear characteristics. However, since the algorithm of this chaotic search, chaos driving heuristic algorithm, is a kind of hybrid dynamical system, it is difficult to straight-forwardly calculate the Lyapunov exponent, which is usually estimated to



Chaotic Search | Tabu Search | Stochastic Search

Figure 1: Solving performances of the chaotic search which drives the 2-opt, the tabu search and the stochastic search, on a 200-city TSP.

evaluate orbital instability of the chaotic dynamics. Therefore, in order to estimate such orbital instability approximately, expansions of small differences, which correspond to the maximum Lyapunov exponent, are calculated and compared with other heuristic methods.

In order to evaluate such characteristics, a similarity between the searching dynamics with and without a small additional difference is calculated. First, an algorithm is run from some initial condition. Second, using the same initial condition, the algorithm is run again, but at 1000th iteration, one additional 2-opt exchange, which is selected randomly, is applied. Average of similarity of the solutions obtained by these two searches, the original run and the run with such an additional exchange, is calculated, and variation of the similarity is observed with increasing the iterations after the additional exchange. In the following experiments, similarity is defined as the number of the same links in the obtained tours by the two searches at the same iteration.

Average similarity calculated by the procedure described above is shown in Figs. 2 and 3. For each original run, 10000 different random exchanges are applied at 1000th iteration, and the average of the similarity over 1000 different initial conditions with such different addtional moves is taken as the obtained result. Variation of the similarity is observed with changing τ, which is the number of iterations after an additional exchange. In Figs. 2 and 3, the parameters corresponding to the best performance in Fig. 1 are selected. From the Fig. 2 expansion of the difference for the chaotic search is slow but increases continuously as in Fig. 3. For the stochastic search, the difference does not expand so large even when a small additional move is added. For the tabu search, the difference expands more faster than the chaotic search. From these results, chaotic dynamics seems to enable both intensification in small searching area and diversification to other area of the searching space.

In Fig. 4, the average similarities of the chaotic search with different parameters are plotted. The parameter value corresponding to the best performance obtained in Fig. 1 is $k_r = 0.99$, which is plotted with plusses. From these results, such a best parameter decreases the similarity the most slowly.

In Fig. 5, relation between the average similarity on $\tau = 30$ and solving ability is shown. From this figure, it can be seen that the searching dynamics which has higher similarity has better solving ability. This means that the chaotic search corresponding to the small Lyapunov exponent realizes high performance.

4. Conclusion

In this paper, the searching dynamics of a chaotic optimization approach, which utilizes the chaotic dynamics to drive a heuristic method, is analyzed. Since straightforward calculation of the Lyapunov exponent for such a hybrid dynamical system is difficult, orbital instability of the dynamics is quantified by expansion of a slight addtional difference, which corresponds to the largest Lyapunov exponent. By such an analysis, it has been clarified that the chaotic dynamics expands a small difference slower, and gradually makes it larger than the other heuristic approaches, the stochastic search and the tabu search. The dynamics of chaotic searches with different parameter values are also analyzed and it is also shown that the chaotic dynamics, which expands the difference the most slowly, has the best performance, that corresponds to smaller Lyapunov exponent. This is same conclusion as the analyses in chaotic neural network approach in Refs. [2, 4], that the chaotic dynamics close to the edge of chaos, whose Lyapunov exponent is almost 0, is effective. From these

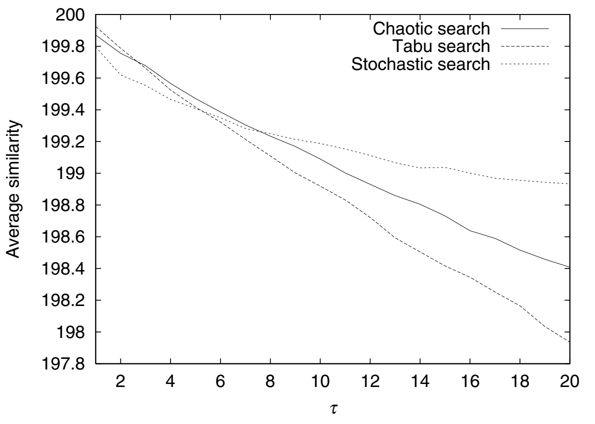


Figure 2: Average similarity of the searching dynamics of the chaotic search, the tabu search and the stochastic search, for smaller τ.

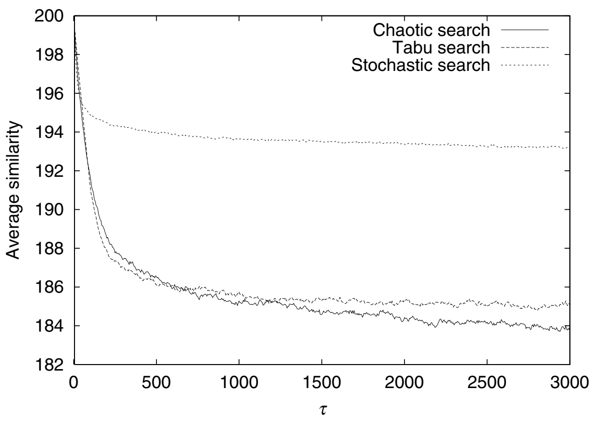


Figure 3: Average similarity of the searching dynamics of the chaotic search, the tabu search and the stochastic search, for larger τ.

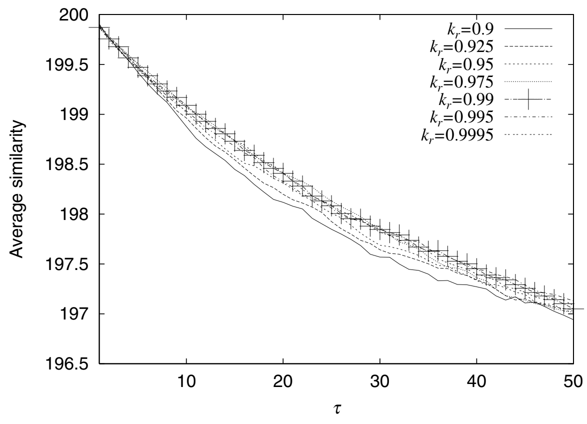


Figure 4: Average similarity of the searching dynamics of the chaotic search, with various values of the parameter k_r.

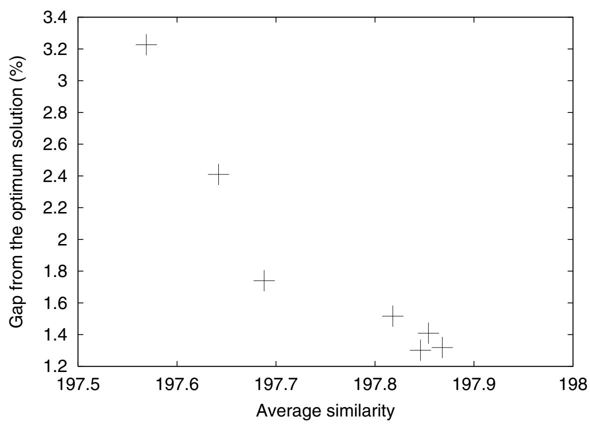


Figure 5: Relation between solving ability and the average similarity of the chaotic search on $\tau = 30$.

results, the chaotic dynamics can be considered important factor also for the third chaotic optimization approach, the chaotic dynamics driving heuristic algorithms.

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