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Synchronization of Coupled Chaotic Circuits with Parameter Dispersion in Small-World Network

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Abstract—This paper considers the synchronization of coupled chaotic circuits with the parameter dispersion in three topologies obtained from the small-world network model. In particular, we focus on the parameter dispersion pattern based on the number of parameter mismatched circuits. By means of the computer calculations, the synchronization probabilities of each parameter dispersion pattern in three network topologies are investigated. From the simulation results, the small-world topology is shown to be effective for the synchronization in the entire network.

1. Introduction

Complex networks have attracted a great deal of attention from various fields since the discovery of "small-world" network [1] and "scale-free" network [2]. In particular, how network topological structure influences its dynamical behaviors, is a hot topic for understanding the structural function on the networks and suitable for practical application in many disciplines. As the dynamics on the networks, the synchronization is one of the typical phenomena. Especially, the synchronization phenomena of coupled chaotic systems are very interesting [3]. However, there are not many studies for complex networks of continuous-time real physical systems such as electrical circuits. In our previous work, we have investigated the synchronization phenomena of coupled chaotic circuits on a complex network with local bridge [4]. We have focused on local bridge structure observed from the small-world network. However, the circuit parameters were fixed with same parameters for all chaotic circuits and the only one network model were considered.

In this study, we investigate the global synchronization of coupled chaotic circuits in the small-world network. Wan and Chen reported the synchronization in the small-world coupled Chua's circuits [5]. We focus on "dispersion" of the parameter mismatched chaotic circuits, the synchronization of coupled chaotic circuits in three network topologies are studied. From the simulation results, the synchronization probability of each parameter dispersion pattern in three network topologies are investigated. Thereby, the small-world topology is shown to be effective for the synchronization in the entire network compared with the regular network and the random network.

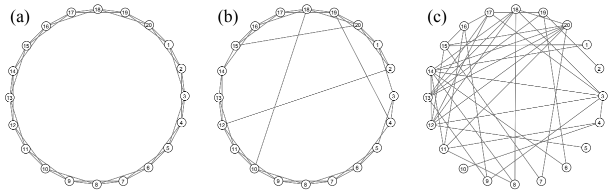


Figure 1: Illustration of the WS model for $N = 20$ and $k = 4$. (a) : Regular network, $p = 0$. (b) : Small-world network, $p = 0.1$. (c) : Random network, $p = 1$.

2. Small-World Network Model

In 1998, Watts and Strogatz introduced very interesting small-world network model, called the WS model [1]. The WS model can be generated as shown in Fig. 1. Starting from a ring lattice with N nodes and k edges per nodes in Fig. 1(a), each edge is rewired at randomly with probability p. The small-world network is known as the graph which is characterized by highly clustering coefficient like a regular graph and small path length like a random graph.

Topological structures in complex networks of N nodes and E edges can be evaluated by the typical three structural metrics (degree, clustering coefficient and path length). First, degree (k) shows the number of edges on a node. Second, clustering coefficient (C) shows the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. This is given as follows:

$$C = \frac{1}{N}\sum_{n=1}^{N} C_n = \frac{1}{N}\sum_{n=1}^{N}\frac{2E_n}{k_n(k_n-1)}. \quad (1)$$

Third, path length (L) shows the shortest path in the network between two nodes. This is given as follows:

$$L = \frac{2}{N(N-1)}\sum_{m=1}^{N-1}\sum_{n=m+1}^{N} l(m,n). \quad (2)$$

In this research, we consider coupled chaotic circuits in three network topologies obtained from WS model in Fig. 1. Each topologies is called regular, small-world and random, respectively. Table 1 shows the properties of three networks as shown in Fig. 1.

Table 1: Properties of three networks as shown in Fig. 1.

	Regular	Small-world	Random
p	0	0.1	1
C	0.500	0.358	0.216
L	2.895	2.458	2.221

3. Coupled Chaotic Circuit

Figure 2 shows the chaotic circuit which is three-dimensional autonomous circuit proposed by Shinriki *et al.* [6][7]. This circuit is composed by an inductor, a negative resistor, two capacitors, and dual-directional diodes. In this study, we propose 20 coupled chaotic circuits in three network topologies as shown in Fig. 1. In these network models, chaotic circuits are applied to each node of the network and each edge corresponds to a coupling resistor R.

First, the circuit equations are given as follows:

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n + i_{dn}, \end{cases} \quad (3)$$

where $n = 1, 2, 3, ..., 20$ and S_n is the set of nodes which are directly connected to the node n. We approximate the $i - v$ characteristics of the nonlinear resistor consisting of the diodes by the following three-segment piecewise-linear function:

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \le V) \\ G_d(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \quad (4)$$

By using the parameters and the variables:

$$\begin{cases} i_n = \sqrt{\dfrac{C_2}{L}}Vx_n,\ v_{1n} = Vy_n,\ v_{2n} = Vz_n \\ t = \sqrt{LC_2}\tau,\ \text{“}\cdot\text{”} = \dfrac{d}{d\tau},\ \alpha = \dfrac{C_2}{C_1} \\ \beta = \sqrt{\dfrac{L}{C_2}}G_d,\ \gamma = \sqrt{\dfrac{L}{C_2}}g,\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C_2}}, \end{cases} \quad (5)$$

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x}_n = z_n \\ \dot{y}_n = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta\displaystyle\sum_{k\in S_n}(y_n - y_k) \\ \dot{z}_n = f(y_n - z_n) - x_n, \end{cases} \quad (6)$$

The nonlinear function $f(y_n - z_n)$ corresponds to the $i - v$ characteristics of the nonlinear resistor consisting of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ \beta(y_n - z_n + 1) & (y_n - z_n < -1). \end{cases} \quad (7)$$

This circuit generates asymmetric chaotic attractor as shown in Fig. 3. The values y and z in Fig. 3 correspond to v_1 and v_2 of the circuit in Fig. 2, respectively.

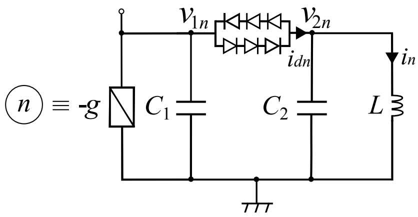


Figure 2: Chaotic circuit.

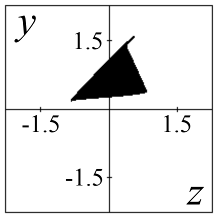


Figure 3: Chaotic attractor of the circuit as shown in Fig. 2. $\alpha = 0.5, \beta = 20$ and $\gamma = 0.5$.

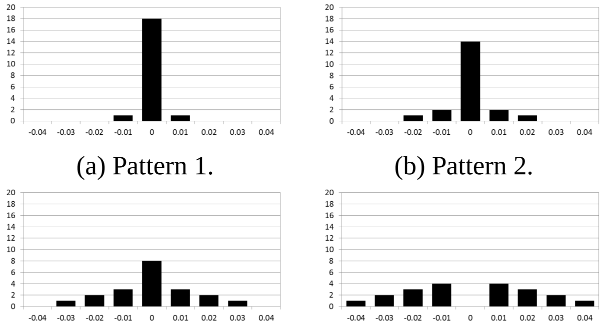
4. Parameter Dispersion

In this research, we fix the circuit parameters as $\alpha = 0.5$, $\beta = 20$, $\gamma = 0.5$ and $\delta = 0.7$ for all chaotic circuits. Additionally, the parameter mismatches $\Delta\alpha$ are added for each circuit parameter α which is relating to the chaos degree. Namely, the parameter α of each circuit is shown as $\alpha = 0.5 + \Delta\alpha$, respectively. We propose four patterns of the parameter dispersion as shown in Fig. 4. Each pattern is different in the number of the parameter mismatched circuits and the range of the parameter mismatches. By using the proposed four patterns of the parameter dispersion, we add the parameter mismatches for the circuits in the computer simulations.

5. Synchronization

5.1. Definition of Synchronization

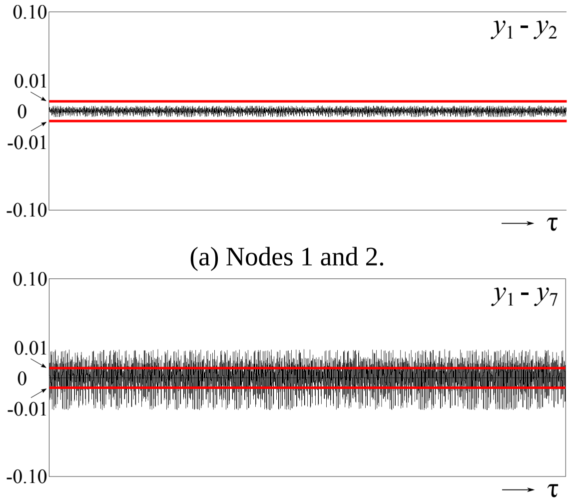
Figure 5 shows the example of the computer simulation results when the parameter dispersion pattern 4 (see Fig. 4(d)) in the small-world network. The vertical axes are the difference between the voltages (corresponding to v_1 of the circuit in Fig. 2) of the nodes 1 and 2 or 7. Namely,



(a) Pattern 1. (b) Pattern 2.

(c) Pattern 3. (d) Pattern 4.

Figure 4: Proposed four pattern of the parameter dispersion. Horizontal axis: the number of circuits. Vertical axis: parameter mismatches $\Delta\alpha$.



(a) Nodes 1 and 2.

(b) Nodes 1 and 7.

Figure 5: The example of the voltage difference between two circuits when the parameter dispersion pattern 4 in the small-world network and the definition of the synchronization. (a) : Synchronization. (b) : Asynchronization.

if the two nodes are synchronized, the value of the graph should be almost zero. In order to analyze the synchronization state, we define the synchronization by the following equation:

$$|y_i - y_j| < 0.01 \quad (i \neq j). \tag{8}$$

By means of the above definition of the synchronization, we define that the nodes 1 and 2 in Fig. 5(a) are synchronized perfectly. However, the nodes 1 and 7 in Fig. 5(b) are almost evaluated as the asynchronization in this definition. Thus, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as (τ= $1,000,000$ and *step* = 0.01τ) and statistically investigate the synchronization probability in the entire network of 20 coupled chaotic circuits.

5.2. Synchronization Probability

Figure 6 shows the investigation results of the synchronization probability and the combinational sample of each parameter dispersion pattern in three network topologies. Each pattern corresponds to the four pattern in Fig. 4. The vertical axes denote the synchronization probability in the entire network during the certain time interval. If the synchronization probability equals 100%, we can consider that the entire network is synchronized perfectly. The horizontal axes denote the combinational samples considered from each parameter dispersion pattern. The combinational of the parameter dispersion is considered a large number. Therefore, we choose the 10 samples randomly as the combinational of the parameter dispersion in each pattern of Fig. 4. From Fig. 6, we confirm that the networks become to be difficult for the global synchronization by increasing the number of the parameter mismatched circuits. On the other hand, the synchronization probability in each pattern depends on the combinational sample. Especially, when the parameter mismatched circuits are the small number, the global synchronization is strongly influenced from the network topologies and the combinational of the parameter dispersion. Figure 7 shows the average synchronization probability in each network of Fig. 6. From this result, we consider that the small-world topology is effective for the synchronization in the entire network.

In addition, the complex relation between the parameter mismatched circuit and the network topological structure can be confirmed in this research. More detailed these relation considering more large number of samples should be investigated for our future works.

6. Conclusion

This paper considered the synchronization of coupled chaotic circuits with the parameter dispersion in three network topologies obtained from the WS model. In particular, we focused on the parameter dispersion pattern based on the number of parameter mismatched circuits. By

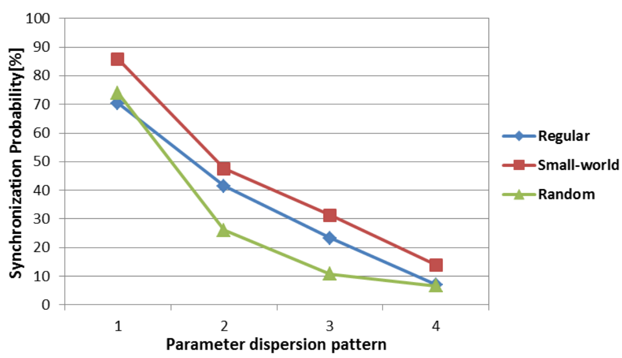
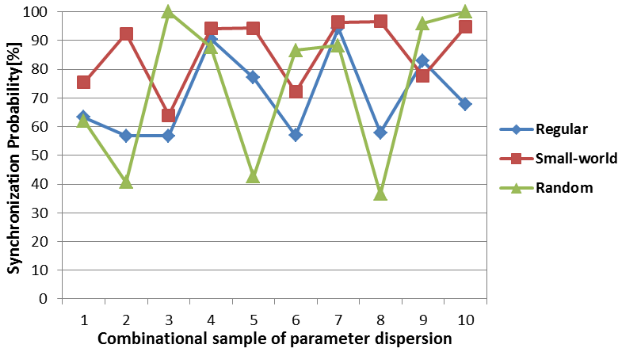
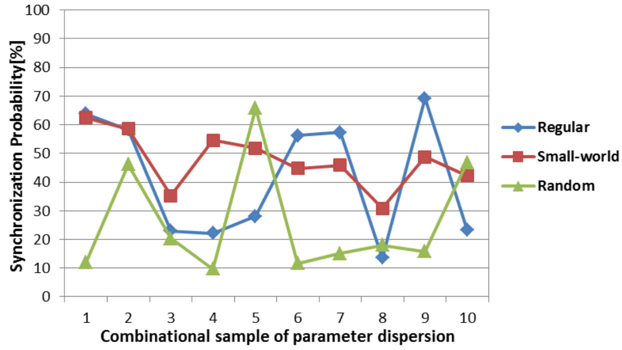


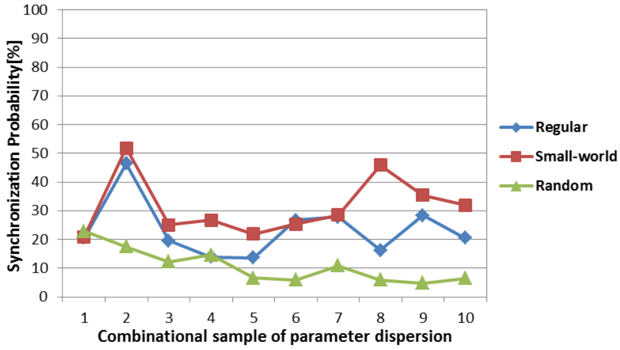
Figure 7: Average synchronization probability of the combinational sample of each parameter dispersion pattern in three network topologies.



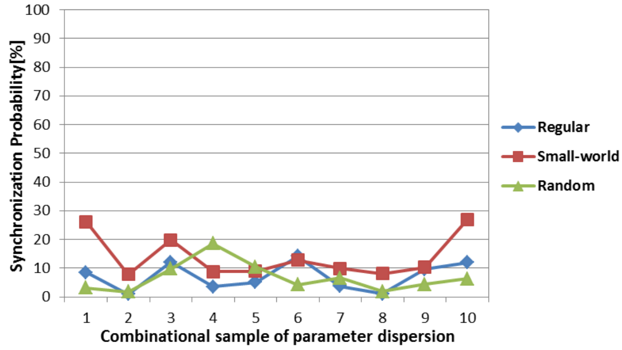
(a) Pattern 1.



(b) Pattern 2.



(c) Pattern 3.



(d) Pattern 4.

Figure 6: Relationship between the synchronization probability and the combinational sample of each parameter dispersion pattern in three network topologies.

means of the computer calculations, the synchronization probabilities of each parameter dispersion pattern in three network topologies were investigated. From the simulation results, we consider that the small-world topology is effective for the synchronization in the entire network. More detailed investigation considering more large-scale networks should be carried out in our future works.

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