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# Rail grid optimization by an algorithm biologically inspired from an amoeba-like organism of true slime mold

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**Abstract**—Optimization of railroad grid was considered by using an algorithm biologically inspired from an amoeba-like organism of plasmodial true slim mold, *Physarum polycepholum*. The organism developed a transportation network consisting of tubular structure to transport protoplasm and cleverly find shortest path connecting multiple food sites. We introduced the path finding algorithm mimicking the adaptation process in the plasmodium developed by Tero et al. (2007) *J.* theor. Biol., **244**, 553 and applied the algorithm to solve the optimizaion of transportation in railroad networks in Tokyo metropolitan. We demonstrate the algorithm works well in sparse mesh type network.

## 1. Introduction

Most of biological organism have transportation network to distribute oxygen, neutrient and etc., into whole body. Animals including human have blood vessels network. Plants have leaf veins and vessels [1]. By expanding this to population, trail pattern by ant-foraging [2], road grid constructed by human [3], and power grid as man-made structures can be included as some of examples for transportation networks. In spite of the species, or, whether the systems is biological or non-biological, the network morphology could play an important role in the function for each system. In this study, we investigate rail grid optimization problem by an algorithm biologically inspired from an amoeba-like organism of true slime mold, *Physarum polycephalum*.

The plasmodium of true slime mold is a giant unicellular organism. The cell size ranges from 10$\mu$ m to 1 m. Even if the single cell is divided into multiple parts, each part can be alive. On the other hand, multiple cells can fuse into a single cell to behave as a single individual. Ability of the cut & paste manipulation in the plasmodium results from unusual cell system; the plasmodium is multinucleated unicellular organism including thousands of nucleus in a single cell.

To maintain such a large cell body, the plasmodium developed a peculiar system in which the cell itself is a transportation network consisting of tubular structure [4]. When the cell body spread into environment by oscillating cell thickness, the tubular network is formed, inside which the shuttle streaming of protoplasm is observed. The streaming transports nutrients, oxygen, organella, and etc., all aver the cell body.

Recently, it was revealed that the plasmodium can solve mazes [5]. In other word, shortest path finding problem can be solved [6]. Tero et al. developed a path finding algorithm by mimicking adaptive dynamics of the transportation network in the plasmodium [7,8]. In this study, we applied this algorithm to the rail grids in Tokyo metropolitan to find optimized solution for transportation.

## 2. Methods

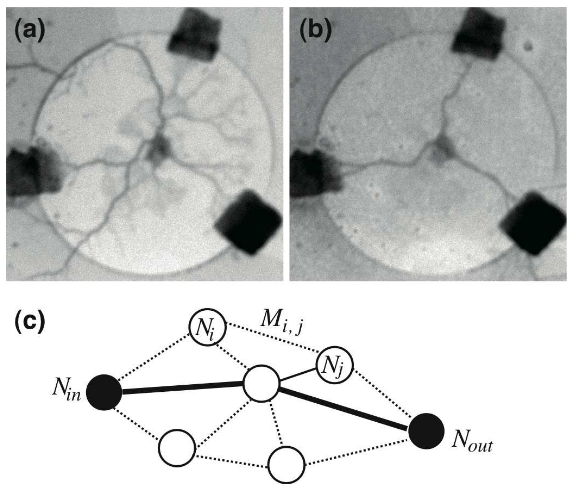


Figure 1: Shortest path finding by plasmodial slime mold. (a) The plasmodium forms a grid network. (b) The plasmodium forms the shortest path connecting the three food sites (dark square images) 8 hours after the food sites setting. (c) Network graph in path finding algorithm. $N_{in}$ and $N_{out}$ denote food sites. $N_i$ and $N_j$ are node numbers without food source. Thick lines show shortest path connecting the nodes $N_{in}$ and $N_{out}$.

## 2.1. Path finding algorithm mimicking plasmodium

When the plasmodium is applied food blocks at multiple sites, it forms a network with shortest path connecting the food sites as shown Fig.1. The shortest path finding algorithm proposed by Tero et al. mimics this adaptation process [7].

In the transportation network, we define nodes $N_i$ at branching points in the tubular network, and links $M_{ij}$ connecting the nodes $N_i$ and $N_j$ as shown in Fig. 1(c). Then we set $N_{in}$ and $N_{out}$ as inlet and outlet of the transported substance,e.g., protoplasm in the plasmodium. This mimics food sites in real system that corresponds to sink/sours of protoplasm during the shuttle streaming. The protoplasm streams inside the tubular link of radius $r_{ij}$ and length $L_{ij}$ with pressure difference among the adjacent nodes. Supposing the pressure $p_i$ at $N_i$ and assuming Poiseuille flow, the flux through the link, $Q_{ij}$, can be defined as follows:

$$Q_{ij} = \frac{\pi r_{ij}^4}{8\kappa}\frac{p_i - p_j}{L_{ij}} = \frac{D_{ij}}{L_{ij}}(p_i - p_j), \qquad (1)$$

where $\kappa$ is viscosity and $D_{i,j}$ is a measure of the conductivity of the tube.

The total amount of fluid in/out of each node must be conserved as follows:

$$\sum_i Q_{ij} = 0. \qquad (2)$$

At the food sites defined as the inlet and the outlet, constant flow $I_0$ is assumed as source and sink. Then the conservation rule should be as follows:

$$\begin{cases} \sum_i Q_{i,in} + I_0 = 0, \\ \sum_i Q_{i,out} - I_0 = 0. \end{cases} \qquad (3)$$

Substituting Eq.(1) for Eq.(2) or Eq.(3), we obtain $p_i$, then calculate $Q_{ij}$ form Eq.(1) under given $L_{i,j}$ and $D_{i,j}$.

In the plasmodium, the tubes grow according to the flux. We assumed the following dynamics for $D_{i,j}$:

$$\frac{dD_{ij}}{dt} = f(|Q_{ij}|) - D_{ij}, \qquad (4)$$

where the first term in the right side of the Eq. (4) represents the expansion of the tubes in response to the flux, and the second term represents degeneration of the tube so that the tube diameter decreases in absence of the flow. We assumed the function $f$ as sigmoidal one like Hill's equation:

$$f(|Q_{ij}|) = \frac{(1+a)Q_{ij}^{\mu}}{1+aQ_{ij}^{\mu}}, \qquad (5)$$

which is generally applied to the biological cooperative process when $\mu > 1$: The tube hardly grow when the flow is extremely slow, while the growing speed is accelerated when it once starts to grow then saturated to $1 + 1/a$. When $\mu = 1$, $f$ is Michaelis–Menten type, i.e., it represents the most simple enzyme reaction. When $\mu < 1$, $f$ suggests fast initial growth and slow saturation.

## 2.2. An application to rail grids

Tero et al. [7] applied the shortest path finding algorithm of plasmodium to maze solving problem [5] by defining nodes at branching point and fixing the inlet/outlet node to the start/goal of the maze. To apply this algorithm to the optimization of rail grids, we defined nodes at stations and links at the rails connecting the adjacent stations. The calculation using Eqs.(1–5) was performed by randomly selecting any two of the all nodes in the railroad network as the inlet/outlet, so that the selection probability of the inlet/outlet nodes is proportional to number of passenger at each station.

We tested the two types of railroad networks in Tokyo metropolitan, JR lines (East Japan Rail way company) with 369 stations and Metro (Tokyo Metro Co., Ltd) with 139 stations. In practical railroads, the network is not closed in Tokyo metropolitan area but connected to outer. Therefore the stations at the ends of the network were selected the ones whose number of passenger is enough small ($< 10,000$). We set $I_0 = 1.0$, and $D_{ij} = 1.0$ to all the links for initial condition. The value of $L_{ij}$ was given as the distance of the rail connecting the adjacent stations obtained from the practical data [9–11]. We calculated time development of $Q_{ij}$ and $D_{ij}$ for the various parameter values of $a, \mu$, and obtained converged value of the flux $Q_{ij}$. Then the flux was compared to the passenger number in the line between the station $i$ and $j$.

# 3. Results

The simulations were performed for the two railroad networks, JR and Metro, independently. Figs.2 and 3 show the simulation result for JR lines and Metro lines, respectively. Comparing the converged flux in the simulation with the number of passengers, the result showed good agreement in JR lines but not in Metro lines: The correlation coefficients between the number of passengers at all lines in the practical systems and the flux at all links obtained in the simulation were 0.94 (after 150,000 iterations) in JR and 0.59 (after 150,000 iterations) in Metro, which are maxima against the tested various parameters of $a, \mu$.

Fig. 4 shows the dependence of the correlation coefficient on the simulation parameters $a, \mu$ in JR. The plots have maxima at $\mu > 1$ for any $a$, e.g. $\mu = 1.48$ when $a = 1.0$. The maximal position moves to lager $\mu$ as $a$ increases. For Metro, the features are same but maximal positions smaller than those in JR, e.g. $\mu = 1.0$ when $a = 1.0$. The simulations shown in Figs. 2 and 3 were performed with the parameters $\mu$ and $a$ maximizing the correlation coefficients.

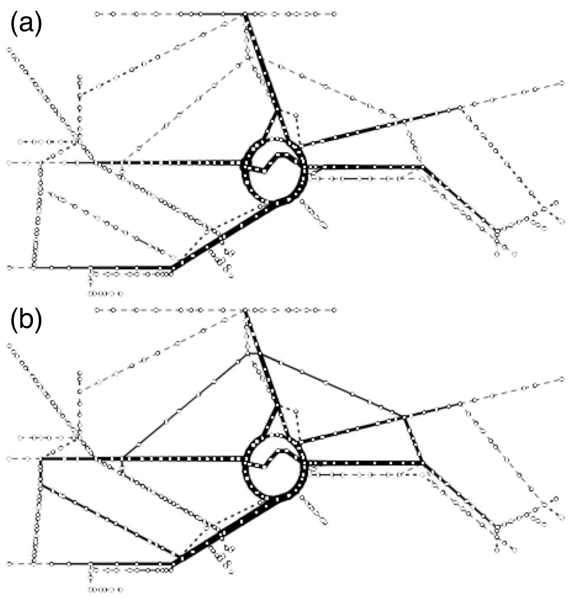


Figure 2: Result for JR lines (East Japan Railway company). (a) Practical system. (b) Simulation. Circles and lines denote railroad stations and lines, respectively. The thickness of the lines are roughly proportional to the number of passengers (reduced value to the simulation result) in (a), and the caluclated flux in (b). Solid lines denoted the flux $Q_{i,j} > 0.05$, dashed lines $0.01 < Q_{i,j} < 0.05$, doted lines $Q_{ij} < 0.01$. The parameters were $a = 1.0, \mu = 1.48$. The reduced value of the number of passengers were calculated as the slope value $c$ by linear regression of the number of passengers and the flux, where it is calculated as $c = 5.494 \times 10^{-10}$.

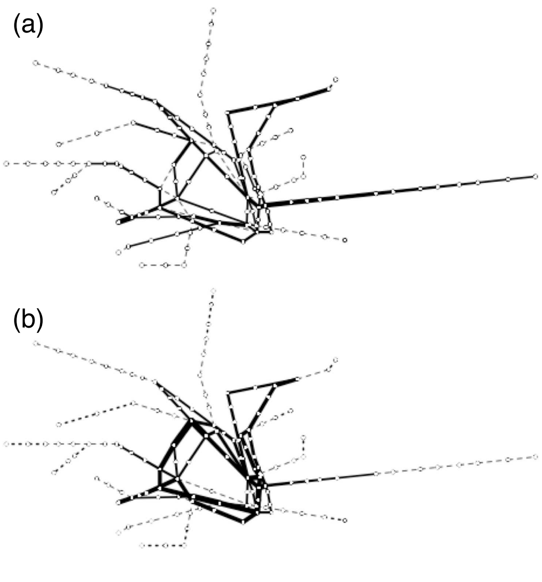


Figure 3: Result for Metro lines (Tokyo Metro Co., Ltd). (a) Practical system. (b) Simulation. All notations are the same as those of Fig.2. $a = 1.0, \mu = 1.0$. $c = 6.451 \times 10^{-10}$.

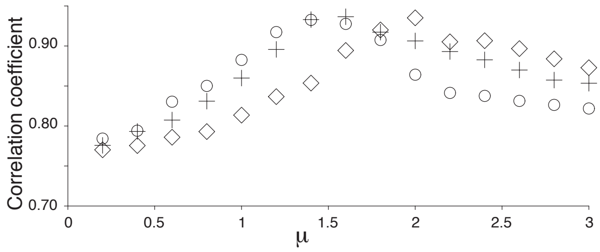


Figure 4: Correlation coefficient between the flux calculated in simulation and the practical systems under various $a$ and $\mu$ in JR lines. Circles, crosses and diamond respectively denote $a = 0.25, 1.00, 4.00$.

## 4. Discussion

We saw good agreement with the results of JR but not in Metro. The difference would result from the one in topology of the networks.

The JR network is sparse mesh: There are many stations with degree 2 (straight lines without any bifurcation). In most cases of the straight lines, the simulation result were highly correlated with the practical value as shown in Fig.5(a). However, a few examples of the short straight lines showed U shaped relation as shown in Fig.5(b), in which junctions existing at the both ends of the straight line affect. Another example of the junction effects can be seen in Fig.5(c): Globally, the result does not show good agreement, however, locally it does. The sub-lines I, II, and III have high correlation each, but the sub-lines are discontinuous at which the other lines such as Metro or local private lines are intersected.

In Metro, the flux in the simulation was concentrated in the central part of the network. The Metro network is tree-graph with circles in center: The number of the stations in the branches is much lager than the ones in the central part. Then the probability selecting two stations in the suburb transversally crossing the center circle area as inlet/outlet might be much higher than expected in the simulation. In reality, little number of passenger would move between the suburbs.

## 5. Conclusion

We tried optimization of rail grid in Tokyo metropolitan using the algorithm biologically inspired from the plasmodium of true slime mold. We found that it works in the sparse mesh network such as JR lines on the whole. Especially, the distribution among the lines with bypass was succeeded. As discussed above, however, the junction disturbs the right distribution. This could be refined by considering mutual entry of JR, Metro, and the other private local lines, while they were separately investigated in this study.

In tree-graph network such as Metro, the optimization

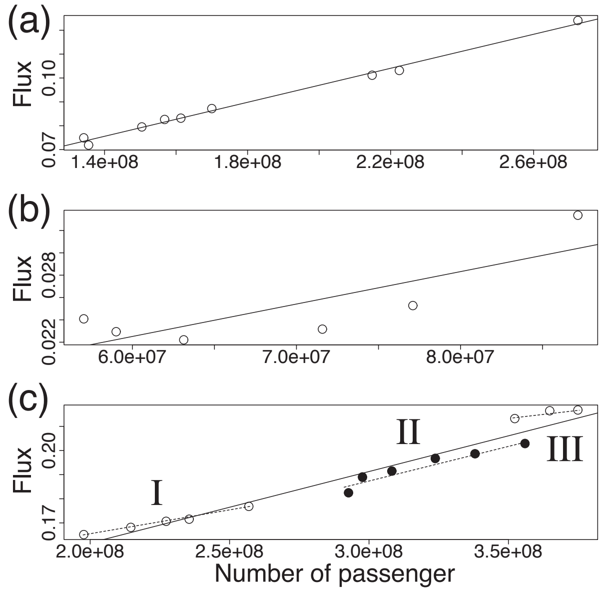


Figure 5: Relation between the number of the passengers and the value of the flux for each line. (a) A long straight line without branching. (b) A short straight line interfered with the branching stations at the both ends. (c) A line with intersected stations. The number of passenger is occasionally almost in order of the station location in each line.

was not succeeded well, since people's preference such that the most commutation location is central area was not considered. These effect should be introduced by using some indices such as the difference between daytime and nighttime populations.

As we saw in Fig.4, the fittest parameters of $\mu$ always larger than 1.0. This could results from the function $f$ defined in Eq.5. Only the biologically appropriated values of $\mu$ are accepted in the optimization process as discussed in the end of the section 2.1. Then the fittest parameters seems to depend also on the network topology. Finding the rules which parameters are appropriate for which type of the networks is left for open question.

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