2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

A Method for Solving Asymmetric Traveling Salesman Problems Using Chaotic Exchange Method and Block Shift Operations

Toshihiro TACHIBANA†* and Masaharu ADACHI†**

†Graduate School of Advanced Science and Technology, Tokyo Denki University
2-2 Kanda–Nishiki–cho, Chiyoda–ku, Tokyo 101–8457, Japan
Email: * 09ude01@ms.dendai.ac.jp, ** adachi@eee.dendai.ac.jp

Abstract—An asymmetric Traveling Salesman Problem is one in which the costs for travel between one city and another are not symmetric. We propose a method for solving such problems based on chaotic search method by Hasegawa *et al*. It uses a chaotic neural network and tabu search, namely, block shift operations and 2-opt exchange are used as exchanging methods. In block shift operations, we consider several cities as a BLOCK, and the whole BLOCK is exchanged with other city. The proposed method switches two types of exchanging method by using chaotic neurodynamics. The proposed method exhibited better solutions than the method with a fixed ratio of the executing the exchanging methods.

1. Introduction

The Traveling Salesman Problem (TSP) is a famous NP-hard problem. It is a combinatorial optimization problem which appears quite frequently in various fields such as transit and control [1]. For example, vehicle routing, designing of VLSI, boring of circuit boards, X-ray crystal structural analysis, and so on. The TSP suffers from combinatorial explosion: the number of feasible solutions increases as $\frac{(n-1)!}{2}$ as the number of cities n increases[1, 2]. In other words, if there are a large number of cities, it is difficult to obtain a good solution to this problem using simple techniques such as the round robin method[2].

Symmetric TSPs have been studied by many researchers. However, in everyday problems of this type, it is rare that the costs are symmetric. Therefore, there are demands for dealing with asymmetric cost problems [3]; however, thus far, asymmetric TSPs have not been extensively studied. In particular, most of the conventional method is applied either to symmetric TSP or asymmetric TSP. Any method that is efficient to both problems does not exist.

Branch and bound method is known as method for solving the asymmetric TSP. However, the branch and bound method requires huge computational costs. It is not practical to solve the asymmetric problem that takes too long computational time. Therefore, in this paper, we propose a method for finding quasi-optimal solution by a metaheuristics approach.

This paper is organized as follows. Section 2 introduces the asymmetric TSP. The chaotic search methods proposed by Hasegawa *et al.*, on which our proposed method is based, is described in Section 3. In Section 4, we describe the proposed method which switches block shift operations and point exchange by using chaotic neurodynamics. In Section 5, results of numerical experiments are shown. Lastly, we present conclusions.

2. Asymmetric traveling salesman problems

The TSP is an optimization problem whose goal is to determine the minimum-cost tour visiting n cities, under the conditions that one visits every cities only at once [3]. For an n-city problem, a set of the cities is

$$V = \{v_1, v_2, \cdots, v_n\} . \quad (1)$$

A tour σ specifies the order in which these cities are visited. The total cost $f(\sigma)$ of a tour σ can be evaluated from the costs for travel from city j to city i, $d_{ij}(v_i, v_j \in V)$.

$$f_{TSP(\sigma)} = \sum_{k=1}^{n-1} d_{\sigma(k),\sigma(k+1)} + d_{\sigma(n),\sigma(1)} \quad (2)$$

The goal of the TSP is to minimize the cost in Eq. (2). The nomenclature for TSPs used in this paper is summarized in Table 1.

Table 1: Nomenclature for TSP

d_{ij}	:	Cost to travel from city i to city j
$f_{TSP}(\sigma)$	:	Total cost of a tour
σ	:	The order of cities visited in a tour
V	:	Set of cities to visit
n	:	Number of the cities
u_{im}	:	Internal state of the neuron im
θ_{im}	:	Threshold of the neuron im
x_{im}	:	Output of the neuron im
$w_{im,jn}$	:	Synaptic weight from the neuron jn to the neuron im

A TSP is asymmetric when the cost of travel between two given cities is not symmetric, that is, $d_{ij} \neq d_{ji}$. In this paper, we do not include cases in which two cities are connected only in one direction.

3. Chaotic search method[4]

The proposed method is based on the chaotic search method [4] which uses a chaotic neural network with tabu search[5] . A feature of this method is that the number of neurons used is equal to the number of cities. We give an outline of the chaotic search method in this section.

3.1. Coding of networks

The updating of the internal state of each neuron is executed by the following set of equations.

$$f(x) = \frac{1}{1+e^{-\gamma x}} \quad (3)$$

$$\xi_i(t+1) = \max_j\{\zeta_j(t+1) + \beta\Delta_{ij}(t)\} \quad (4)$$

$$\eta_i(t+1) = -W\sum_{k=1}^{N} x_k(t) + W \quad (5)$$

$$\zeta_i(t+1) = -\alpha\sum_{d=0}^{t} k_r^d x_i(t-d) + \theta \quad (6)$$

$$x_i(t+1) = f\big(\xi_i(t+1) + \eta_i(t+1) + \zeta_i(t+1)\big) \quad (7)$$

If $x_i(t+1) > \frac{1}{2}$, city i is connected to city j that gives the maximum in Eq. (4) by using 2-opt exchanges. The nomenclature for neural networks used in this paper is summarized in Table 2.

Table 2: Nomenclature for networks

k_r	:	Decay parameter of the gain effect
α	:	Scaling parameter of the tabu effect
β	:	Scaling parameter of the gain effect
Δ_{ij}	:	Gain of the objective function value offered by the 2-opt exchange which links citied i and j.
θ, R	:	Positive biases

3.2. 2-opt exchange

Two-opt exchange is a basic city exchange method for symmetric TSPs in which any two cities exchange their position in the tour. For example, a tour shown in Fig. 1 (a), can be changed to Fig. 1 (b) by cutting two links and creating two new links.

4. Proposed method

4.1. Overview

We propose a method that is a modification of the chaotic search method described in section 3. Although the proposed method uses the same network and the same algorithm, it differs through the use of two exchange methods are switched by chaotic neurodynamics. The two exchange methods are block shift operations and 2-opt exchange in the paper. The block shift operations are described in the next section 4.2. The proposed method for switching between a 2-opt method and block shift operation using chaotic neurodynamics is described in section 4.4.

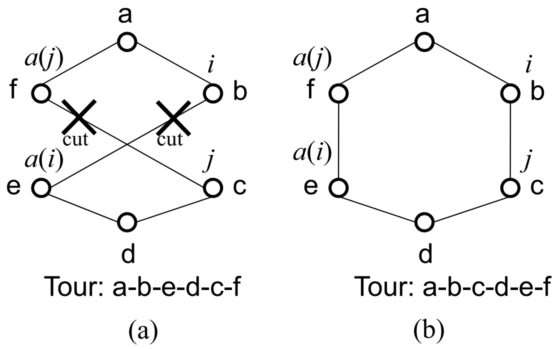

Figure 1: An example of 2-opt exchange: links b–e and f–c are swapped.

4.2. Block shift operations

The 2-opt exchange is a fundamental city exchange method for solving TSPs. When a 2-opt exchange is executed, the direction of a part of the tour is reversed by the city exchange. In symmetric TSPs, this reversal does not alter the cost of the tour. However, in an asymmetric TSP, the 2-opt exchange may cause the cost to rise. To overcome this disadvantage of the 2-opt exchange in asymmetric TSPs, the block shift operation has been proposed [6]. This method considers several cities as a BLOCK, and the whole BLOCK is exchanged with another city. During the exchange, the order of the cities in the BLOCK is maintained. We show an example of a block shift operation in Fig. 2. Although a 2-opt exchange could be viewed as a type of block shift operation, we use the term "block shift operation" in this paper only when a block consists of more than one city.

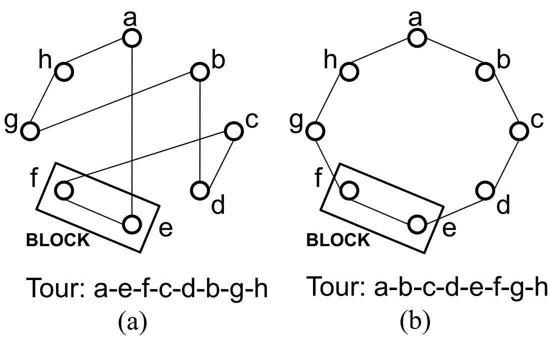

Figure 2: An example of a block shift operation: the BLOCK consists of cities e and f, and the exchange partner is city b.

Table 3: Experimental results of proposed and conventional methods for p43.

Method	Proposed	Proposed (fixed ratio)	Chaotic Search[4]	Local Search[8]	Hybrid GA[9]
Minimum cost	**5620**	**5620**	**5620**	N/A	N/A
Maximum cost	5622	5624	5641	N/A	N/A
Average cost	5620.4	5622.0	5632.3	N/A	N/A
Gap [%]	0.007	0.04	0.2	0.01	0

Table 4: Experimental results of proposed and conventional methods for ry48p.

Method	Proposed	Proposed (fixed ratio)	Chaotic Search[4]	Local Search[8]	Hybrid GA[9]
Minimum cost	14713	15083	15358	N/A	N/A
Maximum cost	15204	15386	17024	N/A	N/A
Average cost	14961.8	15254.4	16146.6	N/A	N/A
Gap [%]	3.7	5.8	12.0	1.92	0.001

4.3. Bidirectional search

In asymmetric TSPs, the cost of a tour in a direction is different from that of the opposite direction. In symmetric TSPs the cost of a clockwise tour is equal to that of counterclockwise one. For example, in Fig. 3 (a), the cost of the tour of clockwise a–b–c–d–e–g–f–g–h and that of counterclockwise h–g–f–g–e–d–c–b–a are the same (they are “58”) . Whereas in asymmetric TSPs, the cost of clockwise tour is not equal to that of counterclockwise one. For example, in Fig. 3 (b), the cost of the clockwise tour a–b–c–d–e–g–f–g–h is equal to “52”, whereas the cost of counterclockwise tour h–g–f–g–e–d–c–b–a is equal to “50”. As shown in the above example, tours generated by the above mentioned operations must be evaluated for both directions.

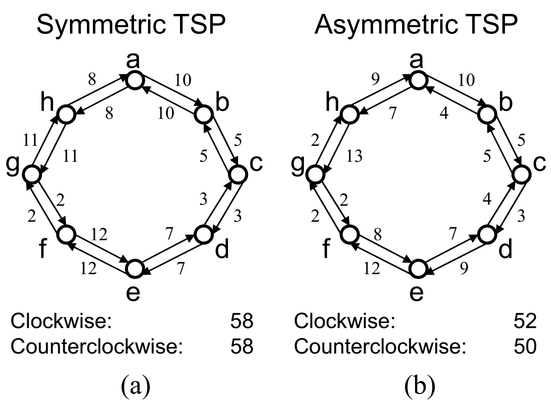

Figure 3: Symmetric and asymmetric TSP by comparing the difference between the cost of visiting direction.

4.4. How to select exchange methods using chaotic neurodynamics

In the proposed method, a switching between two exchange methods are determined by chaotic neurodynamics. It is based on a chaotic search method. The proposed method uses a chaotic neuron in the similar way as it is used in the chaotic search method[4]. The nomenclature for the neuron used in this paper is summarized in Table 5. The updating of the internal state of the neuron is executed by using the following set of equations.

Table 5: Nomenclature for neuron for selecting exchange methods

k_r	:	Decay parameter of the gain effect
α_{ex}	:	Scaling parameter of the tabu effect
β	:	Scaling parameter of the gain effect
$\Delta_{ex}(t)$	:	Gain of the objective function value difference between after the candidate and that of current tour.
A, θ	:	Positive biases

$$\xi_{ex}(t+1) = \beta\Delta_{ex}(t) + A \quad (8)$$

$$\zeta_{ex}(t+1) = -\alpha_{ex}\sum_{d=0}^{t} k_r^d x_{ex}(t-d) + \theta \quad (9)$$

$$x_{ex}(t+1) = f\big(\xi_{ex}(t+1) + \zeta_{ex}(t+1)\big) \quad (10)$$

Where, the sigmoid function of Eq. (3) is used as f. If $x_{ex}(t) > \frac{1}{2}$, the proposed method switches to another exchange method from the current exchange method. In other words, when the current exchange method can reduce costs, the method is continued to use, otherwise, it is switched to the other exchange method.

Table 6: Experimental results of proposed and conventional methods for rbg443.

Method	Proposed	Proposed (fixed ratio)	Chaotic Search[4]	Local Search[8]	Hybrid GA[9]
Minimum cost	**2720**	**2720**	3739	N/A	N/A
Maximum cost	2723	2730	3841	N/A	N/A
Average cost	2720.4	2722.1	3793.0	N/A	N/A
Gap [%]	0.04	0.09	1.16	0.09	0

Table 7: Experimental results of proposed and conventional methods for kroA100.

Method	Proposed	Proposed (fixed ratio)	Chaotic Search[4]
Minimum cost	21369	22373	**21282**
Maximum cost	22288	22852	22134
Average cost	21754.8	22690.6	21440.8
Gap [%]	2.2	6.6	0.7

5. Results of numerical experiments

We numerically evaluated the performance of the proposed method for some benchmark problems of TSPLIB [7], performing 20 trials for each problem. We used asymmetric and symmetric problems. As asymmetric problems, we used 'br17', (17-city) , 'p43', (43-city), 'ry48p', (48-city), and 'rbg443', (443-city) from TSPLIB. Moreover, as a symmetric problem 'kroA100' of 100-city was used for the evaluation. We show results of the performance of the proposed method for 'p43', 'ry48p', and 'rbg443' and 'kroA100' in Table 3, 4, 6 and 7, respectively. In these tables, the performances of conventional methods are also shown. Results in these tables shown that the proposed method with dynamic switching between block shift operations and 2-opt exchanges achieves the exact solution more frequently than the conventional methods.

6. Conclusions

In this paper, we have proposed a method for solving an asymmetric TSP using chaotic neural dynamics. The proposed method with dynamic switching between block shift and spot exchange gives better solutions than the chaotic search method [4], although it is not superior to the existing method of using a hybrid genetic algorithm[9].

A future problem is to investigate a method for selecting efficient parameters automatically.

Acknowledgments

This work was supported by KAKENHI (20300085).

References

[1] RTRI Planning Systems, "Scheduling Algorithm for Railways", NTS, 2005 (in Japanese with English abstract).

[2] M. Yagiura, T. Ibaraki, "Combinatorial Optimization", Asakura Shoten, 2001 (in Japanese with English abstract).

[3] M. Kubo, "Engineering for Logistics", Asakura Shoten, 2001 (in Japanese with English abstract).

[4] M. Hasegawa, T. Ikeguchi, K. Aihara, "Solving large scale traveling salesman problems by chaotic neurodynamics", *Neural Networks*, **15**, 271–283, 2002.

[5] F. Glover, "Tabu Search I", *ORSA Journal on Computing*, **1**, 190–260, 1989.

[6] T. Tachibana, M. Adachi, "A Method for Solving Asymmetric Traveling Salesman Problems Using Neural Networks and Block Shift Operations with Tabu Search", *Proc. of NOLTA 2008*, 69–72, 2008.

[7] TSPLIB (http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/)

[8] J. Cirasella, D. S. Johnson, L. A. McGeoch, W. Zhang, "The Asymmetric Traveling Salesman Problem: Algorithms, Instance Generators, and Tests", Algorithm Engineering and Experimentation, **2153**, 2001.

[9] L.N. Xing, Y.W. Chena, K.W. Yanga, F. Houa, X.S. Shena and H.P. Caia, "A Hybrid Approach Combining an Improved Genetic Algorithm and Optimization Strategies for the Asymmetric Traveling Salesman Problem", Engineering Applications of Artificial Intelligence, **21**, 1370–1380, 2008.