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Throughput and Delay Analysis of IEEE 802.11 String-Topology Multi-hop Networks

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Abstract—The behavior of each node in Wireless Multi-hop Networks (WMNs) is so complex that comprehending network dynamics is challenging problem. As one of the solutions, the analyses of WMNs have attracted attention by many researches. This paper presents throughput and delay analysis of IEEE 802.11 string-topology multi-hop networks. For obtaining those with high accuracy, the operations of each node are expressed in detail. These expressions are associated as a network flow. Additionally, frame-existence probability, which is a new parameter for expressing the property in non-saturated condition, is defined. The validities of obtained expressions are confirmed by comparing with simulation results.

1. Introduction

Recently, the analyses of WMNs have attracted attention by many researchers [1]-[5]. The string-topology network is often selected as an analysis object of WMNs because it is one of the fundamental and simple multi-hop network topologies. The string-topology networks are important in Vehicular Ad-hoc NETworks (VANETs) [5]-[6]. Actually, many multi-hop network analysis techniques were developed from the string-topology multi-hop network analysis [1]-[3].

For obtaining the end-to-end throughput in string-topology multi-hop networks, it was proposed that the Medium Access Control (MAC)-layer operations with respect to each node are expressed by using the 'airtime' expressions. The airtime is defined as time shares of three states, which are transmission, carrier-sensing and channel-idle states at each node. The airtime is effective for expressing of the complex interferences among network nodes. Additionally, by associating the MAC-layer properties of network nodes with a network flow, the maximum end-to-end throughput can be obtained analytically [1]-[3]. However, this procedure is based on the assumption that all the nodes have at least one frame in the buffer, namely network is in saturated condition. Therefore, airtime expressions cannot use in non-saturated condition operation.

On the other hand, delay analyses of WMNs have been also conducted [7]-[8]. The delay is an important factor for evaluating the network in non-saturated condition. The purpose of these analyses is for evaluating MAC protocols. Therefore, it is assumed collision probability of all the network nodes are identical. However, the collision probabilities of network nodes are different one another in WMNs. Therefore, quantitative predication has not been obtained by using the previous procedure of delay analysis of WMNs. For obtaining the end-to-end delay of WMNs with high accuracy, it is necessary to express the MAC-layer operations with respect to each node. Additionally, it is necessary to consider the network flow by associating the expressions with respect to each node.

This paper presents the expressions of throughput and delay of IEEE 802.11 string-topology multi-hop networks. For obtaining two evaluations analytically, the MAC-layer operations with respect to each node are considered by using the airtime expression. For expressing the properties in non-saturated condition, a new parameter, which is frame-existence probability, is defined. The analytical predictions agree with simulation results well, which show validity of the obtained analytical expressions.

2. Throughput and Delay Analysis for IEEE 802.11 String-Topology Multi-hop Networks

In the proposed analytical expressions, all the MAC-layer properties such as frame-collision probability and frame-existence probability are expressed as functions of transmission airtime and offered load. By using the MAC-layer model, the problem of end-to-end throughput derivation is narrowed to the transmission-airtime determinations with respect to each node. For obtaining the transmission airtime, the MAC-layer properties of individual nodes are associated to network flow, which is regarded as Network-layer characteristics. By using the associations, the transmission airtimes of network nodes are fixed uniquely and the end-to-end throughput and delay in the string-topology network can be obtained.

Figure. 1 shows the network topology considered in this paper. In this paper, H-hop string topology is considered. The analysis in this paper is based on the following assumptions [1]-[8]

1. Only the source nodes (Nodes 0) generate fixed sized UDP data frames, payload size of which is P bytes, following Poisson distribution. The destinations of the frames generated by Nodes 0 is Nodes H.
2. Channel conditions of all the links are ideal. Namely, transmission failures occur only due to frame collisions.
3. Node i can transmit DATA and ACK frame only to Nodes $i \pm 1$. Additionally, Nodes $i \pm 1$ and $i \pm 2$ can sense Node-i transmissions. Namely, Nodes i and $i \pm 3$ are in the hidden node relationships [9]

2.1. Airtime

The transmission airtime is the time share of frame transmissions, which includes both the successful- and the

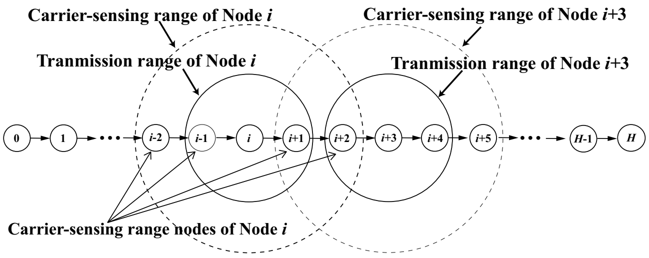

Figure 1: H-hop string-topology network

failure-transmission times. The transmission airtime of Node i is expressed by

$$X_i = \lim_{Time \to \infty} \frac{S_i}{Time}, \tag{1}$$

where S_i is the sum of the durations of the DATA frame (DATA) transmission, ACKnowledgement frame (ACK) transmission, Distributed InterFrame Space (DIFS) and Short InterFrame Space (SIFS) of Node i. By using X_i, the throughput of Node i is expressed as

$$E_i = X_i \times (1 - \gamma_i) \times \frac{P}{T}, \tag{2}$$

where γ_i is the collision probability of Node i and $T = DIFS + DATA + SIFS + ACK$, where $DIFS$ is the duration of the DIFS, $DATA$ is the transmission time of the DATA, $SIFS$ is the duration of the SIFS, ACK is the transmission time of the ACK.

Carrier-sensing airtime is expressed as the sum of frame-transmission durations in all the nodes in the carrie-sensing range. The carrier-sensing airtime of Node i is

$$Y_i = \sum_{j=i-2, j \neq i}^{i+2} X_j - \sum_{j=i-2}^{i-1} \left(\frac{X_j X_{j+3}}{1 - X_{j+1} - X_{j+2}} \right) - \frac{X_{i-2} X_{i+2}}{1 - X_i}. \tag{3}$$

When a node is in neither transmission state nor carrier-sensing states, the channel related with the node is idle. Namely, the channel-idle airtime is expressed as

$$Z_i = 1 - X_i - Y_i \tag{4}$$

2.2. Collision Probability, Transmission Probability, Frame-Existence Probability

In string-topology networks, two types of frame collisions with carrier-sensing range nodes and hidden nodes occur. Because these two collisions are disjoint events, the frame-collision probability of Node i is expressed as

$$\gamma_i = \frac{a(X_{i+3} + X_i)}{1 - X_{i+1} - X_{i+2}} + \left[1 - \prod_{j=i-1, j \neq i}^{i+2} \left(1 - \tau_j \right) \right]. \tag{5}$$

In (5), the first term and the second one indicates hidden node collision probability of Node i and carrier-sensing nodes collision probability of Node i, respectively.

τ_i is transmission probability of Node i. In [4], the simple expression of the Node-i transmission probability in channel-idle state was obtained as

$$G_i = \frac{R_i}{U_i} = \frac{1 + \gamma_i^1 + \gamma_i^2 + \cdots + \gamma_i^L}{w_0 + w_1 \gamma_i^1 + w_2 \gamma_i^2 + \cdots + w_L \gamma_i^L} \tag{6}$$

where R_i is the average number of transmission attempts for Node i and U_i is the average slot number of BT-decrement for one-frame transmission success for Node i. w_s is the expected value of initial BT value for s-th frame retransmission, which is expressed as

$$w_s = \begin{cases} \dfrac{2^s (CW_{min} + 1)}{2}, & \text{for } s = 0, 1, 2, \cdots, L' \\ \dfrac{CW_{max} + 1}{2}, & \text{for } s = L' + 1, L' + 2, \cdots, L \end{cases}, \tag{7}$$

where CW_{min} and CW_{max} are the minimum and maximum values of the contention window, respectively, L is the retransmission limit number and $L' = \log_2 \frac{CW_{max}+1}{CW_{min}+1}$.

G_i is defined based on the assumption that the network is in saturated condition [4]. The frame-existence probability is considered for expressing the non-saturated condition in this analysis. The frame-existence probability q_i is defined as the probability that Node i has at least one frame when it is in the channel-idle state. The BT decrement is carried out only when a node, which is in the channel-idle state, has frames.Therefore, an airtime that Node i decreases the BT in whole time can be expressed as

$$W_i = q_i Z_i. \tag{8}$$

The average spending time of BT decrement for one frame transmission success is expressed as $U_i \sigma$, where σ is system slot time. Therefore, an airtime that Node i decreases the BT in whole time is also expressed as

$$W_i = \lambda_i (1 - V_i) U_i \sigma. \tag{9}$$

where λ_i is frame-reception rate of Node i and V_i is buffer-blocking probability of Node i, which is obtained in Section. 2.4. From (8) and (9), frame-existence probability is obtained as

$$q_i = \frac{\lambda_i (1 - V_i) U_i \sigma}{Z_i} \tag{10}$$

In the string-topology network as shown in Fig 1, it is regarded that the frame-reception rate of Node i is the same as throughput of Node $i-1$. The reception rate for Nodes 0 is network offered load O. Namely, $E_{-1} = O$. From (2), the frame-reception rate of Node i is expressed as

$$\lambda_i = \frac{E_{i-1}}{P} = \frac{X_{i-1}(1 - \gamma_{i-1})}{T} \tag{11}$$

By using frame existence probability, the transmission probability of Node i is expressed as

$$\tau_i = q_i Z_i G_i = \lambda_i (1 - V_i) R_i \sigma \tag{12}$$

2.3. Flow Constraint in Multi-hop Networks

The transmission airtimes of network nodes are fixed by taking into account Network-layer properties. Because each airtime depends on the states of neighbor nodes,

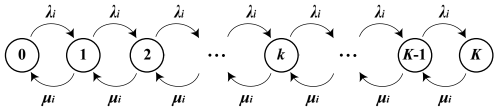

Figure 2: Buffer queueing model of Node i.

transmission airtimes of network nodes are associated with Network-layer properties.

When the retransmission counter reaches the retransmission limit L, the frame is dropped following the DCF policy. Additionally, the frame is dropped when the buffer of receiver is full. Therefore, the throughput of each node should satisfy

$$E_i = E_{i-1}(1-\gamma_{i-1}^{L+1})(1-V_i). \tag{13}$$

The relationship in (13), which is called as the flow-constraint condition, expresses the network-layer property. By eliminating E_i and P from (2), (11), and (13), we have

$$X_i = \frac{\lambda_i(1-V_i)T(1-\gamma_i^{L+1})}{1-\gamma_i} \tag{14}$$

2.4. Buffer-Blocking Probability

The buffer queue is modeled by using airtime expression and queueing theory. Figure 2 shows the buffer queuing model of Node i, where K is the buffer size and μ_i is frame-service rate of Node i. The frame-service time is defined as the average time interval between the instant when a frame reaches the top of the transmission-node buffer and the one when the frame is transmitted successfully to the next node. Namely, the frame-service time is the same as MAC access delay. The frame-existence probability in whole time with respect to Node i is expressed as

$$Q_i = \frac{X_i + q_i Z_i}{1-Y_i} = \frac{X_i + q_i Z_i}{X_i + Z_i}. \tag{15}$$

Because the ratio of the sum of the BT-freezing and BT-decrement durations to transmission duration is $\frac{Q_i Y_i + q_i Z_i}{X_i}$, the frame-service time of Node i is expressed as

$$D_{M_i} = TR_i\left(1 + \frac{X_i + q_i Z_i}{X_i}\right) = \frac{(TR_i + \sigma U_i)(1-V_i)}{X_i + Z_i} = \frac{1}{\mu_i} \tag{16}$$

The utilization rate of Node i is obtained as

$$\rho_i = \frac{\lambda_i}{\mu_i} = \frac{X_i + q_i Z_i}{(X_i + Z_i)(1-V_i)} = \frac{Q_i}{1-V_i}. \tag{17}$$

From the buffer-queueing model in Fig. 2, the steady state probability that the Node i has k frame is expressed as

$$\pi_{i,k} = \frac{\rho_i^k - \rho_i^{k+1}}{1-\rho_i^{K+1}}. \tag{18}$$

Table 1: System Parameters

Data rate	18 Mbps
ACK bit rate	12 Mbps
$DATA$	128 μsec
ACK	32 μsec
$SIFS$	16 μsec
$DIFS$	34 μsec
slot time(σ)	9 μsec
CW_{min}	15
CW_{Max}	1023
Buffer size K	100
Retransmission limit(L)	7

Because the buffer-blocking probability is the same as the steady state probability that the Node i has K frame, namely

$$V_i = \pi_{i,K} = \frac{\left(\frac{Q_i}{1-V_i}\right)^K - \left(\frac{Q_i}{1-V_i}\right)^{K+1}}{1-\left(\frac{Q_i}{1-V_i}\right)^{K+1}}. \tag{19}$$

From (5), (11), (12), (14) and (19), $7H$ algebraic equations are obtained. These equations contain $5H$ unknown parameters, which are X_i, τ_i, γ_i, λ_i, and V_i, for $i = 0, 1, 2, \ldots, H-1$. It is possible to fix the $5H$ unknown parameters and the offered loads are given. In this paper, Newton's method is applied for obtaining the $5H$ unknown parameters.

2.5. End-to-End Delay

In the string-topology multi-hop networks as shown in Fig. 1, the end-to-end delay is defined as the duration from the instant when a frame is generated at the source node to the one when the frame is received at the destination node, which is the sum of the single-hop transmission delay from Node 0 to Node $H-1$. Each single-hop transmission delay consists of two parts, which are the MAC access delay and the queueing delay.

By using the buffer-state probability, queueing delay of Node i is expressed as

$$D_{Q_i} = \sum_{k=1}^{K}\left[\frac{D_{M_i}}{2} + (k-1)D_{M_i}\right]\pi_{i,l} \tag{20}$$

Because the end-to-end delay is the sum of the single-hop transmission delay from Node 0 to Node $H-1$, the end-to-end delay of string-topology network is

$$D = \sum_{i=0}^{H-1}(D_{M_i} + D_{Q_i}). \tag{21}$$

3. Simulation Verification

In this section, the validity of the obtained analytical expressions are discussed by comparing with the results from ns-3 simulator [10]. Table 1 gives system parameters based on the IEEE 802.11a standards. The payload size is 100 bytes.

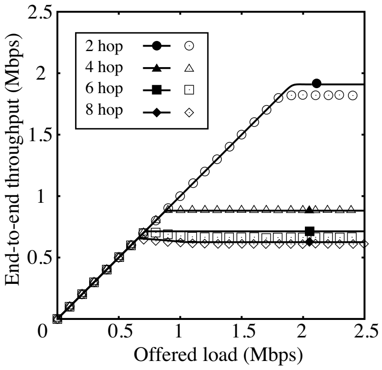

Figure 3: End-to-end throughput of analytical results (lines) and simulation results (plots) in fixed hop network as a function of offered load.

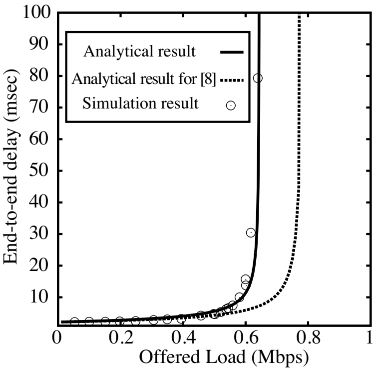

Figure 4: End-to-end delay of nine-hop network as a function of offered load.

Figure 3 shows end-to-end throughput in fixed hop network as a function of offered load. It is confirmed from Fig. 3 that the analytical predictions agree with the simulation results quantitatively. This means that the analytical equations presented in this paper can express both non-saturated and saturated condition described above.

Figure 4 shows end-to-end delays of the nine-hop network as a function of offered load. In Fig. 4, analytical results from the proposed analytical expressions and from the model in [8] are plotted. It is seen from Fig. 4 that analytical results in [8] have differences from simulation results. This is because it is assumed that all the properties are identical for all the network nodes in the conventional analysis approach. It is seen from Fig. 4 that analytical results from the proposed expressions agree with simulation result well. This is because the MAC-layer properties with respect to each node can be expressed individually in the presented analysis.

4. Conclusion

This paper has presented the expressions of throughput and delay for IEEE 802.11 string-topology multi-hop networks. For obtaining two evaluations analytically, the MAC-layer operations with respect to each node are considered by using the airtime expression. For expressing the properties in non-saturated condition, a new parameter, which is frame-existence probability, is defined. The analytical predictions agree with simulation results well, which show validity of the obtained analytical expressions.

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