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A Study of Stochastic Resonance as a Mathematical Model of Stationary Electrogastrography

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Abstract– Electrogastrography (EGG) is the recording of the electric activity of the stomach as measured on the abdominal surface. In this study, we aim to obtain a mathematical model of EGG during supine position and then speculate factors to describe the diseases resulting from constipation and erosive gastritis. Initially, we applied the Wayland algorithm to the EGG in order to measure the degree of determinism. As a result, we could not decide whether or not a chaotic process is appropriate for the mathematical model of the EGG. On the other hand, the waveform of the electric potential in the interstitial cells of Cajal is similar to the graphs of numerical solutions to the Van der Pol equation. We herein added the Van der Pol equation to a periodic function and random white noises named after the intestinal motility and other biosignals. The EGG and numerical solutions were compared and evaluated on the basis of the translation error in the Wayland algorithm and the maximum Lyapunov exponent. The EGG was well described by the stochastic resonance in the stochastic differential equations.

1. Introduction

Percutaneous electrogastrography (EGG) is a simple method employed to examine gastrointestinal activity without constraints. This method is generally applied to the measurement of human gastric electrical activity.

In 1921, Walter C. Alvarez reported performing EGG for the first time in humans [1]. In EGG, the electrical activity of the stomach is recorded by placing electrodes on the surface of the abdominal wall. In the stomach, a pacemaker placed on the side of the greater curvature generates electrical activity at a rate of 3 cycles per minute (3 cpm); the electrical signal is then transferred to the pyloric side [2]. Previously, it was difficult to measure this electrical activity because the EGG signal was composed of low-frequency components and high-frequency noise caused by the electrical activity of the diaphragm and heart. However, the accuracy of EGG measurements has improved recently, and the gastroenteric motility can be evaluated by means of spectrum analyses of the EGG signals [3].

Gastric electrical potential is generated by the interstitial cells of Cajal (ICCs) [4]. ICCs are pacemaker cells that spontaneously depolarize and repolarize at a rate of 3 cpm. These cells exhibit low-amplitude, rhythmic, and circular contractions only if the electrical potential is over a threshold.

In this study, the gastrointestinal motility was measured with the aim of obtaining a mathematical model of EGG and speculating factors to describe the diseases resulting from constipation and erosive gastritis.

Some studies have discussed solutions to the forward and inverse problems associated with the dynamics generating the gastric electrical potential [5]. These studies suggest that it is convenient to use current dipoles in an ellipsoid and to use computer simulations to generate a mathematical model for the stomach. However, results available on non-linear analyses of the EGG are insufficient. In order to examine whether or not a mathematical model describes EGG data appropriately, we have proposed a projection of time series on a two-dimensional plane, E_{trans}-λ, estimated by using the Wayland and Rosenstein algorithms.

The Wayland algorithm has been developed in order to evaluate the degree of determinism for dynamics that generate a time series [6]. This algorithm can estimate a parameter called translation error E_{trans} to measure the smoothness of flow in an attractor, which is assumed to generate the time-series data. In general, the threshold of the translation error required for classifying the time-series data as being deterministic or stochastic is 0.5, which is half that of the translation error resulting from a random walk. On the other hand, anomalous signals are introduced through nonstationary processes; for example, in the degeneration of singular points in the potential function involved in dynamical equation systems (DESs), dynamics's degrees of freedom increase or stochastic factors are added to them. The visible determinism in the latter case would be different from that in the case where random variables do not exist.

Chaos processes generate complexity in the attractor, which can be reconstructed from a time series [7]. The processes have sensitive dependence on the initial conditions and can be quantified using the Lyapunov exponent [8]. If the Lyapunov exponent has a positive value, the dynamics is regarded to be a chaos process. In this study, Rosenstein's algorithm [9][10] was used to calculate the maximum Lyapunov exponent λ (MLE). The chaos processes are sensitively dependent on the initial conditions and can be quantified using the Lyapunov exponent [10]. If the Lyapunov exponent has a positive value, the dynamics are regarded to be a chaos process.

According to the analysis of the degree of determinism for the time series dynamics, EGG data obtained for 30 min after postural change (Fig.1) were significantly different from the other EGG [11]. Autonomic nervous system during the later period could be represented by a stationary process because the autonomic nervous system controls the gastric electrical activity, which can be measured by EGG.

In this study, we suggest a mathematical model of the stationary EGG, and we examine whether numerical solutions to the mathematical model are fit to the EGG data.

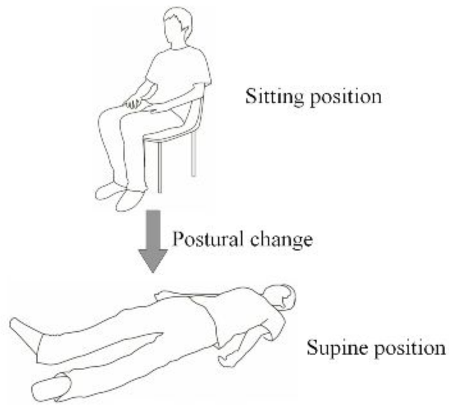

Fig.1 Postural change

2. Submission instructions

2.1. Mathematical Model and Numerical Simulations

The waveform of the electric potential in ICCs is similar to the graphs of numerical solutions to the Van der Pol equation (VPE). To construct a mathematical model of the EGG during supine position, we propose the following SDEs in which the periodic function is added to the VPE system as follows.

$$\dot{x} = y - \alpha\, grad f(x) + s(t) + \mu w_1(t) \quad (1.1)$$

$$\dot{y} = -x + \mu w_2(t) \quad (1.2)$$

$$s.t. \quad f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^2 \quad (2)$$

The function $s(t) = \sin \omega t$ and the white noise $w_i(t)$ represent intestinal movements and other biosignals, for example, myenteric potential oscillations that are weak and random, respectively $(i = 1,2)$. The double-well potential $f(x)$ generates depolarizations and repolarization in the interstitial cells.

In most cases, there exists an optimum value for the noise amplitude, which is named "stochastic resonance (SR)" for rather counterintuitive phenomena. In other words, the SR occurs when the S/N ratio (SNR) of a nonlinear system is maximized for a moderate value of noise intensity [12]. In this study, we numerically solve Eq.(1) and verify the SR in the SDEs. Further, we investigate the effect of SR and evaluate the SDEs as a mathematical model of the EGG.

1) We converted Eq.(1) into difference equations and obtained numerical solutions by using the Runge–Kutta–Gill formula as the numerical calculus; the initial values were (0, 0.5). Pseudorandom numbers were substituted for $w_i(t)$ $(i = 1,2)$. The pseudorandom numbers were generated by using the Mersenne Twister [13]. These numerical calculations were performed in N = 24000 time steps. Each unit of the time step was 0.05.
2) The values of the numerical solutions were recorded after every 20 time steps, which is related to a signal sampling rate of 1Hz. For each value of μ, we obtained 20 numerical solutions to Eq.(1).

2.2. Physiological Procedure

The subjects were 14 healthy people (7 males and 7 females) aged 21–25 years. Sufficient explanation of the experiment was provided to all subjects, and a written consent was obtained from them.

EGGs were obtained at 1 KHz for 150 min in the supine position by using an A/D converter (AD16-16U (PCI) EH; CONTEC, Japan). The EGGs were amplified by using a bio-amplifier (MT11; NEC Medical, Japan) and recorded by using a tape recorder (PC216Ax; Sony Precision Technology, Japan).

To remove the noise from the time series of EGG $\{y_j | j = 0,1,2\cdots, N-1\}$ data obtained at 1 kHz, resampling was performed at 1 Hz. For analysis, we obtained a resampled time series $\{x_i | i = 0,1,2\cdots,(N/1000)-1\}$ as follows:

$$x_0 = \frac{1}{1000}\sum_{j=0}^{999} y_j, \quad x_1 = \frac{1}{1000}\sum_{j=1\times 1000}^{1999} y_j, \cdots, \quad x_i = \frac{1}{1000}\sum_{j=i\times 1000}^{i\times 1000+999} y_j$$

The delay coordinates used are as follows:

$$\{\vec{x}_t = (x_t \quad x_{t+1} \quad \cdots \quad x_{t+(m-1)})\}$$

In this experiment, nine disposable electrodes (Blue Sensor; Medicotest Co. Ltd., Ølstykke, Denmark) were affixed on ch1–ch8 and e, as shown in Fig. 2. The electrode affixed on e was a reference electrode. Prior to the application of electrodes, the skin resistance was sufficiently reduced by using SkinPure (Nihon Kohden Inc., Tokyo, Japan). The EGG data obtained on ch5, which is the position closest to the pacemaker of gastrointestinal motility, were analyzed.

2.3. Calculation Procedure

1) Using the Wayland and Rosenstein's algorithms, estimate the translation errors (E_{trans}) and MLEs (λ) in the attractors generating EGG data except for 30 min after the postural change (Fig.1), respectively. Then, project the stationary EGG onto the E_{trans}–λ plane.
2) Calculate the mean values (m(i)) of the E_{trans} and λ for all the projections obtained in 1). According to statistical theory, 95.5 % of the EGGs would project onto the region

$\{\Re_s^2 | m(E_{trans}) \pm \sigma(E_{trans}) \times m(\lambda) \pm \sigma(\lambda)\}$ as shown in Fig.3.

3) Calculate the standard deviations (σ(i)) of E_{trans} and λ for all the projections obtained in 1).
4) Project the numerical solutions of Eq.(1) onto the E_{trans}–λ plane obtained in 1).
5) Count the number of numerical solutions (Ns) projected onto the region $\Re_s^2$ on the E_{trans} –λ plane.
6) Calculate the conformity ratio of Ns to 20, the number of numerical solutions for each value of μ.

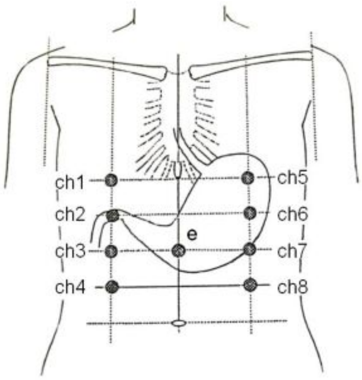

Fig.2 Positions of electrodes

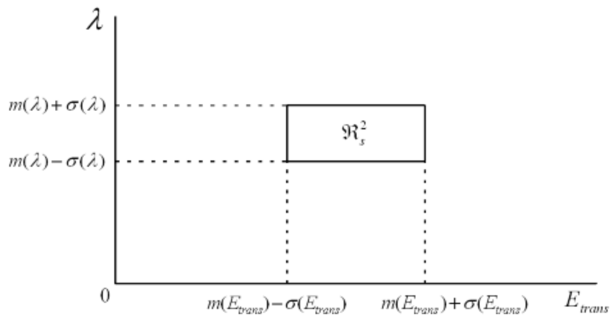

Fig.3 Region $\Re_s^2$

3. Results and Discussion

In the 24000 time steps, there was no exception wherein the numerical solutions did not diverge for $\mu = 1,2,\cdots,20$; the value of τ derived from the first component of the numerical solution was not different from that derived from the second component. We compared this numerical solution (Fig.4) with the EGG data (Fig.5). Temporal variations in the numerical solutions were similar to those in the EGG data.

With regard to Eq.(1), the SR occurred for an appropriate coefficient μ. Some biosystems are based on the nonlinear phenomenon of SR in which the detection of small afferent signals can be enhanced by the addition of an appropriate amount of noise [12]. Furthermore, this mechanism would facilitate behavioral, perceptive, and autonomic responses in animals and humans, such as information transfer in crayfish mechanoreceptors [14], tactile sensation in rats [15], and human hearing [16]. We examined whether or not the SR generated using Eq.(1) could describe the EGG time series.

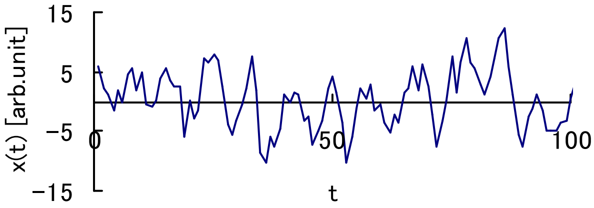

Fig.4 Example of numerical solutions (μ = 12)

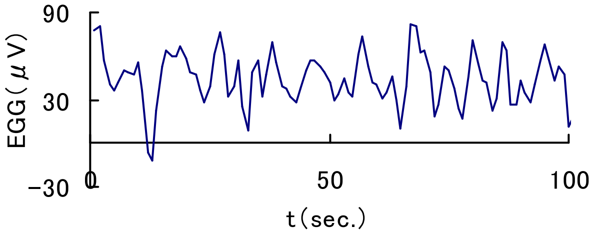

Fig.5 Typical EGG time series.
(Sampling frequency is 1 Hz.)

A cross-correlation coefficient $\hat{\rho}_{xs}$ between the observed signal *x(t)* and the periodic function *s(t)* was calculated as a substitute for the SNR in previous studies in which the occurrence of the SR was investigated. Fig.6 shows the $\hat{\rho}_{xs}$ between the numerical solutions and the periodic function in Eq.(1.1). The cross-correlation coefficient was maximized for a moderate value of noise intensity μ=12 (Fig.6). Thus, the SR could be generated using Eq.(1) with μ=12. All the EGG data and numerical solutions were projected onto the E_{trans} –λ plane (Fig.7).

With respect to the EGG data for 30 min after the postural change, the number of EGG data projected onto the region $\Re_s^2$ was less than the statistical standard as shown in Table 1. On the contrast, 100 % of the stationary EGG data were projected on the following region.

$$\{m(E_{trans}) \pm 2\sigma(E_{trans}) \times m(\lambda) \pm 2\sigma(\lambda)\}$$

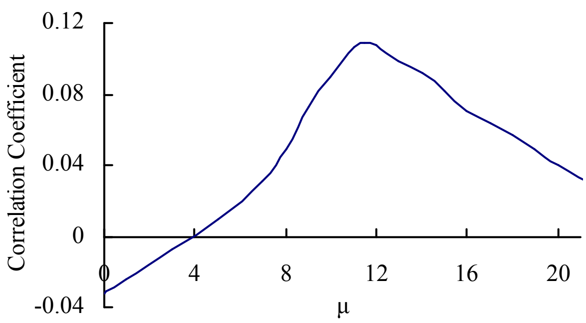

Fig.6 Autocorrelation function for each component of the numerical solution

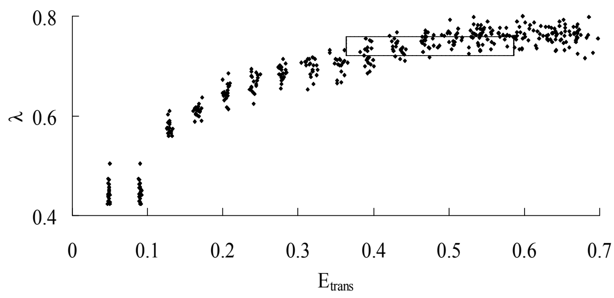

Fig.7 E_{trans}-λ plane

Table.1 The rate of data projected onto the region $\{m(E_{trans}) \pm b\sigma(E_{trans}) \times m(\lambda) \pm b\sigma(\lambda)\}$

b	Statistical standard	For 30 min after the postural change	The other
1	68.3%	54.8 %	57.1 %
2	95.5%	95.2 %	100 %

We quantitatively examined the conformity of the numerical solutions in the region $\Re_s^2$ on the E_{trans} -λ plane. The conformity ratio for μ=11 was the highest (Fig.8). Eq.(1) for μ=11 could be regarded as a mathematical model of the stationary EGG. Therefore, the SR describes the stationary EGG data appropriately.

Numerical solutions involved in the SR are highly correlated with the periodic function $s(t)$, which represent the intestinal movements (6cpm). The gastric electrical activity in a healthy person might synchronize with the intestinal activity.

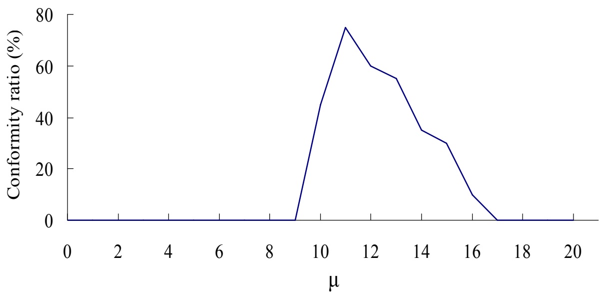

Fig.8 Conformity ratio for $\mu = 1, 2, \ldots, 20$

4. Conclusion

By projecting the stationary EGG data during supine position and the numerical solution onto the E_{trans} -λ plane, we qualitatively evaluated the affinity between them. The SR was statistically the most appropriate with regard to the mathematical model of the stationary EGG.

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