2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Realizing Ideal Chaotic Dynamics for Combinatorial Optimization using a Spatiotemporal Filter

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Abstract—This paper proposes an optimization algorithm, which utilizes the most stable spatiotemporal chaotic dynamics for solution search in a high dimensional space. Such chaotic dynamics is generated by a FIR filter, which has been applied to the chaotic CDMA in previous researches to minimize the cross-correlation among the sequences. In the proposed method, such filters are introduced at the output of decision functions of combinatorial optimization algorithms to realize an ideal chaotic search, which generates ideally complicated searching dynamics. In this paper, the proposed scheme is applied to two combinatorial optimization approaches, the Hopfield-Tank neural network with additive noise and a heuristic algorithm based on the neighboring solution search, which solve the Traveling Salesman Problems and the Quadratic Assignment Problems. Simulation results show that the proposed approach using the ideal chaotic dynamics simply improves the performance of the chaotic algorithms without searching appropriate parameter values even for large-scale problems.

1. Introduction

Chaotic dynamics have been shown effective for combinatorial optimization problems by many researches [1]–[8]. There are two major approaches using chaotic dynamics to avoid trapping at undesirable local minimum solutions. The first one introduces the chaotic fluctuation to the Hopfield-Tank neural network [9], and the second one drives local search heuristics by the chaotic dynamics. Although the first approach is applicable only to very small toy problems, the second approach can solve much more difficult and large-scale problems by introducing simple neighboring solution search heuristics and has been shown more effective than the conventional heuristic algorithms, such as the stochastic searches and the tabu searches [3, 4, 5].

In the previous researches, effectiveness of such chaotic dynamics for combinatorial optimization has been analyzed, and several important characteristics have been found [6, 8]. In the approach that adds chaotic sequences to each neuron in the Hopfield-Tank neural network solving an optimization problem [7], it has been clarified that the most important factor for high performance of the chaotic noise is a specific autocorrelation of the chaotic dynamics, by an analysis based on the method of surrogate data [8]. This effective chaotic noise has the autocorrelation with a negative value in lag 1 and damped oscillation. Such chaotic dynamics with negative autocorrelation has been also utilized in the chaotic CDMA [10, 11]. In those researches, the chaotic dynamics, whose autocorrelation of the sequences becomes $C(\tau) \approx C \times (-r)^{\tau}$, $r = -(2 - \sqrt{3})$, is used because the cross-correlation among the sequences becomes smallest by such autocorrelation. In Ref. [12], effects of the sequences, which has negative autocorrelation with damped oscillation, has been applied also to the combinatorial optimization problems, and it has been shown that the noise sequences having negative autocorrelation improves the performance of the neural networks to the same level as those with the chaotic noise. Since such negative autocorrelation noise minimizes cross-correlation, lower cross correlation may be also important for realizing complex spatiotemporal solution search in combinatorial optimization problems.

In the chaotic CDMA [11], such ideal chaotic noise, whose autocorrelation is $C(\tau) \approx C \times (r)^{\tau}$, $r = -(2 - \sqrt{3})$ has been generated by a FIR filter. It has been shown that such a filter improves bit error rate of the CDMA communication system for various spreading sequences, such as stochastic sequences and deterministic sequences.

In this paper, such a FIR filter making negative autocorrelation is applied to the combinatorial optimization methods, for realizing ideal spatiotemporal searching dynamics. Our proposed algorithm minimizes cross-correlation among the updating dynamics of neighboring solution search, and realizes ideally complex spatiotemporal searching dynamics. The proposed scheme is applied to two optimization approaches, the Hopfield-Tank neural networks with additive noise and the 2-opt heuristic method, solving the Traveling Salesman Problems (TSPs) and the Quadratic Assignment Problems (QAPs). Effectiveness of the proposed approach is investigated also for the large-scale problems up to 1173-city TSP.

2. Performance of the Optimization Neural Networks with Ideal Spatiotemporal Chaotic Dynamics

First, the effectiveness of the ideal chaotic dynamics is evaluated in the Hopfield-Tank neural network approach. This approach is based on the minimization of the energy function of the neural networks by asynchronous update of each neuron. However, since the original Hopfield-Tank neural networks stop search at a local minimum, the chaotic noise and other stochastic dynamics has been added to the neurons to avoid trapping at such undesirable states and to achieve much higher performance [7, 8].

In this paper, such a neuronal update function with additive noise is defined as follows,

$$x_{ik}(t+1) = f[\sum_{j=1}^{N}\sum_{l=1}^{N} w_{ikjl}x_{jl}(t) + \theta_{ik} + \beta z_{ik}(t)], \quad (1)$$

where $x_{ik}(t)$ is the output of the (i,k)th neuron at time t, w_{ikjl} is the connection weight between the (i,k) th and (j,l) th neurons, θ_{ik} is the threshold of the (i,k) th neuron, N is the number of cities, $z_{ij}(t)$ is a noise sequence added to the (i,j)th neuron, β is the amplitude of the noise, and f is the sigmoidal output function, $f(y) = 1/(1+\exp(-y/\epsilon))$. The noise sequence used for $z_{ij}(t)$ is normalized to zero mean and unit variance.

To apply this neural network to the TSP, the connection weights w_{ijkl} and the thresholds $theta_{ij}$ are set as the follows,

$$w_{ikjl} = -A\{\delta_{ij}(1-\delta_{kl}) + \delta_{kl}(1-\delta ij)\}$$
$$-Bd_{ij}(\delta_{lk+1} + \delta_{lk-1}), \quad (2)$$
$$\theta_{ij} = 2A, \quad (3)$$

where d_{ij} is the distance between the cities i and j, A and B are the weight of the constraint term (formation of a closed tour) and the objective term (minimization of total tour length), and δ_{ij} is the Kronecker delta, respectively.

For solving the QAPs, whose objective function is

$$F(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}b_{p(i)p(j)}, \quad (4)$$

By the Hopfield-Tank neural networks, the connection weights and the thresholds have to be set as follows,

$$w_{ijkl} = -A\{\delta_{ij}(1-\delta_{kl}) + \delta_{kl}(1-\delta ij)\} - Ba_{ij}b_{kl}, \quad (5)$$
$$\theta_{ij} = 2A. \quad (6)$$

Figs. 1 and 2 show the solvable performances of the above neural networks applied to the TSP and the QAP, with various noise sequence for $z_{ij}(t)$, such as the white Gaussian noise, the Chebyshev map chaos with the same dimension for all of neurons, that with different dimension for each neuron, and the logistic map chaos, $z^c_{ij}(t+1) = az^c_{ij}(t)(1-z^c_{ij}(t))$, with $a = 3.82$, $a = 3.92$ and $a = 4$. The abscissa axis is an amplitude of the noise β in Eq. (1). The solvable performance on the ordinate is the percentage of successful runs obtaining the optimum solution in 1000 runs with different initial conditions. The successful run obtaining of the optimum solution means that the optimum solution state is found at least once in a fixed iteration. The cutoff iterations for each run are set to 1024 for TSP and to 4096 for QAP, respectively. The parameters of the neural networks are $A = 1, B = 1, \epsilon = 0.3$ for the TSP, and $A = 0.35, B = 0.2, \epsilon = 0.075$ for the QAP, respectively. The problems introduced in Figs. 1 and 2 are a 20-city TSP in [8] and a QAP with 12 nodes, Nug12 in QAPLIB [13].

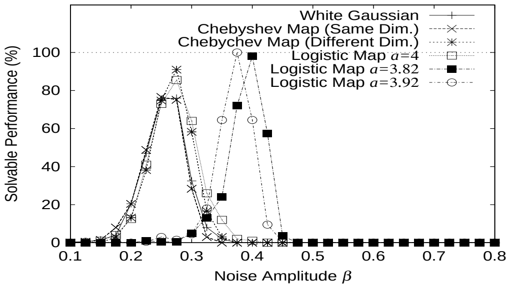

Figure 1: Solvable performance of the neural networks with the stochastic noise and the chaotic noise on a 20-city TSP.

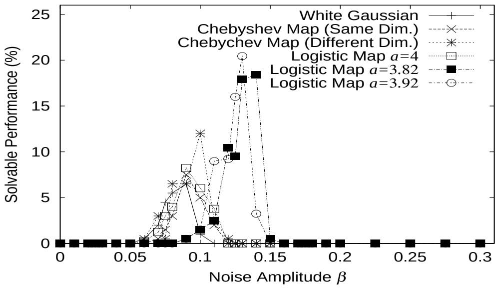

Figure 2: Solvable performance of the neural networks with the stochastic noise and the chaotic noise on a QAP with 12 nodes (Nug12).

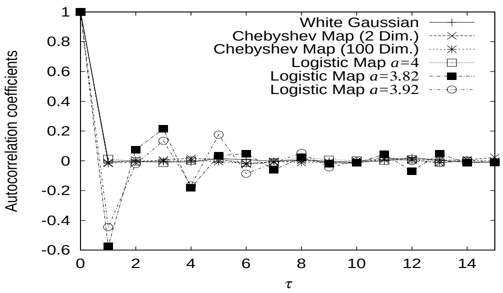

Figure 3: Autocorrelation of the noise sequence applied to the optimization neural networks.

The results in Figs. 1 and 2 show that the neural networks with the logistic map with $a = 3.82$ and $a = 3.92$ perform better than other noise, for both problems. From Fig. 3 showing autocorrelation coefficients of each noise, the sequences having the better performance, the logistic map with $a = 3.82$ and $a = 3.92$, have negative autocorrelation. On the other hand, autocorrelation of the others are almost zero. From these results, it is clear that the negative autocorrelation is very important for improving the performance of the combinatorial optimization algorithm based on the Hopfield-Tank neural networks.

3. Realizing Ideal Spatiotemporal Searching Dynamics by FIR Filter

In order to realize ideal searching dynamics which has negative autocorrelation, this paper proposes a novel scheme to apply the FIR filter to the decision function of the optimization algorithms. In the followings, first such a filter is applied to the Hopfield-Tank neural network approach, and then it is also applied to the 2-opt heuristics for the large-scale TSPs.

3.1. Ideal Spatiotemporal Searching Dynamics for the Hopfield-Tank Neural Networks

In order to make the outputs of the neurons having lower cross-correlation and to realizing ideally complex searching dynamics, the following FIR filter is introduced to make outputs having negative autocorrelation, which have been applied to the chaotic CDMA in Ref. [11],

$$\hat{f}(t) = \sum_{u=0}^{M} r^u f(t-u). \tag{7}$$

By setting $r = -(2 - \sqrt{3})$, ideal sequences to minimize the cross correlation have been generated for the chaotic CDMA. This paper introduces such a filter to the output function of each neurons, which can be described as follows, by replacing the output function f in Eq. (1) with the following equation,

$$f(y(t)) = 1/(1 + \exp(-\sum_{u=0}^{M} r^u y(t-u)/\epsilon)). \tag{8}$$

In the following experiments, $M = 8$ for each noise.

The solvable performance of the neural networks with the novel output function generating the ideal spatiotemporal chaotic sequence is shown in Figs. 4 and 5. From these results, we can see that the performances of the neural networks with the noise, whose autocorrelation is almost zero, could be much improved for all of the cases. In Fig. 5 for the QAP, the performance becomes even better than the logistic map chaos with $a = 3.82$ or $a = 3.92$, that have the best performance when the filter is not applied shown in Fig. 2.

From Figs. 4 and 5, for the noise sequences which already have negative autocorrelation, the logistic map with $a = 3.82$ and $a = 3.92$, $r = 0$ is the best. On the other hand, for other noise sequences having zero autocorrelation, the best value of autocorrelation parameter r for the FIR filter is around $-(2 - \sqrt{3})$, which is the same value as that used to generate optimal sequences to minimize the cross correlation in the chaotic CDMA researches. From this result, such negative autocorrelation may ideally minimize the cross correlation in this asynchronously updated neural network. Minimization of the cross correlation among the neurons makes ideally complex search in the searching space and the performance is much improved.

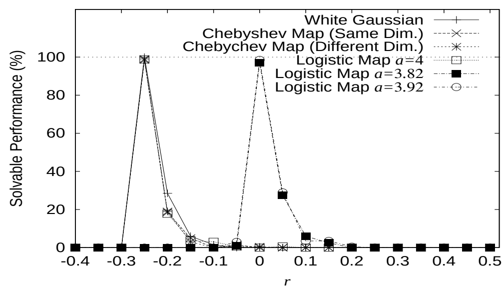

Figure 4: Solvable performance of a neural network with a FIR filter, on a 20-city TSP.

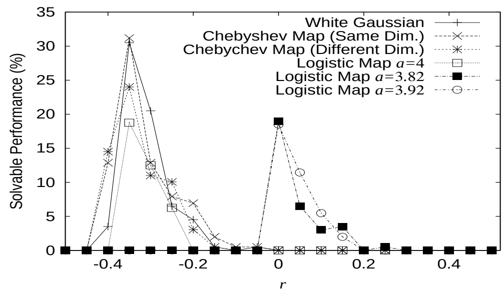

Figure 5: Solvable performance of a neural network with a FIR filter, on a QAP having 12 nodes.

3.2. Ideal Searching Dynamics for the 2-opt solving the TSP.

The FIR filter generating ideal spatiotemporal searching dynamics is also applied to the heuristic algorithms. This paper introduces the 2-opt method for the TSP, which is very simple but possible to be applied to very large scale TSPs.

In order to apply the FIR filter described above, the 2-opt method is defined with two dimensional decision output function, $s_{ij}(t+1) = \Delta_{ij}(t)$, where $\Delta_{ij}(t)$ is the improvement of the total tour length by the 2-opt exchange connecting the city i and j. In the original 2-opt method, $s_{ij}(t+1)$ is asynchronously calculated, and the solution is really updated when $s_{ij}(t+1) > 0$. In this paper, the FIR filter generating negative autocorrelation is introduced to the 2-opt by using the following decision equation,

$$\hat{s}_{ij}(t) = \sum_{u=0}^{M} r^u s_{ij}(t-u). \tag{9}$$

When $\hat{s}_{ij}(t+1) > 0$, the solution is really updated by the 2-opt exchange connecting the city i and j.

The results of the 2-opt with the FIR filter on a 100-city, 200-city, 318-city, 442-city and 1173-city TSPs, KroA100, KroA200, Lin318, Pcb442, Pcb1173 [13], are shown in Fig. 6, with changing the parameter r in the FIR filter. The best performance is the case with negative r around $-(2 - \sqrt{3})$, which minimizes cross correlation among $s_{ij}(t)$. From these results, it is clarified that the FIR filter which realizes ideal spatiotemporal searching dynamics is effective also for the heuristic methods.

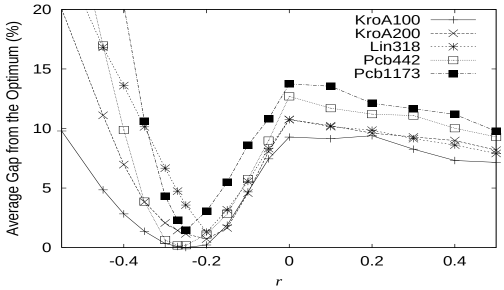

Figure 6: Solvable performance of the 2-opt with a FIR filter, on the large TSPs.

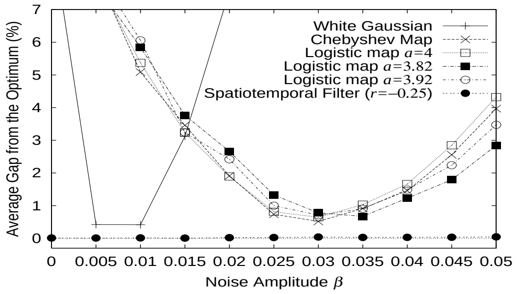

Figure 7: Solvable performance of the 2-opt with a FIR filter, on a 100-city TSP, KroA100.

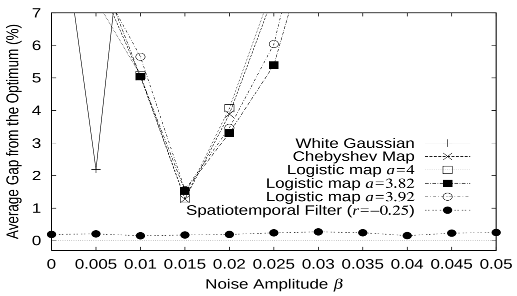

Figure 8: Solvable performance of the 2-opt with a FIR filter, on a 442-city TSP, Pcb442.

In Figs. 7 and 8, the results of the 2-opt with FIR filter are compared with the 2-opt method with five kinds of additive noise sequences, which was introduced in the previous section, on a 100-city and a 442-city TSPs, KroA100 and Pcb442, respectively. From both results, the logistic map noises with negative autocorrelation, $a = 3.82$ and $a = 3.92$ do not provide better solutions than other noise. On the other hand, the proposed filtered search is effective also for the heuristic methods, and improves the performance of the algorithm much better than other algorithms.

4. Conclusion

This paper proposes a scheme to apply a FIR filter to the output of each decision function for solving combinatorial optimization problems. Our algorithm realizes ideal complex spatiotemporal searching dynamics, by minimizing the cross correlation among the decision functions using negative autocorrelation. The conventional researches on the chaotic CDMA [11] have shown that such filtered sequences having autocorrelation $C(\tau) \approx C \times (-r)^{\tau}$ with r around $-(2 - \sqrt{3})$, has the lowest cross-correlation. In the proposed algorithm, such lower cross-correlation makes very complicated searching dynamics, and the performance of the algorithms with such filter can be improved.

Our algorithm is very simple and no need to find the appropriate parameter values carefully. Moreover, it is possible to be applied to various algorithms. Therefore, it should be an interesting future work to apply this proposed approach to various problems and various algorithms.

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