2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Synchronization in a Discrete Vibrate-and-Fire Neural Network

Hidehiro Nakano† and Arata Miyauchi†

†Faculty of Knowledge Engineering, Tokyo City University
1-28-1, Tamazutsumi, Setagaya-ku, Tokyo, 158-8557 Japan
Email: {nakano, miyauchi}@ic.cs.tcu.ac.jp

Abstract—This paper presents a discrete vibrate-and-fire neuron and its pulse-coupled neural network. A single neuron has two discrete state variables; radius and angle, and can generate various periodic pulse-trains. The pulse-coupled neural network of the neurons can exhibit various synchronous and quasi-synchronous phenomena. In this paper, some typical phenomena are presented, and applications of the proposed model are discussed.

1. Introduction

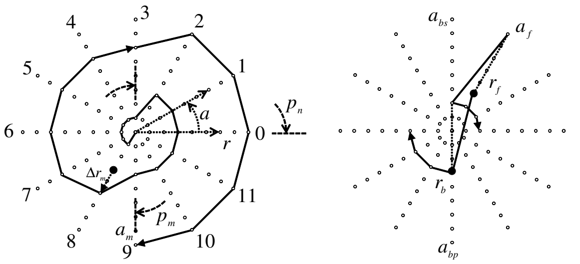
Integrate-and-Fire Neurons (IFNs) are known as simple neuron models. Applying an input to an IFN, a state of the IFN changes. If the state reaches a threshold, the IFN fires and generates a single spike signal. Repeating in this manner, the IFN generates spike-trains. IFNs can exhibit various periodic and aperiodic phenomena applying sinusoidal stimuli[1]. Then, the IFNs can generate various spike-trains. Coupled by the spike-trains, a Pulse-Coupled Neural Network (PCNN) consisting of plural IFNs can be constructed. PCNNs can exhibit various synchronous and asynchronous phenomena[2][3]. Many applications of the PCNNs have been proposed, e.g., associative memories[4]-[6], image processing[7][8], and wireless sensor networks[9][10]. Also, analog chips of IFNs and their PCNNs have been developed[11]. Since PCNNs can be composed by the small number of transistors, they can be implemented on various engineering systems. However, it should be noted that a simple single IFN without stimuli can exhibit simple periodic phenomena only, and its PCNNs can exhibit simple periodic synchronization only. One of the reasons is that a single IFN has only one state variable. Adding one more state variable to IFNs, Vibrate-and-Fire Neurons (VFNs) can be constructed. VFNs can exhibit various chaotic phenomena, and can generate various spike-trains with chaotic interspike-intervals. Coupled by the spike-trains, a Chaotic Pulse-Coupled Neural Network (CPCNN) consisting of plural VFNs can be constructed. The CPCNNs can exhibit various synchronous and quasi-synchronous phenomena[12]-[14].

CPCNNs has been applied to a Data Gathering Scheme (DGS) in wireless sensor networks[10]. Using VFNs as timers to control sleep time of each sensor node, power supply of transceivers can be turned off when sensor nodes do not transmit or relay sensor information, and the number of transmitting and receiving sensor information can be reduced. Hence, energy consumption of each sensor node can be saved. Considering resource limitation of each sensor node, this DGS does not assume to have complex routing algorithms, but to use simple broadcast-based communication. Then, quasi-synchronization in CPCNNs can reduce the number of redundant transmitting and receiving sensor information. However, the conventional works have presented only simulation results; implementation of CPCNNs on sensor nodes have not been discussed. Therefore, the implementation considering resource limitation of sensor nodes should be considered. The implementation methods are classified into the following three patterns: (1) analog hardware implementation, (2) digital hardware implementation, and (3) software implementation. In order to realize the methods (2) and (3), discrete neuron models and their PCNN are suitable. Recently, an interesting digital neuron model has been proposed[15]. This model has two discrete state variables with simple dynamics, and can exhibit rich phenomena. In this model, the trajectory of the neuron rotates around an origin on a phase plane, and moves on the phase plane with a constant velocity. However, in order to synchronize plural neurons in a PCNN, each neuron should have a constant self angular frequency rather than a constant velocity. Then, error between each trajectory of neurons decreases when they are coupled by spike-trains[12]. Purpose of this study is the development of such a discrete VFN model.

This paper presents a Discrete VFNs (DVFNs) and its PCNN. DVFN consists of two discrete state variables; radius and argument. The trajectory vibrates with a constant self angular frequency under a threshold. If the state reaches the threshold, the state is reset to a base state. Repeating in this manner, the DVFN can exhibit various phenomena. Also, these phenomena can coexist depending on initial states. PCNN consisting of plural DVFNs can exhibit various synchronous and quasi-synchronous phenomena. In this paper, some typical phenomena are presented, and applications of the proposed model are discussed.

2. Discrete Vibrate-and Fire Neuron (DVFN)

This section presents a model of a Discrete Vibrate-and-Fire Neuron (DVFN), and typical phenomena in the model are shown. DVFN consists of two discrete state variables; radius r and argument a. The basic dynamics of a single DVFN is described by the following equation.



(a) (b)

Figure 1: Trajectories of DVFN. (a) Rotating dynamics. (b) Resetting dynamics.

(A) Rotating dynamics: if $r(n) < r_f \ \vee \ a(n) \neq a_f$

$$r(n+1) \quad = \quad \begin{cases} r(n) + \Delta r_m, & \text{if } (a(n) \bmod p_m) = a_m \\ r(n), & \text{otherwise} \end{cases} \tag{1}$$

$$a(n+1) \quad = \quad (a(n)+1) \bmod p_n \tag{2}$$

(B) Resetting dynamics: if $r(n) \geq r_f \ \wedge \ a(n) = a_f$

$$r(n+1) \quad = \quad |r(n) - r_f - r_b| \tag{3}$$

$$a(n+1) \quad = \quad \begin{cases} a_{bp}, & \text{if } r(n) - r_f - r_b \geq 0 \\ a_{bs}, & \text{otherwise} \end{cases} \tag{4}$$

The parameters of the DVFN are summarized in Table 1, and trajectories of the DVFN are illustrated in Fig. 1. As shown in Fig. 1(a), the trajectory rotates around the origin, and the radius r increases periodically for the angle a. This period is decided by the parameters p_m and a_m. As shown in Fig. 1(b), if the radius exceeds the threshold r_f at the angle a_f, the DVFN fires and the radius and angle are reset to a base state. The values of the radius and angle in this case are decided by the parameters r_b, a_{bp} and a_{bs}. This resetting dynamics are represented by the injective function. Hereafter, we select r_b as a control parameter. The other parameters are fixed as shown in Table 1.

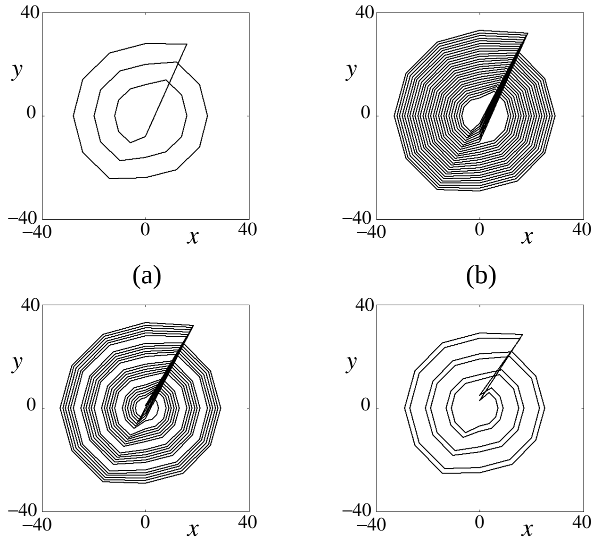
Fig.2 shows typical attractors from a single DVFN. In the figure, the variables x and y are described by

$$x = r\cos\left(\frac{2\pi a}{p_n}\right), \quad y = r\sin\left(\frac{2\pi a}{p_n}\right) \tag{5}$$

As the parameter r_b varies, the DVFN can exhibit various periodic phenomena. Since the DVFN is a discrete model,

Table 1: Parameters of DVFN.

Parameter	Denotation	Value
Δr_m	Incremental radius	4
p_m	Period for increment	6
a_m	Offset of angle for increment	3
p_n	Period for rotation	12
r_f	Threshold of radius for firing	30
a_f	Angle for firing	2
r_b	Offset of radius for resetting	(variable)
a_{bp}	Primary angle for resetting	9
a_{bs}	Secondary angle for resetting	3



(a) (b) (c) (d)

Figure 2: Typical attractors from a single DVFN. (a) $r_b = -6$. (b) $r_b = -3$. (c) $r_b = 3$. (d) $r_b = 6$.

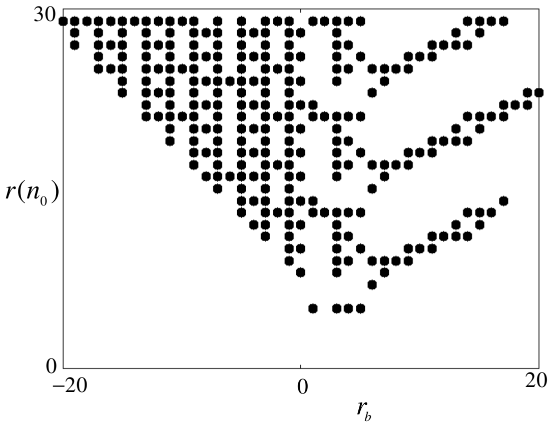
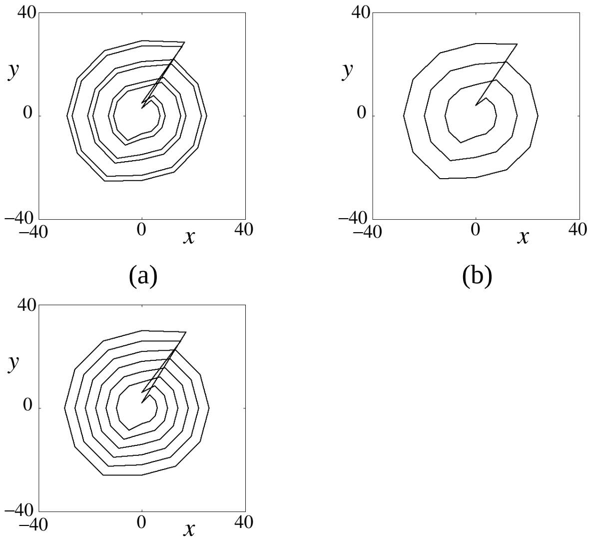


Figure 3: Bifurcation diagram for r_b.

it can never generate chaos. However, as $r_b = -3$ and $r_b = 3$, the trajectories have complex behavior with long period. In order to analyze characteristics of the DVFN, trajectories on the section $a(n) = 0$ are focused on. As a trajectory starts from $(r(n), a(n)) = (r(n_0), 0)$, it must returns to $(r(n_1), 0)$. we then obtain the series of $r(n_0)$. Fig.3 shows the bifurcation diagram. Horizontal axis is the parameter r_b, and vertical axis is the series of $r(n_0)$. Depending on the parameter r_b, the DVFN can exhibit various periodic phenomena, and the structure seems not to be simple. Also, coexistence of plural attractors can be observed depending on initial states. Typical coexisting phenomena are shown in Fig. 4. As $r_b = 6$, at least three attractors coexist depending on initial states. For the other parameters, the DVFN can also exhibit various coexisting phenomena. The single DVFN can exhibit rich phenomena.



(c)

Figure 4: Coexistence of plural attractors ($r_b = 6$).

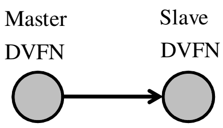


Figure 5: A master-slave PCNN of 2 DVFNs.

3. Master-slave pulse-coupled neural network (PCNN) of DVFNs

Next, a simple master-slave pulse-coupled neural network (PCNN) consisting of 2 DVFNs is considered. The dynamics of the PCNN is described by the following equation.

(A1) Rotating dynamics of the master:
if $r_1(n) < r_f \;\vee\; a_1(n) \neq a_f$

$$r_1(n+1) = \begin{cases} r_1(n) + \Delta r_m, & \text{if } (a_1(n) \bmod p_m) = a_m \\ r_1(n), & \text{otherwise} \end{cases} \quad (6)$$

$$a_1(n+1) = (a_1(n) + 1) \bmod p_n \quad (7)$$

(B1) Resetting dynamics of the master:
if $r_1(n) \geq r_f \;\wedge\; a_1(n) = a_f$

$$r_1(n+1) = |r_1(n) - r_f - r_b| \quad (8)$$

$$a_1(n+1) = \begin{cases} a_{bp}, & \text{if } r_1(n) - r_f - r_b \geq 0 \\ a_{bs}, & \text{otherwise} \end{cases} \quad (9)$$

(A2) Rotating dynamics of the slave:
if $r_2(n) < r_f \;\vee\; a_2(n) \neq a_f$

$$r_2(n+1) = \begin{cases} r_2(n) + \Delta r_m, & \text{if } (a_2(n) \bmod p_m) = a_m \\ r_2(n), & \text{otherwise} \end{cases} \quad (10)$$

$$a_2(n+1) = (a_2(n) + 1) \bmod p_n \quad (11)$$

(B2) Resetting dynamics of the slave:
if $r_2(n) \geq r_f \;\wedge\; a_2(n) = a_f$

$$r_2(n+1) = |r_2(n) - r_f - r_b| \quad (12)$$

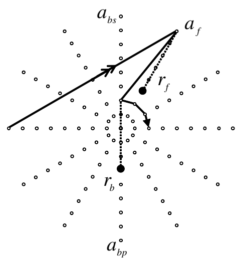
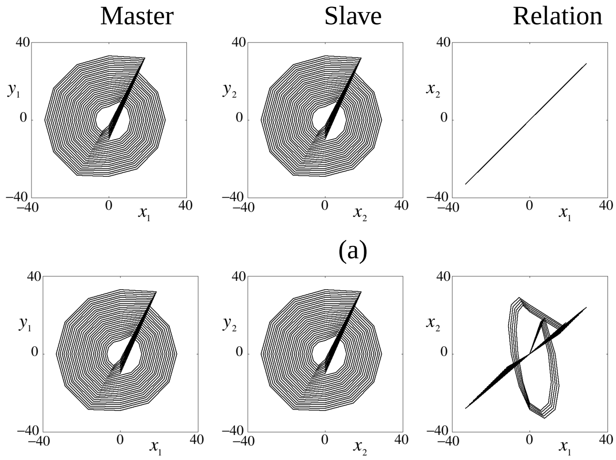


Figure 6: Trajectory of a slave DVFN: coupling dynamics.



(b)

Figure 7: Typical synchronous and quasi-synchronous phenomena from a master-slave PCNN ($r_b = -3$). (a) Synchronization. (b) Quasi-synchronization.

$$a_2(n+1) = \begin{cases} a_{bp}, & \text{if } r_2(n) - r_f - r_b \geq 0 \\ a_{bs}, & \text{otherwise} \end{cases} \quad (13)$$

(C2) Coupling dynamics of the slave:
if $r_1(n) \geq r_f \;\wedge\; a_1(n) = a_f$

$$a_2(n) = a_f \quad (14)$$

If the coupling dynamics (C2) does not exist, each DVFN behaves independently. Coupling is realized in the firing timings of the master. As the master DVFN fires, the slave DVFN is stimulated and the angle of the slave DVFN is instantaneously set to a_f, holding the radius of the slave DVFN as shown in Fig. 6. Since a_f is the parameter of angle for firing, this dynamics can be regarded as a kind of excitatory coupling.

Fig. 7 shows typical phenomena in the master-slave PCNN. As shown in Fig. 7(a), the master and slave exhibit synchronous phenomena. On the other hand, as shown in Fig. 7(b), for different initial states the master and slave exhibit quasi-synchronization. In the simulations, we have confirmed the other quasi-synchronous patterns. These results do not mean that synchronization is unstable. Responses of the slave DVFN for the master DVFN with long period can be classified into some synchronous and quasi-synchronous patterns.

Such a quasi-synchronization is important for an application to wireless sensor networks[10]. In a Data Gathering Scheme (DGS) by using a CPCNN, VFNs are used as timers to control sleep time of each sensor node. A sink node gathering sensor information from all sensor nodes broadcasts stimulus spikes to synchronize all timers of sensor nodes. Using the DGS, power supply of transceivers can be turned off when sensor nodes do not transmit or relay sensor information, and the number of transmitting and receiving sensor information can be reduced. Considering resource limitation of each sensor node, this DGS does not assume to have complex routing algorithms, but to use simple broadcast-based communication. Then, quasi-synchronization in CPCNNs can reduce the number of redundant transmitting and receiving sensor information. Therefore, the observed quasi-synchronization in the PCNN consisting of the DVFNs may contribute to efficient data gathering schemes in the wireless sensor networks.

4. conclusion

This paper has proposed a simple Discrete Vibrate-and-Fire Neuron (DVFN) and its Pulse-Coupled Neural Network (PCNN). A single DVFN can exhibit various periodic phenomena and their coexisting phenomena depending on initial states. A master-slave PCNN of the DVFNs can exhibit synchronous and quasi-synchronous phenomena depending on initial states. In the simulation experiments, typical phenomena have been shown.

Future problems include (a) analysis of bifurcation phenomena based on return maps, (b) analysis of stability of synchronous phenomena, and (c) application to data gathering scheme in wireless sensor networks.

References

[1] J. P. Keener, F. C. Hoppensteadt, and J. Rinzel, “Integrate-and-fire models of nerve membrane response to oscillatory input,” *SIAM J. Appl. Math.*, vol. 41, pp. 503-517, 1981.

[2] R. E. Mirollo and S. H. Strogatz, “Synchronization of pulse-coupled biological oscillators,” *SIAM J. Appl. Math.*, vol. 50, pp. 1645-1662, 1990.

[3] E. Catsigeras and R. Budelli, “Limit cycles of a bineuronal network model,” *Physica D*, vol. 56, pp. 235-252, 1992.

[4] E. M. Izhikevich, “Weakly pulse-coupled oscillators, FM interactions, synchronization, and oscillatory associative memory,” *IEEE Trans. Neural Networks*, vol. 10, no. 3, pp. 508-526, 1999.

[5] G. Lee and N. H. Farhat, “The bifurcating neuron network 1,” *Neural Networks*, vol. 14, pp. 115-131, 2001.

[6] G. Lee and N. H. Farhat, “The bifurcating neuron network 2: An analog associative memory,” *Neural Networks*, vol. 15, pp. 69-84, 2002.

[7] J. J. Hopfield and A. V. M. Herz, “Rapid local synchronization of action potentials: Toward computation with coupled integrate-and-fire neurons,” *Proc. Natl. Acad. Sci.*, vol. 92, no. 15, pp. 6655-6662, 1995.

[8] S. R. Campbell, D. Wang, and C. Jayaprakash, “Synchrony and desynchrony in integrate-and-fire oscillators,” *Neural Comput.*, vol. 11, pp. 1595-1619, 1999.

[9] N. Wakamiya and M. Murata, “Synchronization-based data gathering scheme for sensor networks,” *IEICE Trans. Communications*, vol. E88-B, no. 3, pp. 873-881, 2005.

[10] H. Nakano, A. Utani, A. Miyauchi, and H. Yamamoto, “Synchronization-Based Data Gathering Scheme Using Chaotic Pulse-Coupled Neural Networks in Wireless Sensor Networks,” *Proc. of WCCI*, 2008. (accepted)

[11] T. Asai and Y. Amemiya, “Frequency-and temporal-domain neural competition in analog integrate-and-fire neurochips,” *Proc. of IJCNN*, 2002, pp. 1337-1342.

[12] H. Nakano and T. Saito, “Basic dynamics from integrate-and-fire chaotic circuits with a periodic input,” *IEICE Trans. Funds.*, vol. E84-A, no. 5, pp. 1293-1300, 2001.

[13] H. Nakano and T. Saito, “Basic dynamics from a pulse-coupled network of autonomous integrate-and-fire chaotic circuits,” *IEEE Trans. Neural Networks*, vol. 13, no. 1, pp. 92-100, 2002.

[14] H. Nakano and T. Saito, “Grouping Synchronization in a Pulse-Coupled Network of Chaotic Spiking Oscillators,” *IEEE Trans. Neural Networks*, vol. 15, no. 5, pp. 1018-1026, 2004.

[15] T. Hishiki and H. Torikai, “A Novel Rotate-and-Fire Digital Spiking Neuron and its Neuron-like Bifurcations and Responses,” *IEEE Trans. Neural Networks*, Vol. 22, No. 5, pp. 752-767, 2011.