



Effects of chaotic fluctuation on the Hopfield model for the traveling salesman problem

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Abstract—In previous studies, it has been shown that the chaotic or stochastic fluctuation which has a negative and declining autocorrelation contributes to closer reach to the optimum solution in the Hopfield net. In this study, we examine this property from another viewpoint of evaluation. We evaluate the performance by reaching speed to the neighborhood of the optimum solution for a constant learning period. We made the Hopfield net solve the traveling salesman problem with chaotic fluctuations, which are generated from the logistic map and Bernoulli map. The results show that negative autocorrelations of fluctuations may contribute to better performance in both maps, which corresponds to the results of previous studies. In addition, they suggest that even periodic signals can contribute to the performance, but intermittency never does.

1. Introduction

Hopfield neural network [1] has been used for solving combinatorial optimization problems with the theory of minimizing the energy. However, in most of the cases, the solution tends to be dropped into local minimum and cannot get out of it, so that we cannot attain the optimum solution. In order to avoid such situation, many methods in which various fluctuations are supplied to Hopfield neurons have been proposed. In recent years, the chaotic fluctuation helps to get better performance for searching an optimum solution in neural network [2], [3], and [4]. For example, Hayakawa and Sawada showed that the chaotic fluctuation which was generated from logistic map is able to gain the better performance [2]. Furthermore, using the logistic map, Hasegawa et al. insisted that the negative autocorrelation should be the most important factor to improve the performance [5]. In this study, we examine this property from another viewpoint of evaluation. We evaluate the performance by reaching speed to the neighborhood of the optimum solution for a constant learning period. We made the Hopfield net solve the traveling salesman problem (abbr. TSP) with chaotic fluctuations. The fluctuation signal sequence is generated from modified Bernoulli map [6] as well as the logistic map. The results show that negative autocorrelations of fluctuations may contribute to better performance in both maps, which corresponds to the results of previous studies. In addition, they suggest that even

periodic signals can contribute to the performance, but intermittency never does.

2. Methods

2.1. Neural Network Model

In Hopfield Network, we aim to minimize an energy function which is defined in the network as follows:

$$E = A \sum_{i=1}^N \left(\sum_{k=1}^N x_{ik}(t) - 1 \right)^2 + B \sum_{k=1}^N \left(\sum_{i=1}^N x_{ik}(t) - 1 \right)^2 + D \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N d_{ij} x_{ik}(t) x_{jk+1}(t), \quad (1)$$

where N is the number of cities, d_{ij} is the distance between i th city and j th city, and A , B , and D are constants. The first and second terms are the constraint terms and the third is the objective term, respectively. In order to decrease this energy, all neurons are updated by the following equation:

$$x_{ik}(t+1) = f \left[\sum_{j=1}^n \sum_{l=1}^n w_{ijkl} x_{jl}(t) + \theta_{ik} \right] + \beta z_{ik}(t), \quad (2)$$

where $x_{ik}(t)$ is the output of the (i, k) th neuron, w_{ijkl} is a connection weight between the (i, k) th and (j, l) th neurons, θ_{ik} is the threshold of the (i, k) th neuron, β is the amplitude of a fluctuation, and z_{ik} is the fluctuation for the (i, k) th neuron. The (i, k) th neuron represents that i th city is visited at k th order. According to Eqs.(1) and (2), the connection weight w_{ijkl} and a threshold θ_{ij} are shown in following equations:

$$w_{ijkl} = -A\delta_{ij}(1 - \delta_{kl}) + B\delta_{kl}(1 - \delta_{ij}) - Dd_{ij}(\delta_{k+1} + \delta_{l-k-1}), \quad (3)$$

$$\theta_{ij} = A + B, \quad (4)$$

where $\delta_{ij} = 1$ when $i = j$, otherwise $\delta_{ij} = 0$.

2.2. Fluctuation

In the previous research, the chaotic fluctuation generated by the logistic map helps neural network to raise the frequency of getting the optimum solution [4]. In this study, we use two kinds of fluctuations generated by two nonlinear dynamical maps: the logistic map and the modified Bernoulli map. The logistic map is shown in the following equation:

$$z(t+1) = az(t)(1-z(t)) \quad (5)$$

where a is a bifurcation parameter. As a changes with this range, $0 \leq a \leq 4$, $z(t+1)$ behaves either chaotically or not. We focus on the range of $a = 3.6$ to 4.0 , which generates chaotic or periodic fluctuations. The modified Bernoulli map [6], which is shown in the following equation:

$$z(t+1) = \begin{cases} z(t) + 2^{b-1}(1-2\epsilon)z(t)^b + \epsilon & (0 \leq z(t) \leq \frac{1}{2}) \\ z(t) - 2^{b-1}(1-2\epsilon)(1-z(t))^b - \epsilon & (\frac{1}{2} < z(t) \leq 1), \end{cases} \quad (6)$$

where b is a bifurcation parameter and as b changes with this range, $0 \leq b \leq 3$, $z(t)$ behaves intermittently chaotically or not, and ϵ is equal to 1.0×10^{-13} .

2.3. Evaluation of Performance

After we executed the iterations of updates for 100 times in each TSP, we counted the number when the energy will decrease below $E_0=9.0$. We regard the rate of counts to the total number of trials as the performance in this study.

3. Simulation Results

3.1. Effects of Fluctuations

The energy after learning depends on what kind of fluctuations we choose. We compared the performance with five different fluctuations: uniformed random numbers, chaotic fluctuations from logistic map with parameter $a = 3.92$ and $a = 3.95$, and those from Bernoulli map with parameter $b = 0.2$ and $b = 2.0$. 18-city TSP is solved with $A = 1.0$, $B = 1.0$, and $D = 1.0$.

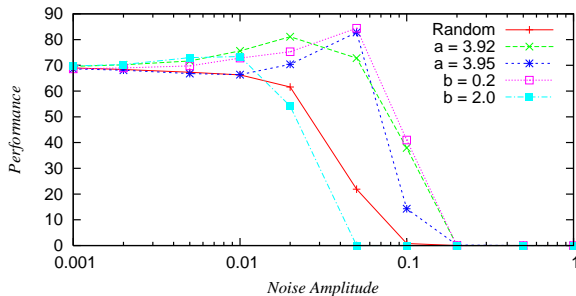


Figure 1: The effects of chaotic fluctuations in 18-city TSP.

Figure 1 shows the relationship between the amplitude of fluctuations and performance. Where the amplitude is 0.05, there are peaks when $a = 3.95$ and $b = 0.2$. On the other hand, the performance with random fluctuations monotonically decreases. This result shows that chaotic fluctuation promotes the performance better than random fluctuation does.

3.2. Effects of Stochastic Properties

Now, we use the surrogate data method [7] in order to clarify which stochastic property of chaotic fluctuations contributes to raise the performance. We adopted three kinds of data. We call the fluctuations which remains to be produced by logistic map "Original data (abbr. Original)", the one which is displaced randomly "Random shuffled (abbr. RS)", and the one which is shuffled but the frequency spectrum is remained "Fourier transformed (abbr. FT)".

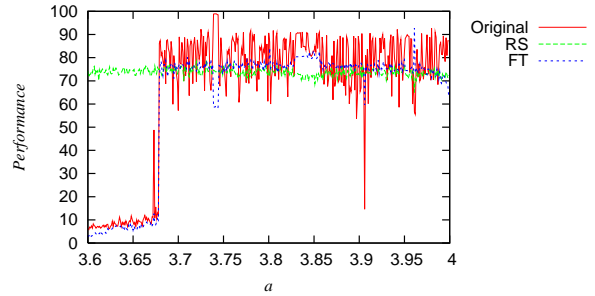


Figure 2: The effects of modified logistic-fluctuation. 18-city TSP is solved with $A = 1.0$, $B = 1.0$, $D = 1.0$, and $\beta = 0.05$.

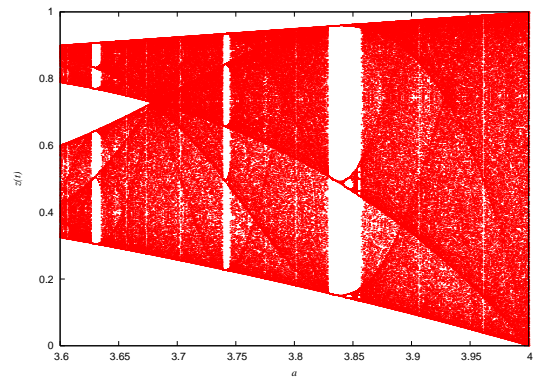


Figure 3: The bifurcation diagram of logistic map.

Figure 2 shows that FT fluctuation succeeded secondly. Also especially during the periodical fluctuation ($3.83 \leq a \leq 3.86$), the performance gets greater. We are also able to compare Fig. 2 with Fig. 3 to see the performance is better when $z(t)$ behaves periodically. However, with RS fluctuation

tuation, because its autocorrelation is destroyed, the performance holds low level. Therefore the result shows that keeping an autocorrelation makes better performance.

Next, we try the other nonlinear dynamical map, Bernoulli map. First, we show how the performance changes as the bifurcation parameter b increases.

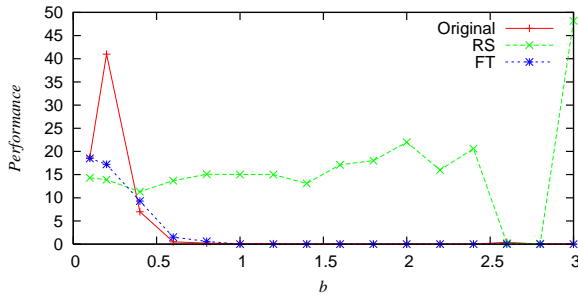


Figure 4: The effects of modified Bernoulli-fluctuations.

Figure 4 shows that the smaller parameter b is, the better performance is gained. This may be because the intermittency increases as the value of b increases. Now, we also adopt the surrogate data method to the chaotic fluctuations from Bernoulli map to see which stochastic property contributes to gain high performance. RS performed in the same way regardless of parameter b . On the other hand, Original and FT show high performance only when b is small.

3.3. Autocorrelation of Fluctuations

To analyze the reason why chaotic and FT-surrogate fluctuations are better than RS, we calculated the autocorrelations of the fluctuation signals. Fig.5 shows the autocorrelations of logistic fluctuations when $a = 3.65, 3.82,$ and 3.84 .

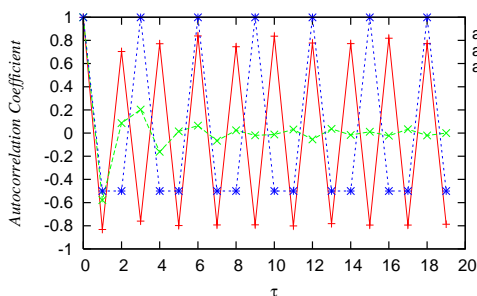


Figure 5: Autocorrelations of various logistic-fluctuations.

When $a=3.82$, negative and declining autocorrelation clearly emerges. At that time, the performance is also the best.

In the case of modified Bernoulli-fluctuations, we plot the autocorrelations when $b = 0.2$ and 0.8 . When $b = 0.2$,

the same property of autocorrelation is clear and the performance is the best.

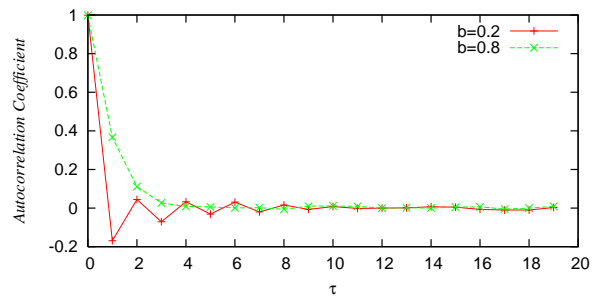


Figure 6: Autocorrelations of various Bernoulli-fluctuations.

4. Conclusion

We made the Hopfield net solve the TSP with chaotic fluctuations, which are generated from the logistic map and modified Bernoulli map. Then we evaluate the performance by reaching speed to the neighborhood of the optimum solution for a constant period. The results show that the negative autocorrelation of fluctuations is able to increase the speed towards the optimum solution in spite of the kind of maps. In addition, they suggest that even periodic signals can contribute to the performance in some cases, while intermittent signals of modified Bernoulli do not contribute at all. Further analyses are needed to these phenomena. And the relationship between the results and the way of evaluation is also one of our future works.

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