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A Trade-off Between Efficiency and Stability in a Simple Coupled System of Switching Power Converters

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Abstract—This paper studies a biobjective optimization problem in a simple coupled system of switching power converters which are widely used in renewable energy supply systems. In order to define the biobjective optimization problem, we define two objectives for circuit stability and power efficiency. In the coupled system, the two objectives and the Pareto front are obtained exactly. The Pareto front guarantees existence of a trade-off between the two objectives

1. Introduction

The multiobjective optimization problems (MOPs) are inevitable and important in various fields of natural science including power electronics. The MOPs require simultaneous optimization of multiple objectives where we often encounter various difficulties such as the presence of conflicting objectives. An improvement in one objective may cause a deterioration in another objective. In such cases, an important task is to find a Pareto front in the objective space that describes the best trade-off. In order to find the Pareto front, efficient evolutionary algorithms have been presented and have been applied to benchmark problems[1][2]. In various engineering systems, a system performance is evaluated by multiple objectives. However, MOPs in concrete engineering systems have not been studied sufficiently.

This paper studies a biobjective optimization problem (BOP, the simplest MOP) in a simple coupled system of switching power converters. The coupled system is suitable to realize ripple reduction and current sharing for reliable and efficient renewable energy supply [3]. The switching power converters are widely used in renewable energy systems [4]. For simplicity, we introduce simple coupled system of switching power converters where the dynamics can be analyze exactly [5]-[7].

First, we define two objectives. The first objective evaluates circuit stability and the second objective evaluates input power efficiency. In the coupled system, the two objectives described exactly and precise analysis is possible. Using the two objectives, we defines the BOP. After simple theoretical calculation, we obtain the Pareto front between the two objectives. The Pareto front is described exactly and guarantees existence of a trade-off between the two objectives.

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2. Simple Coupled System of Switching Power Converters

Fig. 1 shows a simple coupled system of 3 switching power converters. The circuit extracts input power. V_{in} corresponds to the input and V_{out} corresponds to the output load. The switch S_j and the diode D_j (j = 1, 2, 3) can be either of the following states:

State A_1: S_1=on, D_1=off
State B_1: S_1=off, D_1=on
State A_2: S_2=on, D_2=off
State B_2: S_2=off, D_2=on
State A_3: S_3=on, D_3=off
State B_3: S_3=off, D_3=on

The circuit dynamics is described by the following piecewise constant model

$$L_1\frac{di_1}{dt} = \begin{cases} V_{in} & \text{State } A_1 \\ V_{in} - V_{out} & \text{State } B_1 \end{cases}$$
$$L_2\frac{di_2}{dt} = \begin{cases} V_{in} & \text{State } A_2 \\ V_{in} - V_{out} & \text{State } B_2 \end{cases}$$
$$L_3\frac{di_3}{dt} = \begin{cases} V_{in} & \text{State } A_3 \\ V_{in} - V_{out} & \text{State } B_3 \end{cases} \quad (1)$$

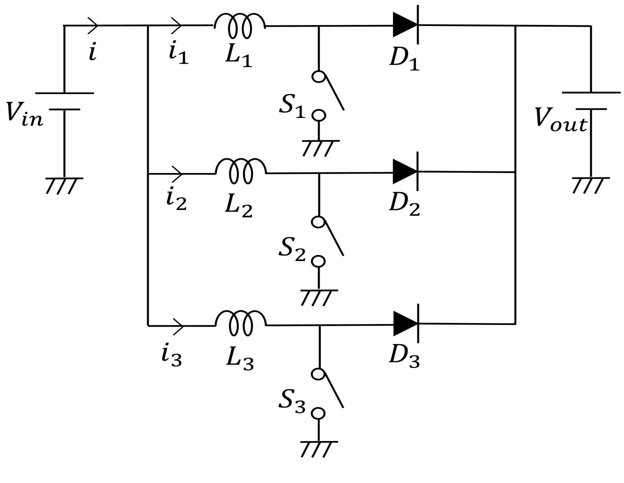

Figure 1: Circuit model

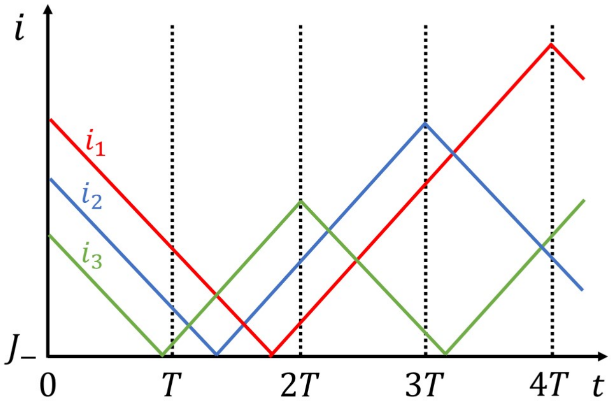

Figure 2: Switching rule

where $V_{out} > V_{in}$ in the boost operation. Fig. 2 illustrates switching rules. The winner-take-all (WTA) rule can realize multiphase synchronization. The switching rules are defined by

$$\begin{cases} \text{State A}_j \to \text{State B}_j \text{ when } i_j \text{ is } MAX \text{ at } t = nT \\ \text{State B}_j \to \text{State A}_j \text{ if } i_j = J_- \end{cases}$$

where J_- is the lower threshold for i_1 (respectively, i_2 and i_3). For simplicity, we assume that the time constant RC of the output load is larger than the clock period T. In this case, we can simplify the RC load into a constant voltage source V_{out}. Using the following dimensionless variables and parameters

$$\tau = \frac{t}{T},\ x_1 = \frac{i_1 - J_-}{I_p - J_-},\ x_2 = \frac{i_2 - J_-}{I_p - J_-},\ x_3 = \frac{i_3 - J_-}{I_p - J_-}$$

$$\begin{aligned} a_1 &= \frac{T}{L_1(I_p - J_-)}V_{in},\ b_1 = \frac{T}{L_1(I_p - J_-)}(V_{out} - V_{in}) \\ a_2 &= \frac{T}{L_2(I_p - J_-)}V_{in},\ b_2 = \frac{T}{L_2(I_p - J_-)}(V_{out} - V_{in}) \\ a_3 &= \frac{T}{L_3(I_p - J_-)}V_{in},\ b_3 = \frac{T}{L_3(I_p - J_-)}(V_{out} - V_{in}) \end{aligned}$$

Eq. (1) is transformed into

$$\begin{aligned} \frac{dx_1}{d\tau} &= \begin{cases} a_1 & \text{State A}_1 \\ -b_1 & \text{State B}_1 \end{cases} \\ \frac{dx_2}{d\tau} &= \begin{cases} a_2 & \text{State A}_2 \\ -b_2 & \text{State B}_2 \end{cases} \\ \frac{dx_3}{d\tau} &= \begin{cases} a_3 & \text{State A}_3 \\ -b_3 & \text{State B}_3 \end{cases} \end{aligned} \tag{2}$$

Switching rule

$$\begin{cases} \text{State A}_j \to \text{State B}_j \text{ when } x_j \text{ is } MAX \text{ at } t = n \\ \text{State B}_j \to \text{State A}_j \text{ if } x_j = 0 \end{cases}$$

Note that, in a likewise manner, we can formulate coupled systems of N converters. Using the exact piecewise solutions, we can calculate waveforms exactly. Fig. 3 shows typical examples of 3-phase synchronized periodic waveforms with period 3; (a) a waveform with strong stability and large ripple, (c) a waveform with weak stability and zero ripple, (b) a waveform with characteristics between (a) and (c), (d) a waveform with ripple. Fig. 3 suggests that there exists a trade-off between stability and efficiency. The trade-off is discussed in the next section. Fig. 4 shows a comparison: 3-phase synchronization of 3 converters $N = 3$, 2-phase synchronization of 2 converters ($N = 2$), and periodic waveform of single converter ($N = 1$). Note that the case $N = 3$ is the criterion of the normalized clock period 1: clock period 3/2 for $N = 2$ and clock period 3 for $N = 1$.

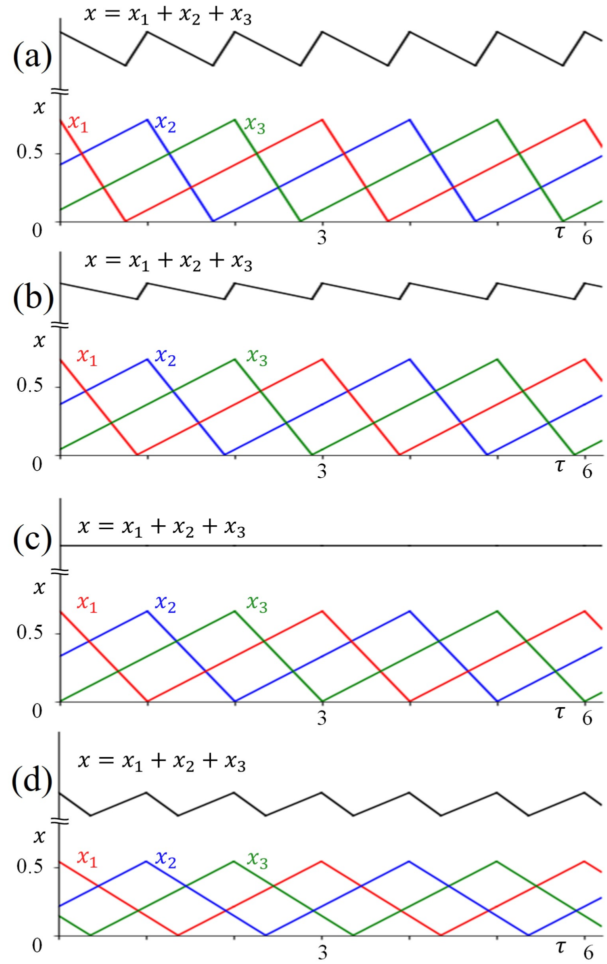

Figure 3: Waveform examples. $a = 0.33$, (a) $b = 1.00$, $|Df(p)| = 0.33$, $Y = 0.25$, (b) $b = 0.80$, $|Df(p)| = 0.42$, $Y = 0.12$, (c) $b = 0.66$, $|Df(p)| = 0.50$, $Y = 0.00$, (d) $b = 0.40$, $|Df(p)| = 0.83$, $Y = 0.20$, $|Df(p)|$ is parameter ratio a/b, and Y is ripple of x

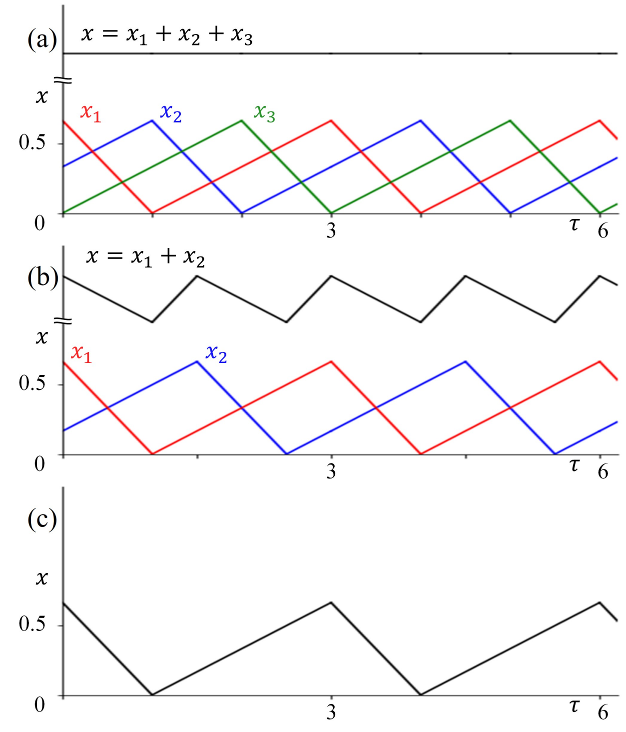

Figure 4: Waveform comparison. $a = 0.33$, $b = 0.66$, $|Df(p)| = 0.50$, (a) 3-phase synchronization with period 3, $Y = 0.00$, (b) 2-phase synchronization with period 3, $Y = 0.33$, (c) Periodic waveform of period 3, $Y = 0.66$, $|Df(p)|$ is parameter ratio a/b, and Y is ripple of x

3. Biobjective Optimization Problems

In this paper, we focus on stable 3-phase synchronization waveforms. In order to evaluate stability of periodic orbit with period 3, we introduce contraction rate $|Df(p)| \equiv |\frac{\Delta x(3)}{\Delta x(0)}|$ near the orbit as show in Fig. 5. p is an initial point of the 3-phase synchronized periodic waveforms with period 3 ($x_1(0) = p, x_1(0) = x_1(3) = p, x_2(1) = x_2(4) = p, x_3(2) = x_3(5) = p$). Fig. 6 illustrates ripple of input current x.

In order to define the BOP, we define two objectives

$$\begin{aligned} F_1(a,b) &= |Df(p)| = |\tfrac{\Delta x(3)}{\Delta x(0)}| = \tfrac{a}{b} \equiv X \\ F_2(a,b) &= (1 - \tfrac{2a}{b})\tfrac{3ab}{a+b} \equiv Y \end{aligned} \tag{3}$$

where $a_1 = a_2 = a_3 \equiv a$, $b_1 = b_2 = b_3 \equiv b$. For simplicity, we consider the case $a = 0.33$. The first objective F_1 evaluate stability: as F_1 approaches 0, stability becomes stronger. The second objective evaluate ripple (efficiency): as F_2 approaches zero, efficiency becomes better. It should be noted that the two objective functions can be calculated precisely using the exact piecewise solutions.

We define biobjective optimization problem.

$$\begin{aligned} &\text{Minimize } F(a,b) = (F_1(a,b), F_2(a,b)) \\ &\text{subject to } (a,b) \in \{(a,b) | a = 0.33, a < b < 100\} \end{aligned} \tag{4}$$

The Pareto front between stability and ripple can be calculated precisely as follows.

$$Y = 3a\tfrac{1-2X}{1+X},\ 0 < X < 1 \tag{5}$$

Pareto front for 2-phase synchronization ($N = 2$) is given by

$$Y = 3a\tfrac{1-X}{1+X},\ 0 < X < 1 \tag{6}$$

Pareto front for periodic waveform ($N = 1$) is given by

$$Y = \tfrac{3a}{1+X},\ 0 < X < 1 \tag{7}$$

Fig. 7 shows these Pareto fronts.

4. Conclusions

A BOP in a simple coupled system of switching power converters has been studied in this paper. Using the piecewise constant model, the Pareto front is obtained exactly. It guarantees existence of a trade-off between circuit stability and efficiency. In our future work, we should consider more detailed analysis of the BPOs in various switching power converters.

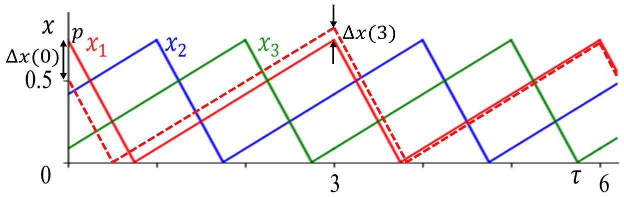

Figure 5: Stability

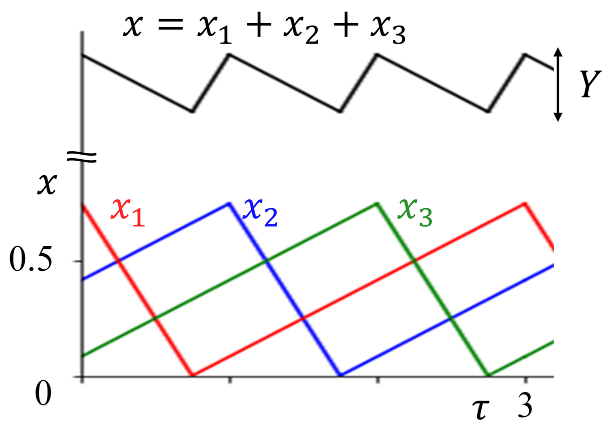

Figure 6: Ripple

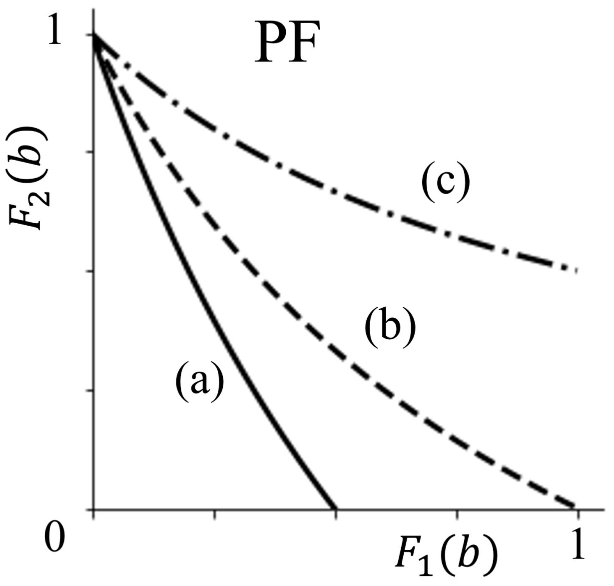

Figure 7: Pareto front(a = 0.33), (a) 3-phase synchronization with period 3, (b) 2-phase synchronization with period 3, (c) Periodic waveform of period 3.

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