2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

Investigation of Asynchronous States in Chaotic Circuits Coupled by Resistors as a Ladder

Yasuteru Hosokawa† and Yoshifumi Nishio‡

†Department of Media and Information Systems, Shikoku University
Furukawa, Ohjin, Tokushima 771–1192, Japan
‡Department of Electrical and Electronic Engineering, Tokushima University
2–1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: hosokawa@keiei.shikoku-u.ac.jp, nishio@ee.tokushima-u.ac.jp

Abstract—There are many studies about coupled chaotic circuits. In these studies, various kinds of interesting phenomena are observed. Especially, interesting behaviours are observed in asynchronous parameter regions near synchronous parameter regions.

In this study, asynchronous states in chaotic circuits coupled by resistors as a ladder are investigated. Especially, relationships among the phase shift, coupling strengths and the number of circuits are investigated.

1. Introduction

In the natural world, there are many coupled systems which have interesting behaviours. For instance, brain and neural networks can process various kinds of informations. Animal skins which consists of many cells show various kinds of patterns. glowing pattern of fireflies, human relations, computer networks and so on also show interesting behaviours. One of interesting behaviours is a synchronization. Especially, chaotic coupled systems show very interesting behaviours.

There are many studies about coupled chaotic circuits. In these studies, various kinds of interesting phenomena are observed. Especially, interesting behaviours are observed in asynchronous parameter regions near synchronous parameter regions. By decreasing a coupling strength, a coupled system does not keep synchronization states and the system becomes asynchronous states. However, in these parameter region, clustering phenomena, switching phenomena between synchronization and asynchronization and so on are observed. These phenomena can be observed vast area of the region. By more decreasing a coupling strength, these phenomena are disappeared and the system becomes asynchronous states perfectly.

In this study, asynchronous states in chaotic circuits coupled by resistors as a ladder are investigated. Especially, relationships among the phase shift, coupling strengths and the number of circuits are investigated. In order to investigate about phase shift in asynchronous states, the lack of an oscillation cycle is defined. By using the definition, the phase shift is treated as the number of the lack.

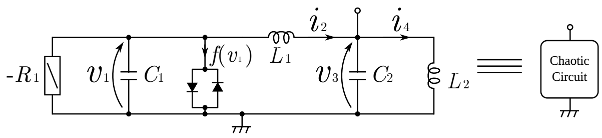
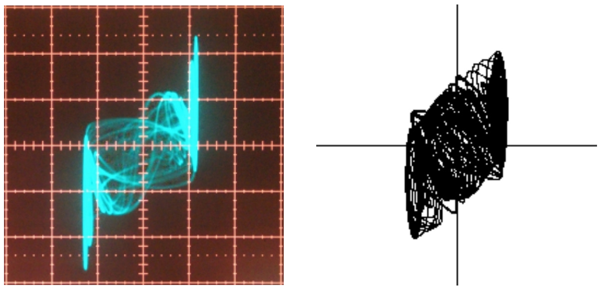


Figure 1: Chaotic circuit applied to the system.



(a) (b)

Figure 2: Attractor of the circuit as shown in Fig. 1. (a) Circuit experimental result. Each axis is set as 0.5[v/div] Horizontal axis: v_1. Vertical axis: v_3. (b) Computer simulation result. Horizontal axis: x_1. Vertical axis: x_3.

2. System Setup

The chaotic circuit applied to the system is shown in Fig. 1. This circuit was proposed and analyzed in [8]. Figure 2 shows a attractor of the circuit. Parameters are set as $C_1 = 0.1$ [μF], $C_2 = 0.068$ [μF], $L_1 = 30$ [mH], $L_2 = 100$ [mH] and $R_1 = 720$ [Ω] in the circuit experiment, $\alpha = 0.75$, $\beta = 1.50$, $\gamma = 0.30$ and $\delta = 5$ in the computer simulation. Chaotic attractor is observed. Largest Lyapunov exponent which calculated by the method in [8] is 0.302.

Figure 3 shows an investigated system. Chaotic circuits are coupled by resistors. Diodes of the chaotic circuit are modeled as a piecewise linear function shown in Fig. 4

In order to investigate the system, system equations are derived.

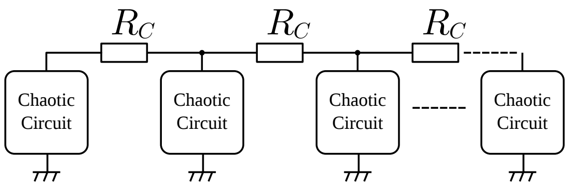


Figure 3: Bidirectionally coupled diode model.

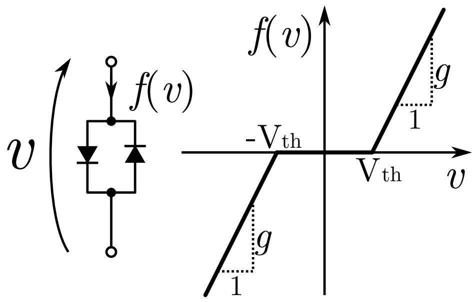


Figure 4: Bidirectionally coupled diode model.

The normalized system equations are described as follows.

· First circuit

$$\begin{cases} \dot{x}_{11} = \alpha_n x_{11} - x_{12} - \delta x_{11} - \delta(|x_{11}-1| - |x_{11}+1|)/2, \\ \dot{x}_{12} = x_{11} - x_{13}, \\ \dot{x}_{13} = p_n\beta\left\{x_{12} - x_{14} + \varepsilon(x_{23} - x_{13})\right\}, \\ \dot{x}_{14} = \gamma x_{n3}. \end{cases} \quad (1)$$

· Middle circuits

$$\begin{cases} \dot{x}_{n1} = \alpha_n x_{n1} - x_{n2} - \delta x_{n1} - \delta(|x_{n1}-1| - |x_{n1}+1|)/2, \\ \dot{x}_{n2} = x_{n1} - x_{n3}, \\ \dot{x}_{n3} = p_n\beta\left\{x_{n2} - x_{n4} + \varepsilon(x_{(n-1)3} - 2x_{n3} + x_{(n+1)3})\right\}, \\ \dot{x}_{n4} = \gamma x_{n3}. \end{cases} \quad (2)$$

· Last circuit

$$\begin{cases} \dot{x}_{N1} = \alpha_N x_{N1} - x_{N2} - \delta x_{N1} - \delta(|x_{N1}-1| - |x_{N1}+1|)/2, \\ \dot{x}_{N2} = x_{N1} - x_{N3}, \\ \dot{x}_{N3} = p_n\beta\left\{x_{N2} - x_{N4} + \varepsilon(x_{(N-1)3} - x_{N3})\right\}, \\ \dot{x}_{N4} = \gamma x_{N3}. \end{cases} \quad (3)$$

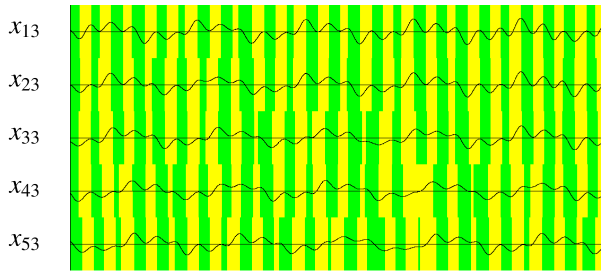


Figure 5: Computer simulation result. $\alpha = 0.75, \beta = 1.50, \gamma = 0.30, \delta = 5$ and $\varepsilon = 0.16$

where

$$\tau = \frac{1}{\sqrt{L_1 C_1}}t, \quad \frac{d}{dt} = \text{“}\cdot\text{”}, \quad x_{n1} = \frac{v_{n1}}{V_{th}}, \quad x_{n2} = \sqrt{\frac{L_1}{C_1}}\frac{i_2}{V_{th}},$$
$$x_{n3} = \frac{v_{n3}}{V_{th}}, \quad x_{n4} = \sqrt{\frac{L_1}{C_1}}\frac{i_4}{V_{th}}, \quad \alpha = \frac{1}{R_1}, \quad \beta = \frac{C_1}{C_2},$$
$$\gamma = \frac{L_1}{L_2}, \quad \delta = g\frac{L_1}{C_1} \quad \text{and} \quad \varepsilon = \frac{1}{R_C}\sqrt{\frac{L_1}{C_1}}.$$
$$(\mathrm{n} = 1, 2, \ldots, \mathrm{N}) \quad (4)$$

Parameter mismatches p_n is added. This is corresponding to parameter mismatches of C_1.

3. Interaction of circuits in asynchronous states

3.1. Lack of the cycle

In order to investigate interactions between coupled two circuits in asynchronous states, a following expression is applied. Figure 5 shows a one of computer simulation results. In this simulation, backgrounds are painted into two colors. Namely, when the value is increasing, a background becomes yellow and when the value is decreasing, a background becomes green. In Fig. 5, green and yellow appear repeatedly as a stripe pattern. Although some gaps occur, all waveforms oscillate with almost same cycle. This stripe pattern is observed in a vast parameter region of asynchronous states. However, some times green or yellow is omitted from the cycle. In x_{43} of Fig. 5, one green is omitted, Namely the lack of the cycle is observed.

In this study, relationship among the number of the lack, a coupling strength and the number of circuits are investigated. Figure 6 shows a procedure of counting the lack. The case of two waveforms A and B, numbers of the slope of a tangent are counted as a and b. When $a = 1$ and $b = 1$, these are reseted as $a = 0$ and $b = 0$. When $a = 2$ or $b = 2$, the lack is counted. Then, a and b are set as 0.

3.2. Relationship the lack and coupling strength

Figure 7 shows a relationship between a number of the lack and coupling strengths in the case of $N = 2$. In this

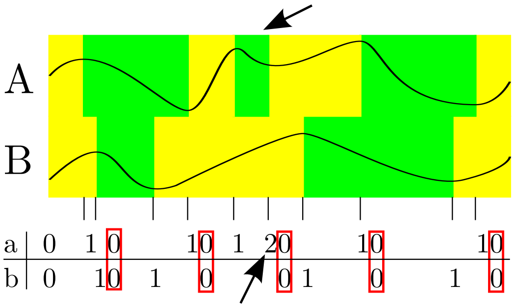


Figure 6: Procedure of counting the lack.

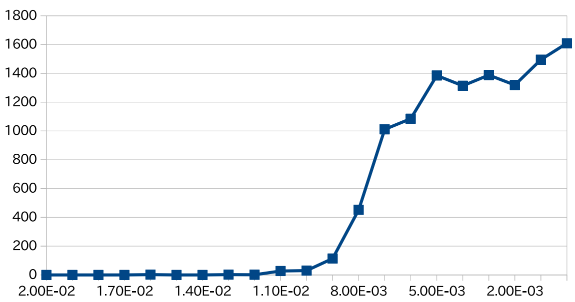


Figure 7: Relationship between the number of the lack and a coupling strength. Horizontal axis: coupling strength ε. Vertical axis: number of the lack. Iteration time is 50000000[τ] in each value ε, Step is 0.001[τ], $p_n = 1 + 0.01(n-1)$, $N = 2$, $\alpha = 0.75$, $\beta = 1.50$, $\gamma = 0.30$ and $\delta = 5$.

parameter region, two circuits are not synchronized at all. The lack is increased by decreasing coupling strength from $\varepsilon = 0.01$.

The case of $N = 6$ is shown in Fig. 8. The lack is observed from $\varepsilon = 0.28$ to 0.20 and from 0.18 to 0. By increasing the number of the circuit, the lack can be observed in a high coupling strength. The number of the lack also becomes high. Especially, the number around $\varepsilon = 0.1$ is higher than the case of $\varepsilon = 0$. In this area, clustering phenomena as shown in Fig. 9 are observed. Upper two oscillators and lower three oscillators have almost same waveforms respectively. However, two groups have different waveforms. Therefore, the number of lacks of 2-3 and 3-4 becomes higher than the others in this area.

The parameter mismatches between both ends of the system are increased by increasing the number of circuits.

In order to investigate the case of keeping parameter mismatches between both ends, Table 1 cases are investigated. Figures 10- 12 show relationships between a number of the lack and coupling strengths. In Figure 10, the lack is observed from $\varepsilon = 0.02$ to $\varepsilon = 0$. In the case of $N = 5$ as shown in Fig. 11, the lack can be observed from $\varepsilon = 0.2$ to $\varepsilon = 0$. In the case of $N = 9$ as shown in Fig. 12, the lack

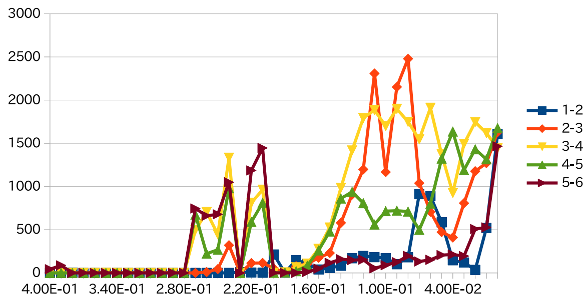


Figure 8: Relationship between a number of the lack and a coupling strength. Horizontal axis: coupling strength ε. Vertical axis: number of the lack. Iteration time is 50000000[τ] in each value ε, Step is 0.001[τ], $p_n = 1 + 0.01(n-1)$, $N = 6$, $\alpha = 0.75$, $\beta = 1.50$, $\gamma = 0.30$ and $\delta = 5$.

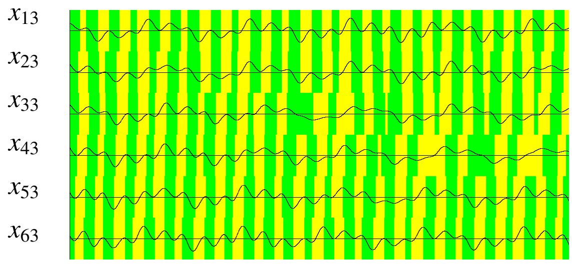


Figure 9: Clustering phenomena. $N = 6$, $p_n = 1 + 0.01(n-1)$, $\alpha = 0.75$, $\beta = 1.50$, $\gamma = 0.30$ and $\delta = 5$.

can be observed in vast area. These results show that increasing the number of circuits means increasing the lack. Additionally, clustering phenomena disappear in very low coupling strengths area (lower $\varepsilon = 0.01$).

4. Conclusion

In this study, asynchronous state in chaotic circuits coupled by resistors as a ladder has been investigated. Waveforms of each circuit were distinguished by an increase or decrease of values. On this distinction, an increase or decrease of values were observed as stripe patterns. By decreasing a coupling strength, lack of patterns were observed.

Table 1: Settings of parameter mismatches

Number of Circuit N	Parameter mismatches p_k
3	0.01
5	0.005
9	0.0025

Relationships among this lack, coupling strengths and the number of circuits were investigated. As results, following behaviours were revealed.

- By decreasing coupling strengths, the number of the lack is increased.
- Clustering phenomena are observed in over four circuits.
- By increasing the number of circuits, the number of the lack is increased.

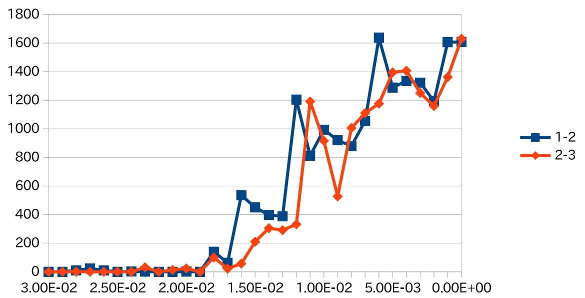


Figure 10: Relationship between the number of the lack and a coupling strength. $p_n = 1 + 0.01(n-1)$, $N = 3$, Horizontal axis: coupling strength ε. Vertical axis: number of the lack. Iteration time is 50000000[τ] in each value ε, Step is 0.001[τ], $\alpha = 0.75, \beta = 1.50, \gamma = 0.30$ and $\delta = 5$.

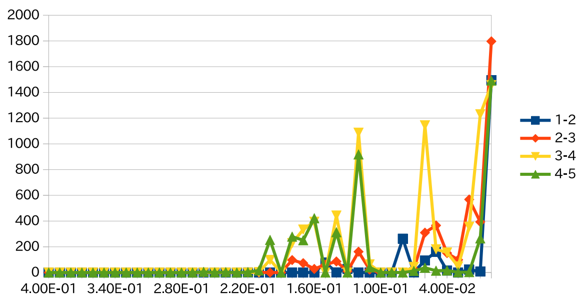


Figure 11: Relationship between the number of the lack and a coupling strength. $p_n = 1 + 0.005(n-1)$, $N = 5$, Horizontal axis: coupling strength ε. Vertical axis: number of the lack. Iteration time is 50000000[τ] in each value ε, Step is 0.001[τ], $\alpha = 0.75, \beta = 1.50, \gamma = 0.30$ and $\delta = 5$.

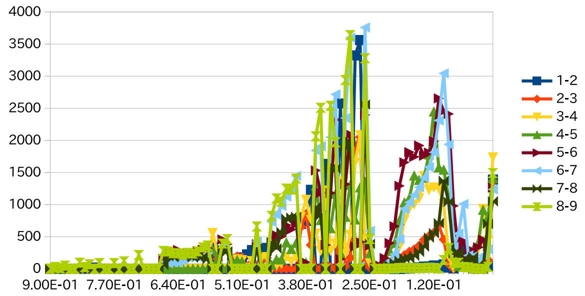


Figure 12: Relationship between the number of the lack and a coupling strength. $p_n = 1 + 0.0025(n-1)$, $N = 9$, Horizontal axis: coupling strength ε. Vertical axis: number of the lack. Iteration time is 50000000[τ] in each value ε, Step is 0.001[τ], $\alpha = 0.75, \beta = 1.50, \gamma = 0.30$ and $\delta = 5$.

References

[1] L. M. Pecora and T. L. Carrol, "Synchronization in Chaotic Systems," *Physical Review Letters*, vol. 64, pp. 821–824, 1990.

[2] T. Endo, and S. Mori, "Mode Analysis of a Ring of a Large Number of Mutually Coupled van der Pol Oscillators," *IEEE Trans. Circuit and Systems*, vol. CAS–25, no. 5, pp. 7–18, 1978.

[3] D. A. Linkens, "Analytical solution of large numbers of mutually-coupled nearly-sinusoidal oscillators," *IEEE Trans. Circuits and Systems*, vol. CAS-21, pp.294–300, Mar, 1974.

[4] H. Sekiya, S. Moro, S. Mori, and I. Sasase, "Synchronization Phenomena on Four Simple Chaotic Oscillators Full-Coupled by Capacitors," *IEICE Trans. Fundamentals*, vol. J82-A, No. 3, pp.375–385, 1999.

[5] M. Miyamura, Y. Nishio and A. Ushida, "Clustering in Globally Coupled System of Chaotic Circuits," *IEEE Proc. ISCAS'02*, vol. 3, pp. 57–60, 2002.

[6] Y. Hosokawa, R. Tsujioka and Y. Nishio, "Relation between Synchronous Rate and Small Variations on an Asymmetrical Coupled Chaotic System," *Proc. NOLTA'05*, pp. 166–169, 2005.

[7] Y. Hosokawa and Y. Nishio, "Asymmetrical Chaotic Coupled System Using Two Different Parameter Sets," *Proc. NOLTA'06*, pp. 543–546, Sep. 2006.

[8] Y. Hosokawa and Y. Nishio, "Chaotic Circuit Generating Two Main Frequency Band Oscillations," *Proc. of RISP International Workshop on Nonlinear Circuits, Communications and Processing*, pp.697–700, 2014.