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On Wien Bridge Oscillators as Modified Multi-vibrators

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Abstract—A tutorial introduction to electrical oscillators. Investigating Wien bridge oscillators as modified multi-vibrators. Introducing chaotic behavior into a Wien bridge oscillator by means of adding a simple nonlinear circuit as a load of one of the amplifier input terminals.

1. Introduction

1.1. Modeling

Electrical circuits are man-made systems. Traditionally they are divided into power circuits for transfer of energy and electronic circuits for transfer of information. Energy is represented by the concepts of electrical charge q and magnetic flux φ. Information is represented by the concept of a bit.

In order to describe and understand electrical circuits we make use of mathematical models. Electrical circuits are nonlinear systems so it is necessary to make assumptions in order to setup analytical models. Our basic approach is to assume quasi-stationary circuits i.e. if the size of the circuits is small compared to the wavelength of the signals then lumped element models may be used.

It is difficult to measure flux φ and charge q. It is more easy to measure voltage $V = d\varphi/dt$ and current $I = dq/dt$. So we choose voltages across $V(X)$ and currents through $I(X)$ the lumped elements X of our circuit as the variables of our model. That is we choose a set of nonlinear differential and algebraic equations as a model for our circuit. It is difficult to solve nonlinear equations analytically so we assume that the signals are small in amplitude so linear models for the elements may be used. For more than 100 years we have designed electrical circuits based on linear models. We have introduced the concept of a time invariant DC bias-point as the reference for the AC signals. We have introduced the concepts of harmonic distortion and noise in order to handle the nonlinear performance.

Linear electronic circuits like filters and amplifiers normally have one signal input port and one signal output port. The DC power supply is a step signal input port of the circuit which create the DC bias-point. We introduce the transfer function of the circuit as the ratio between the output signal and the input signal. The transfer function is a rational function with a numerator and a denominator polynomial: $H(s) = N(s)/D(s)$ where s is the complex frequency. The roots of the denominator polynomial $D(s)$ are the natural frequencies of the circuit also called the poles. The poles are the eigenvalues of the Jacobian of the linear differential equation model for the circuit.

Modeling is the art of creating a model for your system which is able to give answers to your questions concerning your system in the most simple way. If you observe agreement with your measurements you may believe in your model [1].

1.2. Oscillators

Oscillators are a special group of electrical circuits which are able to deliver a steady state signal on the output port for zero signal on the input port. The output signal is the step response of the power supply input i.e. a steady state oscillator do not have a time invariant DC bias point.

A steady state linear oscillator must have a complex pole pair on the imaginary axis. This is impossible in the real world. You can not balance on the razors edge. The poles of the linear circuit must be either in the left half (LHP) or in the right half (RHP) of the complex frequency plane i.e. a linear oscillator is a damped circuit. If the complex pole pair is in RHP the amplitude of the signal goes to infinite which is impossible. A steady state oscillator must be a nonlinear circuit.

The kernel in our circuit analysis programs (SPICE programs) is the iterative solution of a linear set of equations. For each integration step the nonlinearities are replaced with static or dynamic values depending on the iteration schemes (Picard or Newton/Rhapson iteration). That is for each integration step we have an instant bias point and an instant linear small signal model so it make sense to investigate the eigenvalues of the linearized Jacobian, the poles of the small signal model. With time these poles move between RHP and LHP so we have a balance between the energy received from the power source when the poles are in RHP and energy lost when the poles are in LHP.

In an instant solution point a nonlinear dynamic system will behave in accordance with the eigenvalues of the linearized system. If the poles are in RHP of the complex frequency plane the signals will increase in amplitude. If the poles are in LHP the signals will decrease in amplitude. In short:

We may investigate nonlinear circuits as
time varying linear circuits.

Feed-back is an important issue of electrical systems. According to the modified Barkhausen criterion [2] an oscillator is designed as a closed loop of a linear amplifier with gain A and a linear frequency determining feedback circuit with transfer function $H(s) = N(s)/D(s)$. The loop is opened and the circuit is designed with gain 1 and phase-shift $\angle 0$. The loop is closed and regeneration is assumed to start-up oscillations because of positive feed-back. Very often you observe oscillations but unfortunately you have no guarantee of steady state oscillations. The criterion is an observation [3] for placing the complex pole pair of a steady state linear oscillator on the imaginary axis. In order to start up oscillations the circuit is modified i.e. the poles are moved to RHP so the linear circuit becomes unstable. The mechanism is regeneration with positive feedback of the output signal.

2. Bridge Oscillators

The first electrical bridge circuit was invented in 1833 by S. Hunter Christie [4] ten years before it was reported by C. Wheatstone. Very many bridge circuit have been proposed for measurements as e.g. Resonance, Hay, Owen, Maxwell, Wien and Schering [5]. Figure 1 shows a loop of an amplifier and a bridge circuit. The input port of the bridge is nodes $(+,-) = (3,0)$ and the output port is $(+,-) = (2,1)$. When the bridge is in balance $V(2) = V(1)$. If the impedances are resistors the value of one unknown resistor may be calculated from the other three resistors. The input port of the amplifier is $(+,-) = (1,2)$ and the output port of the amplifier is $(+,-) = (3,0)$. The amplifier is inverting so we have a proper Barkhausen topology with positive feed-back and regeneration. If we observe a steady-state signal $V(3)$ on node 3 for $V_{in} = 0$ we have an oscillator. Apparently the first bridge oscillator was reported in 1938 by L.A. Meacham [6]. With reference to fig 1 he introduced a crystal as Z_A (the frequency-controlling resonant element), a thermally controlled resistance (a small tungsten-filament lamp of low wattage rating) as Z_C and fixed resistances as Z_B and Z_D.

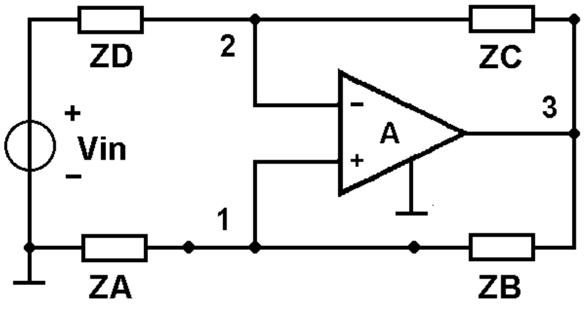

Figure 1: A loop of a bridge-circuit and an amplifier which may be an oscillator if $V_{in} = 0$ and $V(3) \neq 0$

2.1. Multi-Vibrators

If the elements of the bridge circuit are linear the necessary nonlinearity for steady state oscillations must be provided by the amplifier i.e. we can not use an ideal operational amplifier model with infinite gain.

In the following we will assume that the amplifier is a perfect voltage controlled voltage source with infinite input impedance and zero output impedance. The gain A is assumed to be time varying i.e. with reference to fig 1:

$$V(3) = A(t) * (V(1) - V(2))$$

For $V_{in} = 0$ and $V(3) \neq 0$ the following characteristic equation is derived:

$$\frac{1}{A} + \frac{Z_D}{Z_C + Z_D} - \frac{Z_A}{Z_A + Z_B} = 0 \qquad (1)$$

From this equation the characteristic polynomial of the instant linear circuit may be derived.

Each of the impedances define a port of the circuit. Table 1 shows the input impedances of the four ports. If the impedances are resistors and the amplifier is an ideal operational amplifier then it is seen that the input resistor is equal to minus the product of the neighbor resistors divided by the opposite resistor e.g. R_B is loaded by $-R_A R_C/R_D$. In [7] it is reported that we have four common multi-vibrators if one of the four resistors is replaced with a memory element capacitor or inductor: C_D, C_B, L_A or L_C. In these cases the time varying linear circuit has a real pole in RHP for $A > 1$ and we observe oscillations with an amplitude determined by the power rails. It is obvious that multi-vibrator behavior is not violated by inserting a resistor in series with C_B.

Port	Input Impedance	$A = \infty$	$A = 1$	$A = 0$
D	$Z_C\frac{Z_B-Z_A(A-1)}{Z_A+Z_B(A+1)}$	$-\frac{Z_AZ_C}{Z_B}$	$\frac{Z_CZ_B}{Z_A+2Z_B}$	Z_C
B	$Z_A\frac{Z_D-Z_C(A-1)}{Z_C+Z_D(A+1)}$	$-\frac{Z_AZ_C}{Z_D}$	$\frac{Z_AZ_D}{Z_C+2Z_D}$	Z_A
A	$Z_B\frac{Z_C-Z_D(A-1)}{Z_D+Z_C(A+1)}$	$-\frac{Z_DZ_B}{Z_C}$	$\frac{Z_BZ_C}{Z_D+2Z_C}$	Z_B
C	$Z_D\frac{Z_A-Z_B(A-1)}{Z_B+Z_A(A+1)}$	$-\frac{Z_DZ_B}{Z_A}$	$\frac{Z_DZ_A}{Z_B+2Z_A}$	Z_D

Table 1: Input impedance

2.2. Wien Bridge Oscillators [8]

In 1939 W.R. Hewlett [9] created a Wien Bridge oscillator where Z_A is a *RC parallel* circuit, Z_B is a *RC series* circuit, Z_D is a nonlinear resistor R_D (tungsten-filament lamp) and Z_C is a linear resistor R_C.

$$Z_A = \frac{1}{sC_A + \frac{1}{R_A}} = \frac{R_A}{1 + sR_AC_A} \qquad (2)$$

$$Z_B = R_B + \frac{1}{sC_B} = \frac{1 + sR_BC_B}{sC_B} \qquad (3)$$

From the equations (1), (2) and (3) the characteristic polynomial of the instant linear circuit may be derived as

$$s^2 + 2\alpha s + \omega^2 = 0 \qquad (4)$$

where

$$2\alpha = \frac{1}{C_A R_A} + \frac{1}{C_B R_B} + \frac{1}{C_A R_B} \frac{R_D - R_C(A-1)}{R_C + R_D(A+1)} \qquad (5)$$

and

$$\omega^2 = \frac{1}{C_A R_A C_B R_B} \qquad (6)$$

The roots of the characteristic polynomial becomes

$$p_1, p_2 = -\alpha \pm j\sqrt{(\omega^2 - \alpha^2)} \qquad (7)$$

Figure 2 shows a 3.333kHz Wien Bridge oscillator de-

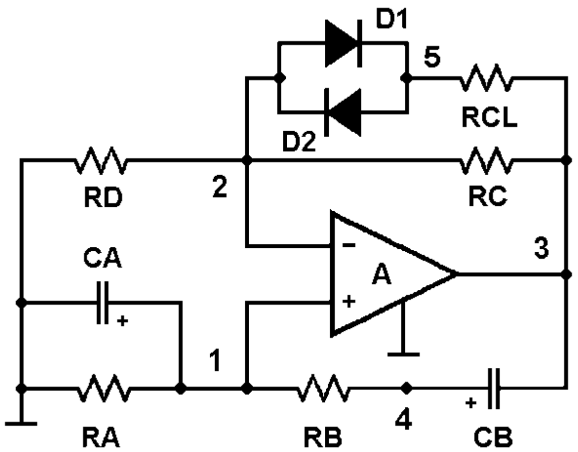

Figure 2: Wien Bridge Oscillator, frequency 3.333kHz, RA = RB = 20kΩ, CA = CB = 2.387324147nF, RC = 6.010kΩ, RCL = 17kΩ and RD = 3kΩ. The diodes D1 and D2 are D1N4148, and the operational amplifier is an AD712.

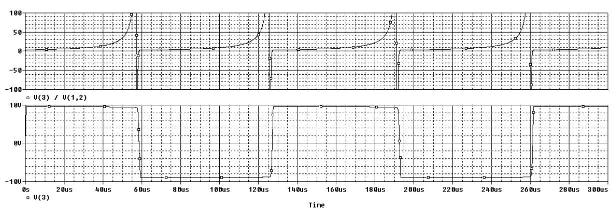

Figure 3: Multi-vibrator, y axis for A ±100 A=V(3)/V(1,2) and V(3) as function of time

signed according to the Barkhausen criterion i.e. $2\alpha = 0$ assuming $A = \infty$. The linear resistor R_C is replaced with R_C in parallel with a nonlinear resistor based on two diodes in antiparallel and a resistor R_{CL} = 17kΩ i.e. the resistor is large for small signals and small for large signals. R_C is made slightly greater than $2R_D$ so the linear circuit becomes unstable with a complex pole pair in RHP ($\alpha < 0$).

Now we remove the diodes, R_{CL} and C_A so we have a multi-vibrator with C_B as the single memory element loaded with a negative resistor $R_B - R_A R_C / R_D = -20.07k\Omega$. The RC time constant becomes $-47.9\mu s$. Figure 3 shows that the pulse width is $64\mu s$ and the amplitude is determined by the power rails ±10 volt.

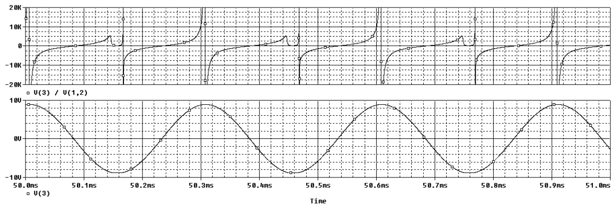

Figure 4: Wien Bridge Oscillator, y axis for A ±20k A=V(3)/V(1,2) and V(3) as function of time

Now the capacitor $C_A = C_B$ is added in parallel to R_A and fig. 4 shows that the amplitude is determined by the power rails.

Finally the diodes and the resistor R_{CL} are added and fig. 5 shows that the amplitude is reduced to 0.6 volt.

The **conclusion** is that multi-vibrator behavior is the ker-

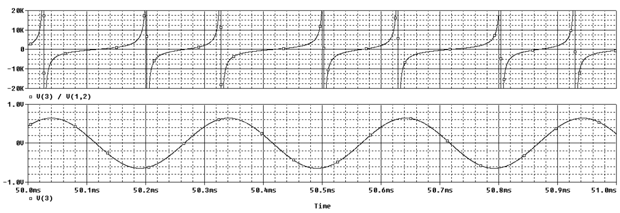

Figure 5: Wien Bridge Oscillator, y axis for A ±20k A=V(3)/V(1,2) and V(3) as function of time

nel mechanism of the wien-bridge oscillator. First when nonlinearity is introduced in the negative feed-back path as Hewlett did the amplitude is controlled and the distortion is minimized by the complex pole pair having an imaginary part as constant as possible in the period.

2.3. Chaotic Wien Bridge Oscillator

In 2005 a simple *non-autonomous* chaotic circuit called the *LMT* circuit was reported [10]. In order to create a simple *autonomous* chaotic circuit the *LMT* circuit on fig. 6 may be connected to the wien bridge oscillator. If the coupling node is the output node of the operational amplifier it corresponds to replacement of the ideal sinusoidal source with the wien bridge oscillator. In this case the feedback to the wien bridge circuit is small and chaotic signals are not observed in the wien bridge circuit but only in the *LMT* circuit. In order to obtain close coupling of the two circuits the *LMT* circuit is connected to the negative input node of the operational amplifier. In this case the amplitude of the node voltage $V(2)$ is not large enough to activate the transistor QXX so the diodes and resistor R_{CL} are removed i.e. the wien bridge oscillator is operating in multi-vibrator mode.

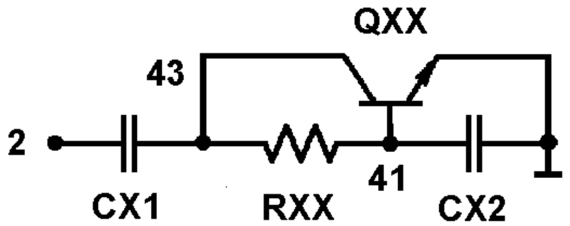

Figure 6: The LMT circuit. CX1 = 4.7nF, RXX = 994kOhm, CX2 = 0.95nF and QXX = Q2N2222A [10]

Figure 7 shows an almost white noise spectrum in the frequency range 0 to 100kHz. Figure 8 and fig. 9 shows two chaotic attractors.

3. Conclusion

Nonlinear circuits may be investigated as time varying linear circuits. It is demonstrated that multi-vibrator behavior is the kernel mechanism of the wien-bridge oscillator. A simple autonomous chaotic oscillator based on coupling of the wien bridge oscillator and the non-autonomous *LMT* circuit is presented.

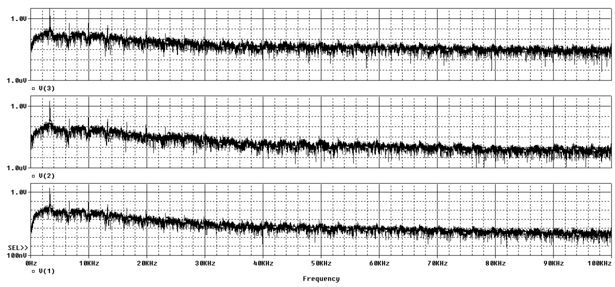

Figure 7: Frequency spectrum of V(1), V(2) and V(3)

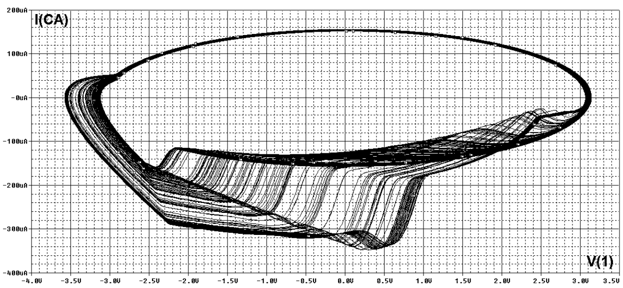

Figure 8: Chaotic attractor I(CA) as function of V(CA)

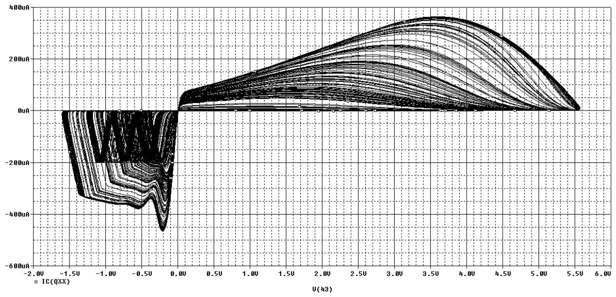

Figure 9: Chaotic attractor, collector current IC(QXX) as function of collector-emitter voltage V(43)

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