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One-to-Many Association Ability of Chaotic Complex-Valued Multidirectional Associative Memory

Akio YOSHIDA and Yuko OSANA

School of Computer Science, Tokyo University of Technology
1404-1 Katakura, Hachioji, Tokyo, 192-0982, Japan Email: osana@cs.teu.ac.jp

Abstract—In this paper, we investigate the one-to-many association ability of the Chaotic Complex-valued Multidirectional Associative Memory (CCMAM). The Chaotic Complex-valued Multidirectional Associative Memory is based on the Multidirectional Associative Memory, and is composed of complex-valued neurons and chaotic complex-valued neurons. In this model, associations of multi-valued patterns are realized by using complex-valued neurons, and one-to-many associations are realized by using chaotic complex-valued neurons.

1. Introduction

Recently, neural networks are drawing much attention as a method to realize flexible information processing. Although a lot of associative memories have been proposed, most of them can deal with only one-to-one associations of binary/bipolar patterns.

As the model which can deal with one-to-many associations of multi-valued patterns, the Chaotic Complex-valued Bidirectional Associative Memory[1][2] and the Chaotic Complex-valued Multidirectional Associative Memory (CCMAM)[3] have been proposed. These models are composed of the complex-valued neurons [4] and the chaotic complex-valued neurons[5]. In these models, associations of multi-valued patterns are realized by using complex-valued neurons, and one-to-many associations are realized by using chaotic complex-valued neurons. In ref.[3], it is confirmed that the CCMAM can realize one-to-many associations of multi-valued patterns. Since the CCMAM is composed of the chaotic complex-valued neurons, one-to-many association ability is very sensitive to the chaotic complex-valued neuron parameters. However, the relation between one-to-many association ability and the chaotic complex-valued neuron parameters has not been investigated.

In this research, we investigate the relation between the one-to-many association ability and the chaotic complex-valued neuron parameters in the Chaotic Complex-valued Multidirectional Associative Memory.

2. Chaotic Complex-Valued Neuron Model

Here, we explain the chaotic complex-valued neuron model[5] which is used in the Chaotic Complex-valued Multidirectional Associative Memory. This model is based on the complex-valued neuron model [4] and the chaotic neuron model[6]. The chaotic complex-valued neuron model is the extended chaotic neuron model in order to deal with complex-value as internal states and output of neurons. It is known that the chaotic complex-valued associative memory composed of chaotic complex-valued neurons can realize dynamic associations of multi-valued patterns[5].

The dynamics of the chaotic complex-valued neuron is given by

$$x(t+1) = f\left(A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta\right) \qquad (1)$$
$$(A(t), x(t), \theta \in \mathrm{C} \quad k, \alpha \in \mathrm{R})$$

where $x(t)$ is the output of the neuron at the time t, $A(t)$ is the external input at the time t, α is the scaling factor of the refractoriness, k is the damping factor ($0 < k_r < 1$), and θ is the threshold of the neuron. $f(\cdot)$ is the output function which is given by

$$f(u) = \frac{\eta u}{\eta - 1.0 + |u|} \qquad (\eta \in \mathrm{R})$$

where η is the constant ($\eta > 1$).

3. Chaotic Complex-Valued Multidirectional Associative Memory

Here, we explain the Chaotic Complex-valued Multidirectional Associative Memory (CCMAM)[3].

3.1. Structure

The Chaotic Complex-valued Multidirectional Associative Memory has more than two layers as similar as the conventional Multidirectional Associative Memory[7]. Figure 1 shows the structure of this model which has three layers. Each layer has two parts; (1) Key Input Part composed of complex-valued neurons and (2) Context Part composed of chaotic complex-valued neurons.

3.2. Learning Process

Generally, the associative memory which is trained by the correlation matrix can not deal with one-to-many as-

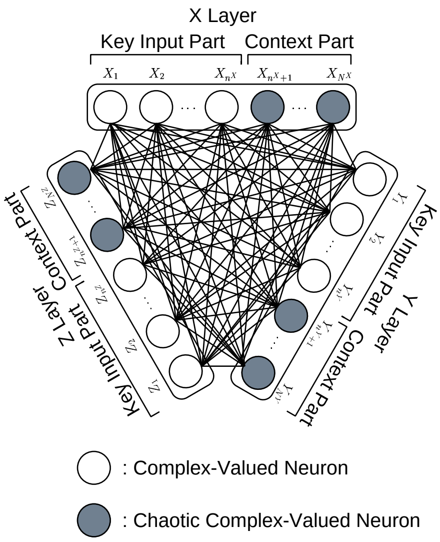


Figure 1: Structure of CCMAM.

sociations because the stored common data cause superimposed patterns. In the Chaotic Bidirectional Associative Memory (CBAM)[8], each training pair is memorized together with its own contextual information in order to memorize the training set including one-to-many relations. In the CCMAM, the same method is used to memorize the training set including one-to-many relations.

In the CCMAM, the patterns with its own contextual information are memorized by the orthogonal learning. The connection weights from the layer y to the layer x, $\boldsymbol{w}^{xy}$ and the connection weights from the layer x to the layer y , $\boldsymbol{w}^{yx}$ are given by

$$\boldsymbol{w}^{xy} = \boldsymbol{X}_y(\boldsymbol{X}_x^*\boldsymbol{X}_x)^{-1}\boldsymbol{X}_x^* \tag{2}$$

$$\boldsymbol{w}^{yx} = \boldsymbol{X}_x(\boldsymbol{X}_y^*\boldsymbol{X}_y)^{-1}\boldsymbol{X}_y^* \tag{3}$$

where * shows the conjugate transpose, and -1 shows the inverse. $\boldsymbol{X}_x$ and $\boldsymbol{X}_y$ are the training pattern matrices which are memorized in the layer x and the layer y, and are given by

$$\boldsymbol{X}_x = \{\boldsymbol{X}_x^{(1)}, \cdots, \boldsymbol{X}_x^{(p)}, \cdots, \boldsymbol{X}_x^{(P)}\} \tag{4}$$

$$\boldsymbol{X}_y = \{\boldsymbol{X}_y^{(1)}, \cdots, \boldsymbol{X}_y^{(p)}, \cdots, \boldsymbol{X}_y^{(P)}\} \tag{5}$$

where $\boldsymbol{X}_x^{(p)}$ is the pth pattern which is stored in the layer x, $\boldsymbol{X}_y^{(p)}$ is the pth pattern which is stored in the layer y and P is the number of the training pattern sets.

3.3. Recall Process

Since we assume that contextual information is usually unknown for users, in the recall process of the CCMAM, only the Key Input Part receives input. For example, in the training sets which is given by

$$\begin{aligned}\{&(\boldsymbol{X}_1\_\boldsymbol{C}_{X1}, \boldsymbol{Y}_1\_\boldsymbol{C}_{Y1}, \boldsymbol{Z}_1\_\boldsymbol{C}_{Z1}),\\ &(\boldsymbol{X}_1\_\boldsymbol{C}_{X2}, \boldsymbol{Y}_2\_\boldsymbol{C}_{Y2}, \boldsymbol{Z}_2\_\boldsymbol{C}_{Z2}),\\ &(\boldsymbol{X}_2\_\boldsymbol{C}_{X3}, \boldsymbol{Y}_3\_\boldsymbol{C}_{Y3}, \boldsymbol{Z}_3\_\boldsymbol{C}_{Z3})\},\end{aligned} \tag{6}$$

$\boldsymbol{X_1}$ is used as an input to the CCMAM. Here, $\boldsymbol{C}_{\mathbf{xx}}$ (such as $\boldsymbol{C}_{X1}$ and $\boldsymbol{C}_{Y1}$) shows the contextual information. In the CCMAM, when $\boldsymbol{X}_1$ is given to the network as an initial input, since the chaotic complex-valued neurons in the Contextual Information Part change their states by chaos, we can expect that they can realize one-to-many associations as follows:

$$\begin{aligned}(\boldsymbol{X}_1\_\boldsymbol{0}, ?, ?) &\rightarrow \cdots \rightarrow (\boldsymbol{X}_1\_\boldsymbol{C}_{X1}, \boldsymbol{Y}_1, \boldsymbol{Z}_1) \rightarrow \cdots\\ &\rightarrow (\boldsymbol{X}_1\_\boldsymbol{C}_{X2}, \boldsymbol{Y}_2, \boldsymbol{Z}_2) \rightarrow \cdots\end{aligned} \tag{7}$$

The recall process of the CCMAM has the following procedures when the input pattern is given to the layer x.

Step 1 : Input to Layer x

The input pattern is given to the layer x.

Step 2 : Propagation from Layer x to Other Layers

When the pattern is given to the layer x, the information are propagated to the Key Input Part in the other layers. The output of the neuron k in the Key Input Part of the layer y $(y \neq x)$, $\boldsymbol{x}_k^y(t)$ is given by

$$x_k^y(t) = f\left(\sum_{j=1}^{N^x} w_{kj}^{yx} x_j^x(t)\right) \tag{8}$$

where N^x is the number of neurons in the layer x, w_{kj}^{yx} is the connection weight from the neuron j in the layer x to the neuron k in the layer y, $x_j^x(t)$ is the output of the neuron j in the layer x at the time t. And $f(\cdot)$ is the output function which is given by Eq.(2).

Step 3 : Propagation from Other Layers to Layer x

The output of the neuron j in the Key Input Part of the layer x $\boldsymbol{x}_j^x(t+1)$ is given by

$$x_j^x(t+1) = f\left(\sum_{y \neq x}^{M}\left(\sum_{k=1}^{n^y} w_{jk}^{xy} x_k^y(t)\right) + vA_j\right) \tag{9}$$

where M is the number of layers, n^y is the number of neurons in the Key Input Part of the layer y, w_{jk}^{xy} is the connection weight from the neuron k in the layer y to the neuron j in the layer x, v is the connection weight from the external input, and A_j is the external input (See **3.4**) to the neuron j in the layer x.

The output of the neuron j of the Contextual Information Part in the layer x, $\boldsymbol{x}_j^x(t+1)$ is given by

$$\begin{aligned}x_j^x(t+1) \;=\; & f\left(\sum_{y \neq x}^{M}\left(\sum_{k=1}^{n^y} w_{jk}^{xy} \sum_{d=0}^{t} k_m^d x_k^d(t-d)\right)\right.\\ & \left. -\alpha \sum_{d=0}^{t} k_r^d x_j^x(t-d)\right)\end{aligned} \tag{10}$$

where k_m and k_r are damping factors. And, α is the scaling factor of the refractoriness.

Step 4 : Repeat

Steps 2 and **3** are repeated.

3.4. External Input

In the CCMAM, the external input A_j is always given so that the key pattern does not change into other patterns.

If the pattern is given to the layer x and the initial input does not include noise, we can use the initial input pattern $A_j = x_j^x(0)$ as the external pattern. However, the initial input pattern sometimes includes noise. So we use the following pattern $\hat{x}_j^x(t_{in})$ when the network becomes stable at t_{in} as an external input.

$$t_{in} = \min \left\{ t \;\middle|\; \sum_{j=1}^{n^x} (\hat{x}_j^x(t) - \hat{x}_j^x(t-1)) = 0 \right\} \tag{11}$$

where n^x is the number of neurons in the Key Input Part of the layer x. $\hat{x}_j^x(t)$ is the quantized output of the neuron j in the layer x at the time t, and is given by

$$\hat{x}_j^x(t) = \arg\min(\omega^s - x_j^x)^*(\omega^s - x_j^x) \qquad (s = 1, 2, ..., S-1) \tag{12}$$

where S is the number of states and ω is given by

$$\omega = \exp(i2\pi/S) \tag{13}$$

where i is the imaginary unit.

4. Computer Experiment Results

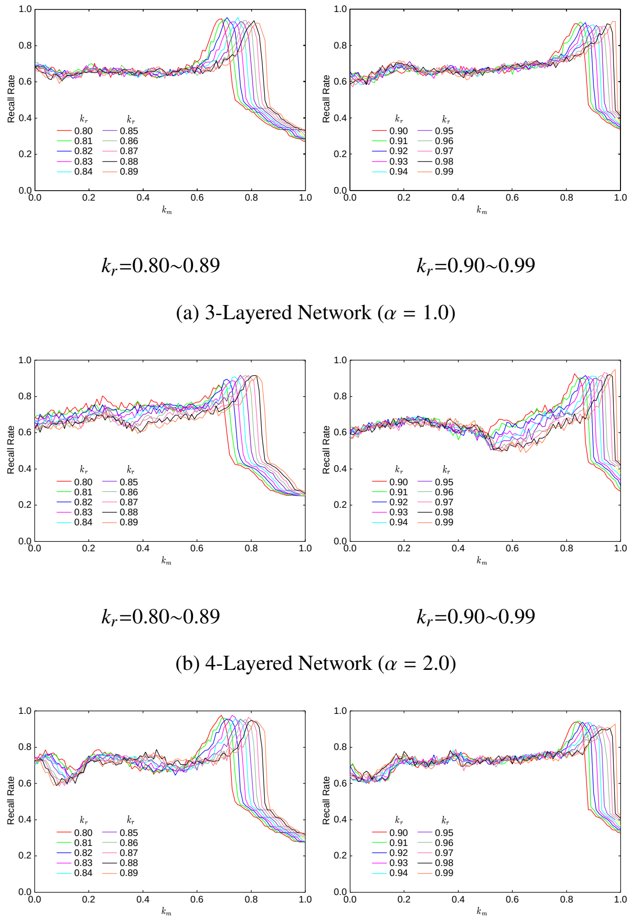
Here, we examined the relation between the one-to-many association ability and the chaotic complex-valued neuron parameters in the Chaotic Complex-valued Multi-directional Associative Memory.

4.1. Relation between One-to-Many Association Ability and Damping Factor k_m

Figure 2 show the relation between the one-to-many association ability and the damping factor k_m in the CCMAM. In this experiment, the network which has 3∼5 layers composed of 500 neurons (400 neurons for Key Input Part and 100 neurons for Context Part) and 4-valued random patterns in 1-to-4 relation were memorized was used. As shown in this figure, high recall rate can be obtained if the appropriate pair of damping factors k_m and k_r is set.

4.2. Relation between One-to-Many Association Ability and Damping Factor k_r

Figure 3 show the relation between the one-to-many association ability and the damping factor k_r in the CCMAM. In this experiment, the network which has 3∼5 layers composed of 500 neurons (400 neurons for Key Input Part and 100 neurons for Context Part) and 4-valued random patterns in 1-to-4 relation were memorized was used. As shown in this figure, high recall rate can be obtained if the appropriate pair of damping factors k_m and k_r is set.



k_r=0.80∼0.89 k_r=0.90∼0.99

(a) 3-Layered Network ($\alpha = 1.0$)

k_r=0.80∼0.89 k_r=0.90∼0.99

(b) 4-Layered Network ($\alpha = 2.0$)

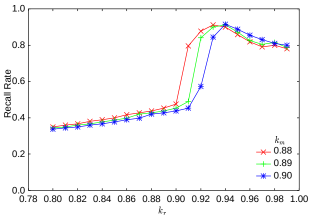
k_r=0.80∼0.89 k_r=0.90∼0.99

(c) 5-Layered Network ($\alpha = 2.0$)

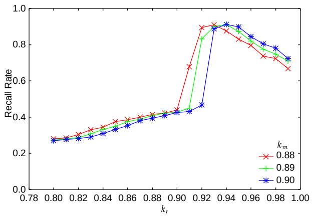
Figure 2: Relation between One-to-Many Association Ability and Damping Factor k_m.

4.3. Relation between One-to-Many Association Ability and Scaling Factor α

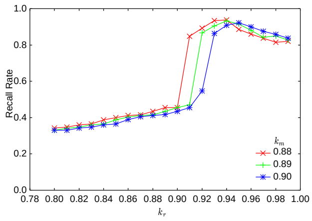
Figure 4 show the relation between the one-to-many association ability and the scaling factor α in the CCMAM. In this experiment, the network which has 3∼5 layers composed of 500 neurons (400 neurons for Key Input Part and 100 neurons for Context Part) and 4/8/16-valued random patterns in 1-to-4(∼6) relation were memorized was used. As shown in this figure, 4-valued patterns can be recalled easier than 8/16-valued patterns in the CCMAM. And the range of the scaling factor α which can give high recall rates is wider when 4-valued patterns are memorized than when 8/16-valued patterns are memorized.



(a) 3-Layered Network



(b) 4-Layered Network

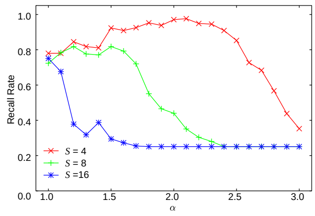


(c) 5-Layered Network

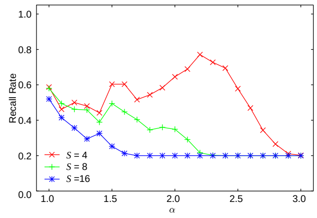
Figure 3: Relation between One-to-Many Association Ability and Damping Factor k_r.

5. Conclusion

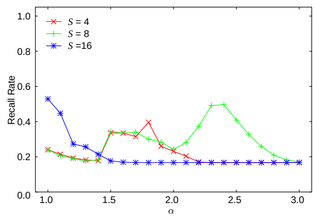
In this paper, we investigated the relation between the one-to-many association ability and the chaotic complex-valued neuron parameters in the Chaotic Complex-valued Multidirectional Associative Memory. We carried out a series of computer experiments and confirmed that the one-to-many association ability is very sensitive to chaotic complex-valued neuron parameters, especially, the scaling factor of the refractoriness α.



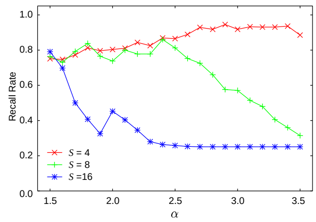
(a) 1-to-4 Patterns (M=3)



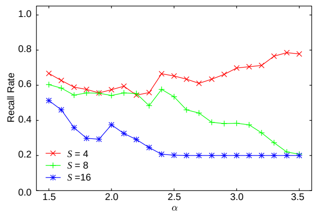
(b) 1-to-5 Patterns (M=3)



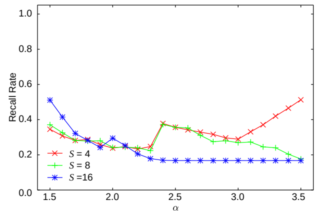
(c) 1-to-6 Patterns (M=3)



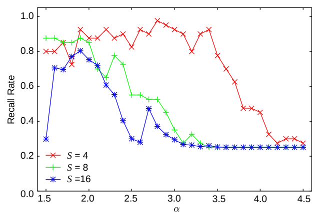
(d) 1-to-4 Patterns (M=4)



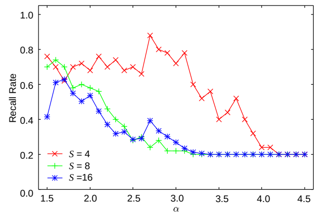
(e) 1-to-5 Patterns (M=4)



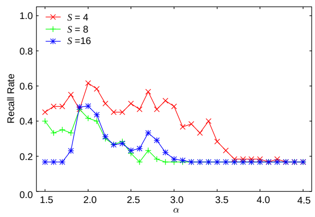
(f) 1-to-6 Patterns (M=4)



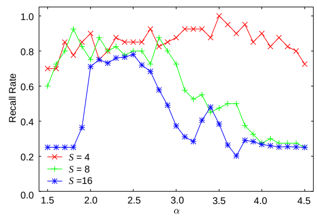
(g) 1-to-4 Patterns (M=5)



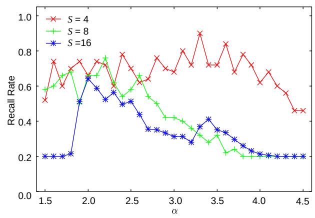
(h) 1-to-5 Patterns (M=5)



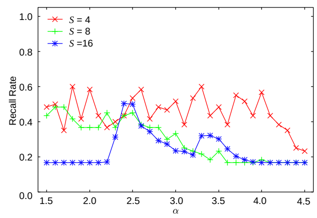
(i) 1-to-6 Patterns (M=5)



(j) 1-to-4 Patterns (M=6)



(k) 1-to-5 Patterns (M=6)



(l) 1-to-6 Patterns (M=6)

Figure 4: Relation between One-to-Many Association Ability and Scaling Factor α (k_m=0.89, k_r=0.92).

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