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Decomposition of Symmetric Almost Periodic Oscillation in Three-Phase Circuit

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Abstract—This paper describes a method to decompose symmetric almost periodic oscillations in a three-phase circuit based on symmetries. The decomposition classifies frequency components exclusively and we calculate the bifurcation diagram of almost periodic oscillations using 2-dimensional harmonic balance.

1. Introduction

Symmetries provides crucial clue to understand general mechanism for nonlinear systems. Specifically, the symmetries of a system can be used for finding typical modes. That is, if a system has symmetries, we can define invariant subspaces and find out typical periodic modes [1, 2]. This paper extends the concept to almost periodic oscillations and proposes a method to decompose symmetric almost periodic oscillations in a symmetric three-phase circuit.

The symmetric three-phase circuit in Fig.1 is a fundamental model of power systems. The nonlinearity of the inductors generates many kinds of nonlinear oscillations, e.g., subharmonic oscillations[3], asymmetric oscillations[4], cnoidal waves[5] and ILMs[6]. The generation of those periodic oscillations is clarified by symmetries in [2]. We consider almost periodic oscillations and decompose the oscillations by the symmetries.

First, we show the symmetry of the three-phase circuit and define the symmetric almost periodic oscillations. Next, we propose a method to decompose the oscillations. Further, we calculate bifurcation diagram of the almost periodic oscillations using 2-dimensional harmonic balance.

2. Equation of Three-Phase Circuit

The normalized equation of the three-phase circuit in Fig.1 is

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{\psi}_{\mathrm{abc}}\\ \boldsymbol{u}_{\mathrm{abc}}\end{pmatrix}=\begin{pmatrix}-\boldsymbol{A}_{\mathrm{abc}}\boldsymbol{u}_{\mathrm{abc}}\quad+\boldsymbol{e}_{\mathrm{abc}}-\boldsymbol{R}_{\mathrm{abc}}\boldsymbol{i}_{\mathrm{abc}}\\ \boldsymbol{A}^{\mathrm{T}}_{\mathrm{abc}}\boldsymbol{i}_{\mathrm{abc}}\end{pmatrix}. \qquad (1)$$

$$\boldsymbol{A}_{\mathrm{abc}}\equiv\begin{pmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{pmatrix},$$

$$\begin{aligned}
\boldsymbol{\psi}_{\mathrm{abc}} &\equiv (\psi_{\mathrm{a}},\psi_{\mathrm{b}},\psi_{\mathrm{c}})^{\mathrm{T}}\in\mathbb{R}^3\\
\boldsymbol{u}_{\mathrm{abc}} &\equiv (u_{\mathrm{a}},u_{\mathrm{b}},u_{\mathrm{c}})^{\mathrm{T}}\in\mathbb{R}^3\\
\boldsymbol{R}_{\mathrm{abc}} &\equiv \boldsymbol{A}^{\mathrm{T}}_{\mathrm{abc}}\boldsymbol{A}_{\mathrm{abc}}R+\boldsymbol{I}r\in\mathbb{R}^3\\
\boldsymbol{i}_{\mathrm{abc}}(\boldsymbol{\psi}_{\mathrm{abc}}) &\equiv (i(\psi_{\mathrm{a}}),i(\psi_{\mathrm{b}}),i(\psi_{\mathrm{c}}))^{\mathrm{T}}\in\mathbb{R}^3\\
\boldsymbol{e}_{\mathrm{abc}}(t) &\equiv E_{\mathrm{abc}}(\cos(\omega_e t),\cos(\omega_e t-\tfrac{2\pi}{3}),\cos(\omega_e t+\tfrac{2\pi}{3}))^{\mathrm{T}},
\end{aligned}$$

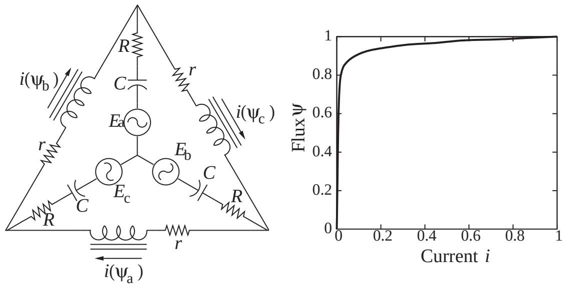

Figure 1: Three-phase circuit and nonlinear characteristics of flux interlinkages $i(\psi)$.

where $\boldsymbol{\psi}_{\mathrm{abc}}$ and $\boldsymbol{u}_{\mathrm{abc}}$ are the flux interlinkages of the inductors and voltages of the capacitors, respectively. The $\boldsymbol{I}$ denotes unit matrix, and $(*)^{\mathrm{T}}$ and $\mathbb{R}$ denote transposition and the set of real numbers, respectively. R, r, ω_e represent normalized circuit parameters which correspond to Y-connected resistors, Δ-connected resistors, and angular frequency of the voltage sources, respectively. We assume that the characteristics of the flux interlinkages $i(\psi)$ are represented by monotone increasing odd function.

In order to describe the symmetry of the circuit easily, we introduce phasor vector $\boldsymbol{E}_{\mathrm{abc}}$ and rewrite (1) by the following autonomous equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\boldsymbol{\psi}_{\mathrm{abc}}\\ \boldsymbol{u}_{\mathrm{abc}}\\ \boldsymbol{E}_{\mathrm{abc}}\end{pmatrix}=\begin{pmatrix}-\boldsymbol{A}_{\mathrm{abc}}\boldsymbol{u}_{\mathrm{abc}}+\mathrm{Re}\,[\boldsymbol{E}_{\mathrm{abc}}]-\boldsymbol{R}_{\mathrm{abc}}\boldsymbol{i}_{\mathrm{abc}}\\ \boldsymbol{A}^{\mathrm{T}}_{\mathrm{abc}}\boldsymbol{i}_{\mathrm{abc}}\\ \mathrm{j}\omega_{\mathrm{e}}\boldsymbol{E}_{\mathrm{abc}}\end{pmatrix},(2)$$

where $\boldsymbol{E}_{\mathrm{abc}}\equiv(E_{\mathrm{a}},E_{\mathrm{b}},E_{\mathrm{c}})^T\in\mathbb{C}^3$, the $\mathbb{C}$ denotes the set of complex numbers, j is imaginary unit and Re[·] denotes the real part of ·. The argument of the initial value $\boldsymbol{E}_{\mathrm{abc}}(0)$ satisfies

$$\begin{pmatrix}\arg[E_{\mathrm{b}}(0)]-\arg[E_{\mathrm{a}}(0)]\\ \arg[E_{\mathrm{c}}(0)]-\arg[E_{\mathrm{b}}(0)]\\ \arg[E_{\mathrm{a}}(0)]-\arg]E_{\mathrm{c}}(0)]\end{pmatrix}=-\mathrm{k}_3\mathbf{1},\quad \mathrm{k}_3\equiv\frac{2\pi}{3}. \qquad (3)$$

where $\arg[\cdot]\in\mathbb{T}$ denotes the argument, the $\mathbb{T}$ denotes torus and $\mathbf{1}=(1,1,1)^{\mathrm{T}}$.

Further, we rewrite (2) as

$$\frac{\mathrm{d}\boldsymbol{x}_{\mathrm{abc}}}{\mathrm{d}t}=\boldsymbol{f}_{\mathrm{abc}}(\boldsymbol{x}_{\mathrm{abc}}), \qquad (4)$$

where $\boldsymbol{x}_{\mathrm{abc}}=(\boldsymbol{\psi}^{\mathrm{T}}_{\mathrm{abc}},\boldsymbol{u}^{\mathrm{T}}_{\mathrm{abc}},\boldsymbol{E}^{\mathrm{T}}_{\mathrm{abc}})^{\mathrm{T}}$.

3. Symmetries of Three-Phase Circuit

In order to describe the symmetries of the three-phase circuit, we introduce the following permutations $\check{\gamma}$:

$$\check{\gamma} = \begin{pmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \chi_{\mathrm{a}} & \chi_{\mathrm{b}} & \chi_{\mathrm{c}} \end{pmatrix}, \quad \begin{array}{l} \chi_n \in \{\mathrm{a,b,c}\}\ (n \in \{\mathrm{a,b,c}\}), \\ \chi_n \neq \chi_m\ (n \neq m \in \{\mathrm{a,b,c}\}). \end{array} .$$

For example, a cyclic permutation is represented by

$$\check{c}_3 \equiv \begin{pmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{b} & \mathrm{c} & \mathrm{a} \end{pmatrix}. \tag{5}$$

The action $\check{c}_3$ satisfies the following commutativity:

$$\check{c}_3 \boldsymbol{f}_{\mathrm{abc}}(\boldsymbol{x}_{\mathrm{abc}}) = \boldsymbol{f}_{\mathrm{abc}}(\check{c}_3 \boldsymbol{x}_{\mathrm{abc}}). \tag{6}$$

This relation shows that the three-phase circuit has the cyclic symmetry. The action of $\check{c}_3$ can be represented also by the following matrix $\boldsymbol{c}_3 \in \mathbb{R}^9 \times \mathbb{R}^9$:

$$\boldsymbol{c}_3 \equiv \begin{pmatrix} \boldsymbol{c}_3' & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c}_3' & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{c}_3' \end{pmatrix}, \quad \boldsymbol{c}_3' \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{7}$$

The matrix $\boldsymbol{c}_3'$ has three eigenvalues $1, a, a^2$ where $a \equiv e^{\mathrm{i}k_3}$ and the eigenvectors are

$$\boldsymbol{w}_0' \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \boldsymbol{w}_+' \equiv \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix}, \quad \boldsymbol{w}_-' \equiv \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix}, \tag{8}$$

respectively. In the same way, the eigenvectors of $\boldsymbol{c}_3$ are $\boldsymbol{w}_0 \equiv (\boldsymbol{w}_0'^T, \boldsymbol{w}_0'^T, \boldsymbol{w}_0'^T)^T$, $\boldsymbol{w}_+ \equiv (\boldsymbol{w}_+'^T, \boldsymbol{w}_+'^T, \boldsymbol{w}_+'^T)^T$ and $\boldsymbol{w}_- \equiv (\boldsymbol{w}_-'^T, \boldsymbol{w}_-'^T, \boldsymbol{w}_-'^T)^T$, respectively.

Next, we consider inversion symmetry based on the odd symmetry of the function $i(\psi)$:

$$\tilde{\boldsymbol{x}}_{\mathrm{abc}} = \check{i}\boldsymbol{x}_{\mathrm{abc}} = -\boldsymbol{x}_{\mathrm{abc}} \tag{9}$$

The action $\check{i}$ satisfies

$$\check{i} \boldsymbol{f}_{\mathrm{abc}}(\boldsymbol{x}_{\mathrm{abc}}) = \boldsymbol{f}_{\mathrm{abc}}(\check{i} \boldsymbol{x}_{\mathrm{abc}}). \tag{10}$$

This relation shows that the three-phase circuit has the inversion symmetry. The action $\check{i}$ can be represented also by the following matrix $\boldsymbol{i} \in \mathbb{R}^9 \times \mathbb{R}^9$

$$\boldsymbol{i} \equiv -\boldsymbol{I} \tag{11}$$

where $\boldsymbol{I}$ denotes 9×9 unit matrix and the eigenvalue of $\boldsymbol{i}$ equals -1.

From the 2 symmetries, the three-phase circuit has the symmetry with respect to the commutative group $\check{\Gamma}$

$$\check{\Gamma} \equiv \left\{ \check{e}, \quad \check{c}_3, \quad \check{c}_3^2, \quad \check{i}, \quad \check{i}\check{c}_3, \quad \check{i}\check{c}_3^2, \right\}. \tag{12}$$

Subgroups of the group $\check{\Gamma}$ are listed in Tab.1 and the lattice of the subgroups is shown in Fig.2.

Table 1: Subgroup of $\check{\Gamma}$

Order 1	$\check{E}$	$\equiv \{\check{e}\}$
Order 2	$\check{I}$	$\equiv \left\{\check{e},\ \check{i}\right\}$
Order 3	$\check{C}_3$	$\equiv \left\{\check{e},\ \check{c}_3,\ \check{c}_3^2\right\}$
Order 6	$\check{\Gamma}$	$\equiv \left\{\check{e},\ \check{c}_3,\ \check{c}_3^2,\ \check{i},\ \check{i}\check{c}_3,\ \check{i}\check{c}_3^2\right\}$

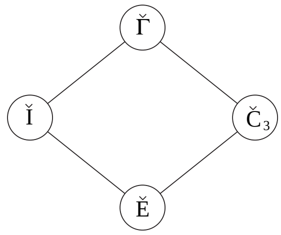

Figure 2: Lattice of symmetries in three-phase circuit

4. Decomposition of Almost Periodic Oscillation

4.1. Definition of symmetric almost periodic oscillation

Almost periodic oscillation with normalized phase $\boldsymbol{\theta} \in \mathbb{T}^2$ on 2 dimensional torus is defined by $\hat{\boldsymbol{x}}(\boldsymbol{\theta})$ and $\boldsymbol{x}_{\mathrm{abc}}(t)$ is represented by

$$\boldsymbol{x}_{\mathrm{abc}}(t) = \hat{\boldsymbol{x}}(\boldsymbol{\theta}),\ \dot{\boldsymbol{\theta}} = \boldsymbol{\omega},\ \boldsymbol{\omega} \in \mathbb{R}^2, \tag{13}$$

where $\boldsymbol{\omega}$ is angular frequency of torus.

Let us consider a subgroup $\check{H} \subset \check{\Gamma}$. If an almost periodic oscillation $\hat{\boldsymbol{x}}(\boldsymbol{\theta})$ satisfies

$$\check{H} = \left\{ \check{\gamma} \in \check{\Gamma} \mid \check{\gamma}\{\hat{\boldsymbol{x}}(\boldsymbol{\theta})\} = \{\hat{\boldsymbol{x}}(\boldsymbol{\theta})\} \right\} \tag{14}$$

for all the actions $\check{\gamma} \in \check{H}$, the almost periodic oscillation has spatio-temporal symmetry[1]. This relation shows that the $\check{H}$-action preserves the trajectory of $\hat{\boldsymbol{x}}(\boldsymbol{\theta})$ and an action $\check{\gamma}$ causes only a shift $\boldsymbol{k} \in \mathbb{T}^2$:

$$^{\forall}\boldsymbol{\theta} \in \mathbb{T}^2,\ \check{\gamma}\hat{\boldsymbol{x}}(\boldsymbol{\theta}) = \hat{\boldsymbol{x}}(\boldsymbol{\theta} - \boldsymbol{k}). \tag{15}$$

We denote the correspondence between $\check{\gamma}$ and $\boldsymbol{k}$ by a map $\boldsymbol{k} = \boldsymbol{\Theta}(\check{\gamma})$. An example of $\boldsymbol{k} = \boldsymbol{\Theta}(\check{\gamma})$ for the group $\check{\Gamma}$ is shown in Tab.2.

Table 2: Example of the map $\boldsymbol{k} = \boldsymbol{\Theta}(\check{\gamma})$

$\check{\gamma}$	$\check{e}$	$\check{c}_3$	$\check{c}_3^2$	$\check{i}$	$\check{i}\check{c}_3$	$\check{i}\check{c}_3^2$
$\boldsymbol{k}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{2\pi}{3} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{4\pi}{3} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \pi \\ \pi \end{pmatrix}$	$\begin{pmatrix} \frac{5\pi}{3} \\ \pi \end{pmatrix}$	$\begin{pmatrix} \frac{\pi}{3} \\ \pi \end{pmatrix}$

4.2. Decomposition of oscillation

We represent $\hat{\boldsymbol{x}}(\boldsymbol{\theta})$ by 2-dimensional Fourier series expansion

$$\hat{\boldsymbol{x}}(\boldsymbol{\theta}) = \sum_{\boldsymbol{j}\in\mathbb{Z}^2} \boldsymbol{X}_{\boldsymbol{j}} \exp(\mathrm{j}\boldsymbol{j}^{\mathrm{T}}\boldsymbol{\theta}) + \text{c.c.}, \quad \boldsymbol{X}_{\boldsymbol{j}} \in \mathbb{C}^9, \tag{16}$$

where c.c. represents complex conjugate and $\mathbb{Z}$ denotes the set of integers. Then, the 2-dimensional Fourier series expansion of Eq.(15) derives

$$\check{\gamma} \in \check{H}, \quad \check{\gamma}\boldsymbol{X}_{\boldsymbol{j}} = \exp(\mathrm{j}\boldsymbol{j}^{\mathrm{T}}\boldsymbol{k})\boldsymbol{X}_{\boldsymbol{j}}. \tag{17}$$

This equation shows that the vector $\boldsymbol{X}_{\boldsymbol{j}}$ is in the eigenspace of the action $\check{\gamma}$ with respect to the eigenvalue $\lambda \equiv \exp(\mathrm{j}\boldsymbol{j}^{\mathrm{T}}\boldsymbol{k})$. For example, the cyclic symmetry $\check{\gamma} = \check{c}_3$ has three eigenvalues $1, a, a^2$ and the $\boldsymbol{X}_{\boldsymbol{j}}$ can be classified into three eigenspaces $\boldsymbol{w}_0$, $\boldsymbol{w}_+$ and $\boldsymbol{w}_-$. That is, from Tab.2 we can classify the vector $\boldsymbol{j}$ into three set

$$\begin{aligned} \kappa_0 &\equiv \left\{\boldsymbol{j} \in \mathbb{Z}^2 \mid j_1 \mod 3 = 0\right\}. \\ \kappa_+ &\equiv \left\{\boldsymbol{j} \in \mathbb{Z}^2 \mid j_1 \mod 3 = 1\right\}. \\ \kappa_- &\equiv \left\{\boldsymbol{j} \in \mathbb{Z}^2 \mid j_1 \mod 3 = -1\right\}. \end{aligned} \tag{18}$$

In the case of the inversion symmetry $\check{\gamma} = \check{i}$, the eigenvalues are -1 and the $\boldsymbol{j}$ belongs to the set κ_{odd}:

$$\kappa_{\mathrm{odd}} \equiv \left\{\boldsymbol{j} \in \mathbb{Z}^2 \mid j_1 + j_2 \mod 2 = 1\right\}. \tag{19}$$

As a result, the flux $\hat{\boldsymbol{\psi}}_{\mathrm{abc}} : \mathbb{T}^2 \mapsto \mathbb{R}^3$ can be decomposed as

$$\begin{aligned} \hat{\boldsymbol{\psi}}_{\mathrm{abc}}(\boldsymbol{\theta}) = & \sum_{\boldsymbol{j}\in\kappa_0\cap\kappa_{\mathrm{odd}}} \Psi_{0,j_1,j_2}\boldsymbol{w}'_0 \exp(\mathrm{i}(j_1\theta_1 + j_2\theta_2)) \\ & + \sum_{\boldsymbol{j}\in\kappa_+\cap\kappa_{\mathrm{odd}}} \Psi_{+,j_1,j_2}\boldsymbol{w}'_+ \exp(\mathrm{i}(j_1\theta_1 + j_2\theta_2)) \\ & + \sum_{\boldsymbol{j}\in\kappa_-\cap\kappa_{\mathrm{odd}}} \Psi_{-,j_1,j_2}\boldsymbol{w}'_- \exp(\mathrm{i}(j_1\theta_1 + j_2\theta_2)) + \text{c.c.} \end{aligned} \tag{20}$$

We call the components in $\boldsymbol{w}'_0, \boldsymbol{w}'_+, \boldsymbol{w}'_-$ common mode, forward mode and backward mode, respectively. This decomposition (20) indicates that the spectra are also decomposed exclusively in the modes.

4.3. Example of decomposition

In order to confirm the decomposition, we apply the method to an almost periodic oscillation with $\check{\Gamma}$ symmetry shown in Fig.3. The figure shows the waveform of fluxes $\psi_a(t), \psi_b(t), \psi_c(t)$. The frequency components of the fluxes and the decomposed frequency components are shown in Fig.4 and Fig.5. The pattern diagram of Fig.5 is shown in Fig.6, where ω_0 and ω_+ correspond to $\boldsymbol{j} = (0, 1)^{\mathrm{T}}$ and $\boldsymbol{j} = (1, 0)^{\mathrm{T}}$, respectively. The common, forward and backward modes enables the exclusive decomposition of the frequency components.

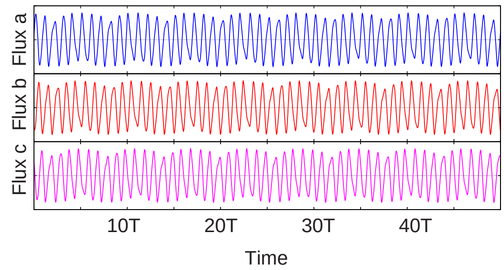

Figure 3: Almost periodic oscillation with $\check{\Gamma}$ symmetry. $(i(\psi) = \psi + 3.8\psi^7, \omega_{\mathrm{e}} = 7, T = 2\pi/\omega_{\mathrm{e}}, E_{\mathrm{abc}} = 2.0, R = 0.01, r = 0.01.)$

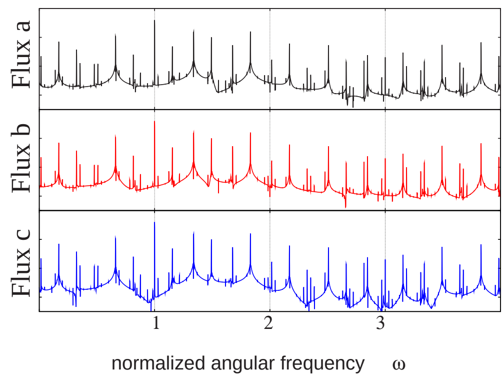

Figure 4: Frequency components of flux $\hat{\boldsymbol{\psi}}_{\mathrm{abc}}$

5. Bifurcation Analysis by 2-Dimensional Harmonic Balance

Almost periodic oscillations can be analyzed by 2-dimensional harmonic balance method [7]. Although the method requires large number of frequency components, the proposed method decomposes the spectra exclusively and reduces the number of unknowns.

Table.3 and Fig.7 show the considered frequency components for the harmonic balance. The bifurcation diagram obtained by the method is shown in Fig.9. The lines A_1 and A_2 represent the bifurcation diagram of periodic oscil-

Table 3: Frequencies for harmonic balance

		j_1	j_2
0	C_1	0	1
	C_2	3	0
positive	F_1	1	0
	F_2	1	2
	F_3	1	-2
negative	B_1	2	-1
	B_2	2	1
	B_3	2	-3

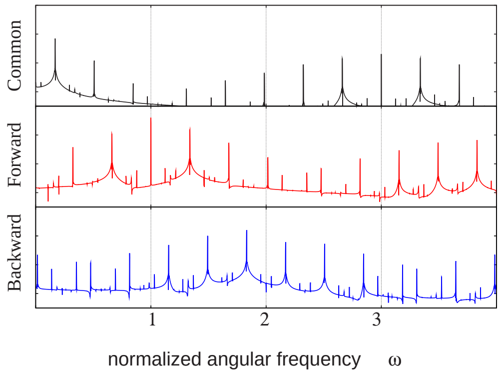

Figure 5: Decomposed frequency components of flux $\hat{\boldsymbol{\psi}}_{\text{abc}}$ by the common, forward and backward modes $\boldsymbol{w}'_0, \boldsymbol{w}'_+, \boldsymbol{w}'_-$.

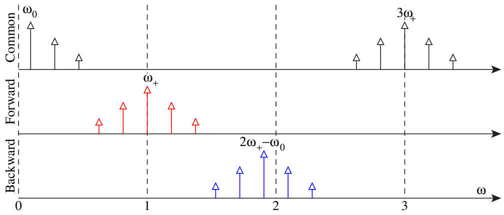

Figure 6: Pattern diagram of frequency components in Fig.5. The ω_0 and ω_+ corresponds to $\boldsymbol{j} = (0,1)^{\text{T}}$ and $\boldsymbol{j} = (1,0)^{\text{T}}$, respectively.

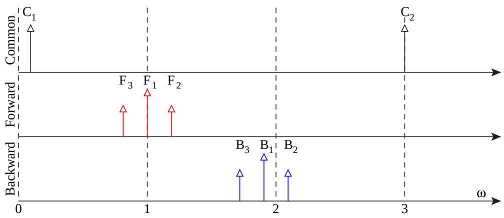

Figure 7: Considered frequency components for harmonic balance method.

lation shown in Fig. 8. The line A_3 shows the bifurcation diagram of the almost periodic oscillation. The bifurcation diagram succeeds to calculate the Neimark Sacker bifurcation points N_1 and N_2 where the almost periodic oscillation is generated.

6. Conclusion

We showed that symmetric almost periodic oscillations can be decomposed into the subspaces which correspond to the eigenspaces. Using the decomposition, we classified also the frequency components of the almost periodic oscillation exclusively. Further, applying the decomposition to 2-dimensional harmonic balance method, we obtained the bifurcation diagram of the almost periodic oscillation.

Although the decomposition is shown in the three-phase circuit, the method can be applied to symmetric almost periodic oscillations in general systems.

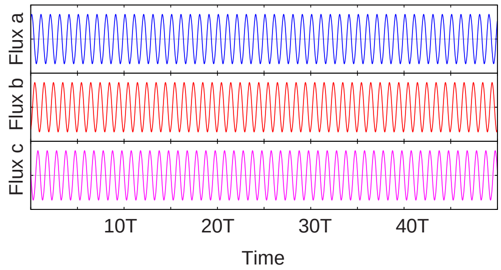

Figure 8: Periodic fundamental harmonic oscillation.

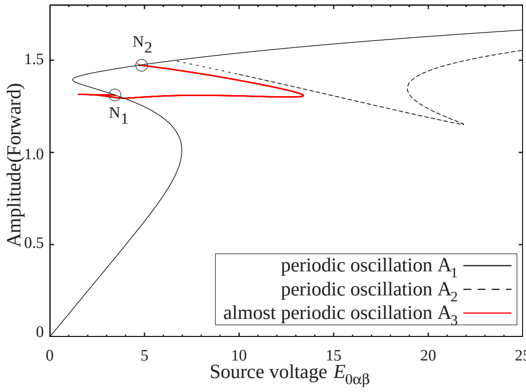

Figure 9: Bifurcation diagram of the fundamental periodic and almost periodic oscillations. ($E_{0\alpha\beta} = 3E_{\text{abc}}/\sqrt{6}$.) N_1 and N_2 represent the Neimark Sacker bifurcation points where the almost periodic oscillation is generated.

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