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Nonlinear Dynamics in Buck-Boost Converter with Spike Noise

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Abstract—This paper approaches to the nonlinear dynamics in buck-boost converter with spike noise from the mathematical point of view. First, we explain the circuit dynamics. Then, the discrete map is expressed in detail for the rigorous analysis. Finally, we derive some bifurcation diagrams, and discuss the dynamic behavior of the waveform in wide parameter space. In particular, we focus on the dynamical effect of spike noise when the circuit dynamics is in discontinuous conduction mode (DCM).

1. Introduction

Analysis of nonlinear dynamics in the piecewise smooth dynamical system is one of a fast developing area of engineering. In the field of electrical engineering as an example, discrete map of the DC/DC power converters is classified into the piecewise smooth map, and have actively studied in the past decade [1, 2, 3, 4].

Most of the previous studies have been conducted under the assumption of the theoretical switching action. Although, unavoidable missed switching actions such as the time delay (switching delay) and high frequency ripple (spike noise) can occur via the switching action in the practical systems; and effect in the circuit's qualitative property [5]. Therefore we have completely clarified the dynamical effect of missed switching action from both of the mathematical and experimental viewpoints in a simplified switched dynamical system with one dimensional discrete map [6]. Also, we have studied the switched dynamical system containing missed switching with attention to the circuit's discontinuous conduction mode (DCM) as the next step [7], e.g., clarified map's property and appearance of some bifurcation phenomena. On the other hand, the dynamical effect of missed switching in a wide parameter space is unclear when the circuit dynamics is in DCM.

In this paper, we focus on the circuit's DCM dynamics in current mode controlled buck-boost converter as an example, and clarify the dynamical effect of spike noise in a wide parameter space; to developing the circuit theory of such kinds of power converter. First, we explain the circuit dynamics. Then, the consecutive waveform is discretized by the clock interval, and discrete map (return map) is mathematically defined for the rigorous analysis. Finally, we derive some bifurcation diagrams and discuss the dynamical effect of spike noise in a wide parameter space.

2. Circuit Dynamics

Figure 1 shows the circuit model. Now, we let the capacitance voltage v be a constant value E_0 under the assumption of the capacitance C and resistance R are large enough against the clock interval T. Thus, the circuit equation is given by

$$L\frac{di}{dt} = \begin{cases} E_{\mathrm{d}}, & : \textbf{state-1} \\ -E_0, & : \textbf{state-2} \\ 0, & : \textbf{state-3} \end{cases}, \qquad (1)$$

where **state-1**, **state-2** and **state-3** are expressed as follows:

state-1: the switch is conducting, diode is blocking and current satisfies $0 < i < i_{\mathrm{ref}}$,

state-2: the switch is blocking, diode is conducting and current satisfies $0 < i < i_{\mathrm{ref}}$,

state-3: the switch and diode are blocking and the current satisfies $i = 0$.

In the following analysis, the dimensionless variables $\tau = E_{\mathrm{d}}t/(LI_{\mathrm{d}})$ and $y = i/I_{\mathrm{d}}$ are used. In addition, we let a variable M that satisfies $M = E_0/E_{\mathrm{d}}$.

Figure 2 shows the circuit behavior with ideal switching and spike noise, respectively. The detailed analysis in the circuit with ideal switching has been already studied in Ref. [4]. However, we show the behavior of waveform in the circuit with ideal switching again for the comparison between the circuit with spike noise. Also, the operation time of spike noise does not influence into the circuit dynamics so much, because it is short enough for the clock interval (see Ref. [5]). Thus, we ignore the operation time of spike noise in the following analysis to study the circuit's fundamental property. The basic switching rule in the circuit falls into three types as follows.

Rule-1: The switch changes from state ON to OFF if the waveform hits the reference value ($y = 1$).

Rule-2: The switch changes from state OFF to ON if the clock pulse is impressed when the switch is set at state OFF.

Rule-3: The switch keeps state OFF until the next clock pulse is impressed if the waveform reaches to zero.

Note that **Rule-2** can not be observed if the maximum size of spike noise h reaches to the reference value, i.e., the switch immediately changes from state OFF to ON and then it keeps state OFF during the clock interval. It also should be mention that the circuit dynamics is in DCM if the waveform reaches to zero.

3. Return Map

The waveform during the clock interval T is discretized into ten types by using the critical values D_1, D_2, D_3 and D_4 in the circuit with spike noise.

$$D_1 = 1 - T,\ D_2 = 1 - T + \frac{1}{M},\ D_3 = 1 - h,\ D_4 = MT \qquad (2)$$

Consequently, we can define the return map in the circuit with spike noise as follows:

$$y_{k+1} = F(y_k) = \begin{cases} y_k + 1, & (x_k, y_k) \in I_{s1} \\ -M(y_k + T - 1) + 1, & (x_k, y_k) \in I_{s2}, I_{s6}, I_{s7} \\ 0, & (x_k, y_k) \in I_{s3}, I_{s8} \\ y_k - MT, & (x_k, y_k) \in I_{s4}, I_{s9} \\ 0, & (x_k, y_k) \in I_{s5}, I_{s10} \end{cases}, \qquad (3)$$

where I_{sj} ($j = 1 \sim 10$) denotes the parameter space of the discretized waveform during the clock interval.

$$\begin{aligned} I_{s1} &= \{y_{k-1}, y_k |\ y_k < D_1\}, \\ I_{s2} &= \{y_{k-1}, y_k | D_1 \le y_k < D_3 \text{ or } (D_3 \le y_k, y_{k-1} \le D_1)\}, \\ I_{s3} &= \{y_{k-1}, y_k | D_2 \le y_k, y_{k-1} \le D_1\}, \\ I_{s4} &= \{y_{k-1}, y_k | D_3 \le y_k, D_1 < y_{k-1}\}, \\ I_{s5} &= \{y_{k-1}, y_k | D_3 \le y_k < D_4, D_1 < y_{k-1}\}, \\ I_{s6} &= \{y_{k-1}, y_k | D_1 \le y_k < D_3 \text{ or } (D_3 \le y_k < D_2, y_{k-1} \le D_1)\}, \\ I_{s7} &= \{y_{k-1}, y_k | D_1 \le y_k < D_2\}, \\ I_{s8} &= \{y_{k-1}, y_k | D_2 \le y_k < D_3 \text{ or } (D_3 \le y_k, y_{k-1} \le D_1)\}, \\ I_{s9} &= \{y_{k-1}, y_k | D_4 \le y_k, D_3 \le y_k, D_1 < y_{k-1}\}, \\ I_{s10} &= \{y_{k-1}, y_k | D_3 \le y_k, D_1 < y_{k-1}\}, \end{aligned} \qquad (4)$$

Let us suppose that we need the information of y_k and y_{k-1} to classify the waveform during the clock interval; that are the waveform at $\tau = kT$ and $\tau = (k-1)T$, respectively. In other words, the circuit with spike noise is divided into the two dimensional system, because of a solution y_{k+1} depends on y_k and y_{k-1}. However the parameter y_{k-1} is only used in the classification of the map in the circuit with spike noise. Note that the detailed process to the derivation of a discrete map is shown in Ref. [7]. On the other hand, the consecutive waveform during the clock interval is classified into three types in the circuit with ideal switching by using the border D_1 and D_2. Also, we can easily derive the return map in the circuit with ideal switching (see Ref. [4]).

Figure 3 shows the examples of waveform and its corresponding return map on y_{k-1}-y_k-y_{k+1} plane in the circuit with spike noise. In the figure, (a), (b) and (c) denote the circuit's possible three dynamics; that the circuit dynamics is in CCM (a), is in CCM and DCM (b) and is in DCM (c), respectively.

4. Analytical Results

Figure 4 shows the two dimensional bifurcation diagrams in the circuit with ideal switching and spike noise, respectively. The blue line denotes the bifurcation set of the period doubling bifurcation described as follows:

$$\prod_{k=1}^{m} \frac{dF(y_k)}{d\tau} + 1 = 0, \qquad (5)$$

where m represents the period-m waveform. Likewise, green, red and yellow lines are the border for classification of the circuit dynamics. Note that the dashed or continuous lines are used for the circuit with ideal switching and the circuit with spike noise, respectively.

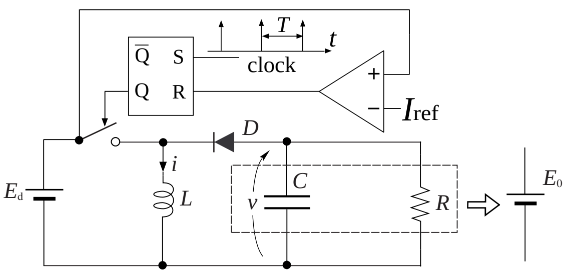
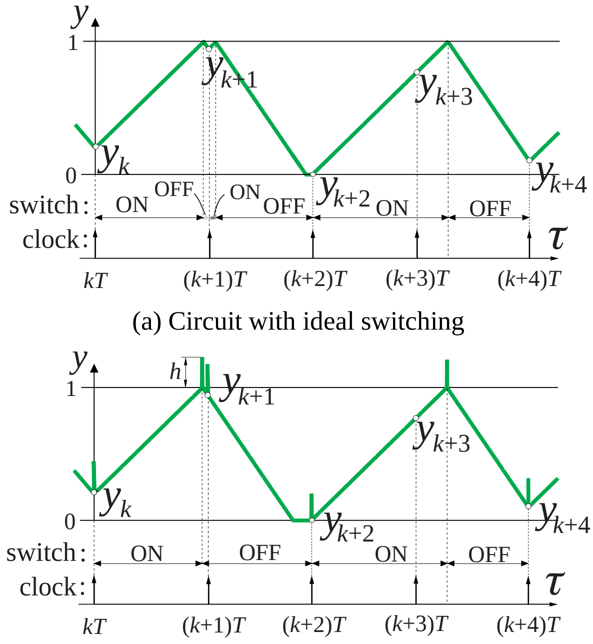


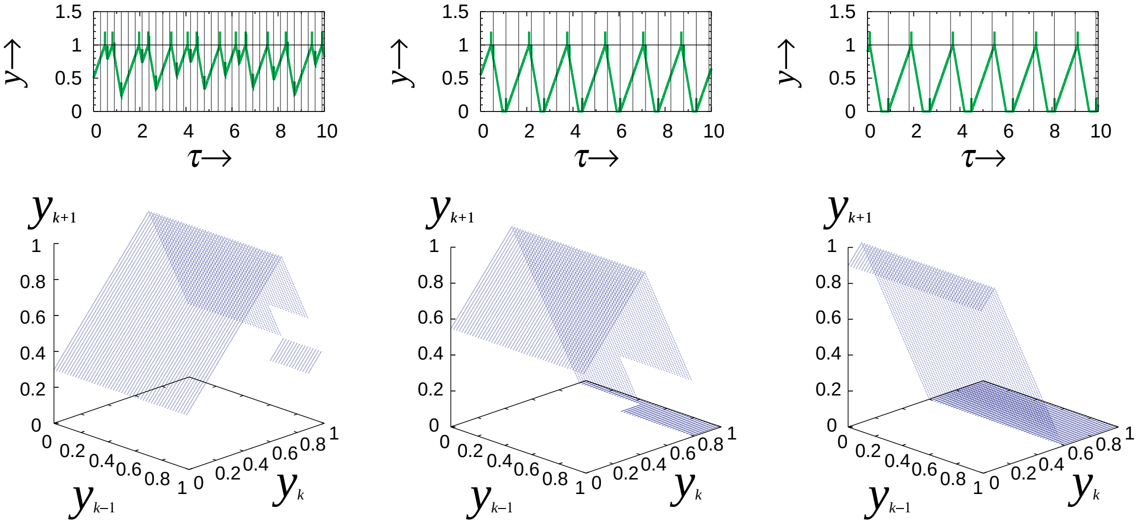
Figure 1: Buck-Boost converter.



(a) Circuit with ideal switching

(b) Circuit with spike noise

Figure 2: Examples of the waveform.



(a) $T = 0.3$ (b) $T = 0.55$ (c) $T = 0.9$

Figure 3: Examples of the waveform and its return map on $y_{k-1} - y_k - y_{k+1}$ plane ($M = 2.0$, $h = 0.2$).

The parameter space in the circuit with spike noise is classified into five regions based on the property of waveform as follows (see Fig. 4 (b)):

$$\begin{aligned}
&\textbf{Region-s0} = \{T, M : T < h, MT > h\} \\
&\textbf{Region-s1} = \left\{T, M : M < 1, 1 - T + \frac{1}{M} > 0\right\} \\
&\textbf{Region-s2} = \left\{T, M : 1 - T + \frac{1}{M} \le 0\right\} \\
&\textbf{Region-s3} = \left\{T, M : MT > 1 - h, M > 1, 1 - T + \frac{1}{M} > 0\right\} \\
&\textbf{Region-s4} = \{T, M : T \ge h, M > 1, MT \le 1 - h\}
\end{aligned} \quad (6)$$

More preciously, we let **Region-s0** be the not practical region, because occurrence of spike noise too much influences into the circuit dynamics in **Region-s0**. **Region-s1** and **Region-s2** are the existence region of the fixed point. Note that the circuit dynamics certainly be in CCM in **Region-s1**; however it is in DCM in **Region-s2**. Also, if the waveform at $\tau = kT$ is classified into the parameter space I_{s3}, I_{s8}, I_{s5} or I_{s10} (see Eq. (4)), the circuit's characteristic multiplier equal to -1 and the super stable periodic waveform is generated. Now, **Region-s3** denotes the existence region of the super stable periodic waveform. Moreover, circuit dynamics surely be in CCM, and all of the waveform is the chaotic attractor in **Region-s4**. Let us suppose that **Region-i1**, **Region-i2**, **Region-i3** and **Region-i4** in the circuit with ideal switching have same properties explained in the above sentence (see Fig. 4 (a)). In the following analysis, we focus on the circuit's DCM dynamics and clarified the dynamical effect of spike noise in a wide parameter space (see **Region-s3** and **Region-i3** in Fig. 4).

Figure 5 shows the enlarged two parameter bifurcation diagram in the circuit with ideal switching and spike noise (it correspond to the pink area in Fig. 4). We notice that the occurrence of spike noise effects in the blue-colored region in Fig. 5. More preciously, it is clear that the circuit dynamics is in DCM at the earlier timing in the circuit with spike noise compared with the circuit with ideal switching (see the dashed or continuous lines in Fig. 5). In other wards, **Region-i3** is expressed by the following equation.

$$\textbf{Region-i3} = \left\{T, M : MT > 1, M > 1, 1 - T + \frac{1}{M} > 0\right\} \quad (7)$$

Thus we can theoretically understand that the occurrence of spike noise makes the existence region of the super stable periodic waveform large, because the parameter h is a positive value in **Region-s3** (see Eq. (6)). Now, Fig. 3 (b) corresponds to the above case, i.e., a part of the return map containing the new switching rule; that keeps state off during the clock internal. Consequently, occurrence of spike noise makes the new switching rule, and effect in the circuit's DCM dynamics.

On the other hand, appearance of spike noise does not effect in the yellow-colored region in Fig. 5. Here, the gray line that is the border between the blue and yellow region is expressed as follows:

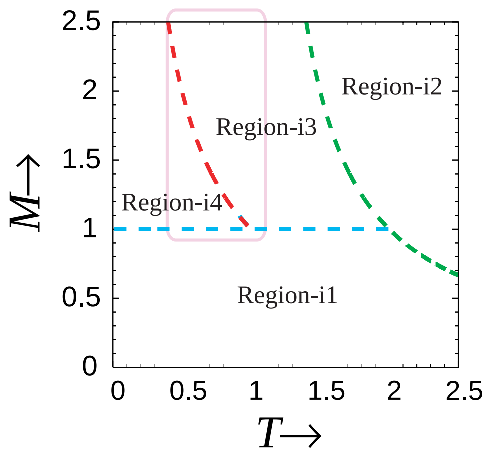
$$-T + \frac{1}{M} + h = 0, \quad (8)$$

where this is the existence condition of the critical value D_3 (see Eq. (2)). In other wards, yellow-colored region satisfies the condition of $D_3 \ge D_2$, i.e., the critical value D_3 is never been observed in the yellow-colored region. Figure 3 (c) corresponds to this case; the new type of switching rule is never been observed in the circuit with spike noise. Therefore the occurrence of spike noise has no influence into the circuit's DCM dynamics in the yellow-colored region.

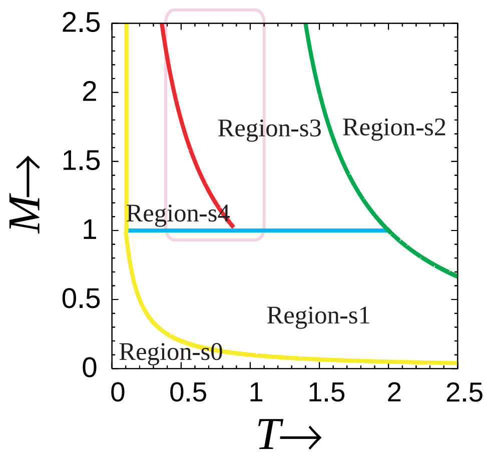
Consequently, we conclude that the appearance of spike noise makes the new switching rule, i.e., the switch keeps state OFF during the clock interval, and can effect into the circuit's DCM dynamics. Although, occurrence of spike noise has no influence into the circuit dynamics; if both of the circuit with ideal switching and spike noise is in DCM.

5. Conclusion

In this paper, we focus on the circuit's DCM dynamics in current mode controlled buck-boost converter, and have clarified the dynamical effect of spike noise in a wide parameter space. First, we explained the circuit dynamics. Then, the return map was mathematically defined for the rigorous analysis. Finally, we derived some bifurcation diagrams and discussed the dynamical effect of spike noise in a wide parameter space. As the results, we concluded that the appearance of spike noise makes the new switching rule; and can effect into the circuit's DCM dynamics. Although, occurrence of spike noise had no influence into the circuit dynamics; if both of the circuit with ideal switching and spike noise was in DCM. Our future work to be studied is the rigorous analysis of the two or more dimensional systems with missed switching.



(a) Circuit with ideal switching



(b) Circuit with switching delay

Figure 4: Two parameter bifurcation diagram.

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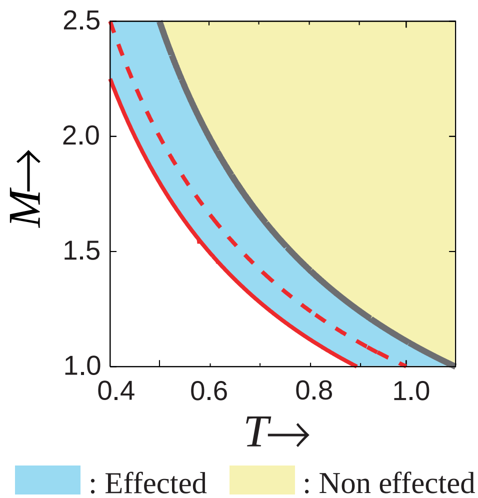

Figure 5: Enlarged two parameter bifurcation diagram of pink area in Fig. 4.