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Adaptive Synchronization in Presence of Noisy Perturbations

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Abstract—Adaptive coupling technique has been widely and successfully used in the research of chaos synchronization and collective dynamics in complex networks in recent years. Also this technique has been applied to identified parameters or topological structures in complex networks. In this work, we, through numerical investigation of several representative examples, show that in presence of noise perturbations, this coupling technique is still useful. We also provide a very brief outline for demonstrating the usefulness of this adaptive coupling technique in a sense of probability one (i.e., in a physical sense). We believe that our findings will further promote the applicability of adaptive coupling technique in the research of collective dynamics in real complex networks.

1. Introduction

It is well known that adaptive coupling technique has been widely and successfully used in the research of chaos synchronization and collective dynamics in complex networks in recent years [1]-[4]. Also this technique has been successfully applied to identified unknown parameters or hidden topological structures in complex networks [5]. The success of the application of this technique is attributed to its character which does not require the explicit information of the manipulated systems. The adaptive law is able to adjust the coupling gain to attain a suitable value at which the synchronization to equilibrium, periodic orbit, or even chaotic attractor can be realized.

Since the noisy perturbations are omnipresent in nature and in manmade systems [1, 6, 7], it is thus reasonable to import noise influence to adaptive synchronization and see whether this importation will destroy or preserve the applicability of adaptive coupling technique in adaptive synchronization. In this work, we, through numerical investigation of several representative examples, show that in presence of noise perturbations, this coupling technique is still useful. We also provide a very brief outline for demonstrating the usefulness of this adaptive coupling technique in a sense of probability one (i.e., in a physical sense). We believe that our findings will further promote the applicability of adaptive coupling technique in the research of collective dynamics in real complex networks.

2. The model: From deterministic to stochastic couplings

To begin with, we consider the following dynamical system:

$$\dot{\boldsymbol{x}}_t = \boldsymbol{f}(\boldsymbol{x}_t), \tag{1}$$

with an initial value $\boldsymbol{x}_0 \in \mathbb{R}^n$. Here, the system vector field $\boldsymbol{f} : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. Assume further that $\boldsymbol{f}$ satisfies the global Lipschitz condition, or more generally, the one-side global Lipschitz condition, i.e., there exists a constant L such that: $|\langle \boldsymbol{x}-\boldsymbol{y}, \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{y})\rangle| \le L|\boldsymbol{x}-\boldsymbol{y}|^2$ for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$. Here, $\langle\cdot,\cdot\rangle$ represents an inner product of two given vectors, and $|\cdot|$ is the Euclidean norm of a given vector. Analogous to the configurations in the literature, once the unidirectionally adaptive coupling technique is taken into account, i.e.,

$$\dot{\boldsymbol{y}}_t = \boldsymbol{f}(\boldsymbol{y}_t) + k_t(\boldsymbol{y}_t - \boldsymbol{x}_t), \quad \dot{k}_t = -\gamma|\boldsymbol{y}_t - \boldsymbol{x}_t|^2, \tag{2}$$

a complete synchronization between $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ could be certainly realized according to the LaSalle invariance principle. Here, $\gamma \ge 0$ is an arbitrary constant that could be adjusted for getting an optimal convergence rate.

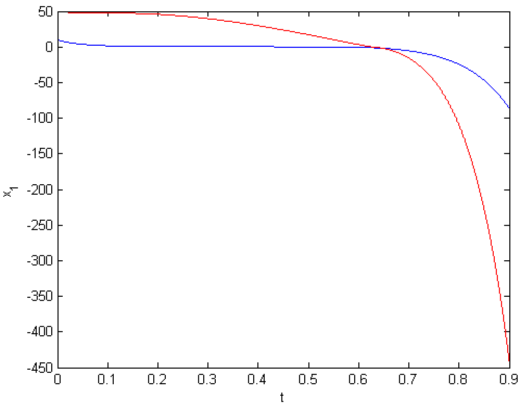
In this work, we intend to investigate whether a complete synchronization between $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ could be preserved when a certain type of noisy perturbation is imported. More precisely, instead of systems (1) and (2), we consider the following noise-perturbed adaptive system:

$$\begin{aligned} \mathrm{d}\boldsymbol{x}_t &= \boldsymbol{f}(\boldsymbol{x}_t)\mathrm{d}t, \\ \mathrm{d}\boldsymbol{y}_t &= \boldsymbol{f}(\boldsymbol{y}_t)\mathrm{d}t + k_t(\boldsymbol{y}_t - \boldsymbol{x}_t)\mathrm{d}B_t, \\ \mathrm{d}k_t &= \gamma|\boldsymbol{y}_t - \boldsymbol{x}_t|^2\mathrm{d}t. \end{aligned} \tag{3}$$

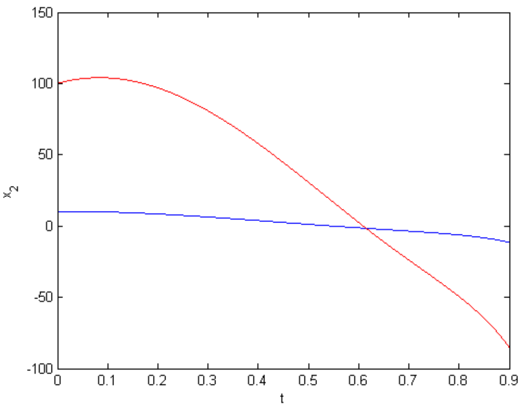
with an initial value $(\boldsymbol{x}_0, \boldsymbol{y}_0, k_0) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}$. Here, B_t is an one dimensional Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\ge0}, \mathbb{P})$, where the filtration $\{\mathscr{F}_t\}_{t\ge0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets).

3. Illustrative examples

First, we use Chua's system as an example. Accordingly, the noise-perturbed adaptive system is governed by the fol-



(a)



(b)

Figure 1: Sample paths in the planes of state-variable versus time, showing divergent dynamics in the uncoupled Chua's system.

lowing equations:

$$
\begin{aligned}
\mathrm{d}x_t &= a(y_t - x_t - g(x_t))\mathrm{d}t, \\
\mathrm{d}y_t &= (x_t - y_t + z_t)\mathrm{d}t, \\
\mathrm{d}z_t &= (-by_t - cz_t)\mathrm{d}t, \\
\mathrm{d}x'_t &= a(y'_t - x'_t - g(x'_t))\mathrm{d}t + k_t(x'_t - x_t)\mathrm{d}B_t, \\
\mathrm{d}y'_t &= (x'_t - y'_t + z'_t)\mathrm{d}t + k_t(y'_t - y_t)\mathrm{d}B_t, \\
\mathrm{d}z'_t &= (-by'_t - cz'_t)\mathrm{d}t + k_t(z'_t - z_t)\mathrm{d}B_t, \\
\mathrm{d}k_t &= \gamma[(x'_t - x_t)^2 + (y'_t - y_t)^2 + (z'_t - z_t)^2]\mathrm{d}t,
\end{aligned} \tag{4}
$$

where

$$
g(x) = \begin{cases} 10x, & x \in [0, 1], \\ x + 9, & x \in (1, +\infty), \\ -2x, & x \in (-\infty, 0). \end{cases}
$$

In the simulations, we take parameters as $a = 10.725$, $b = 10.52$, and $c = 0.26$. Without the coupling, i.e., we set $\gamma = 0$ and $k_t \equiv 0$ in system (4), the synchronization cannot be realized, as shown in Fig. 1. However, the complete synchronization could be realized numerically when we switch on the adaptive technique in presence of the noisy perturbations, as shown in Fig. 2.

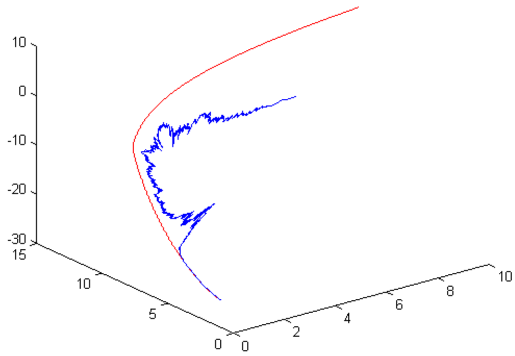


Figure 2: Sample paths (the red line denotes the trajectory of $\boldsymbol{x}_t$ and the blue one denotes $\boldsymbol{y}_t$) in the phase space, showing synchronized dynamics in the adaptively-coupled Chua's systems in presence of noisy perturbation. Here, $\gamma = 1$.

Second, we consider the adaptively-coupled Hindmarsh-Rose neuron models with noise perturbations. The synchronization phenomena can be observed whenever γ is positive or negative. It is noted that the synchronization cannot be realized when γ is positive and the noise perturbations are not taken into account.

As a matter of fact, the adaptive synchronization phenomena can also be numerically observed in some other representative systems with noisy perturbations, including the Lorenz system and the Rössler hyper-chaotic system. Due to the space limitation of this presentation, we do not include all the numerical results for these examples.

4. Outline of theoretical demonstration

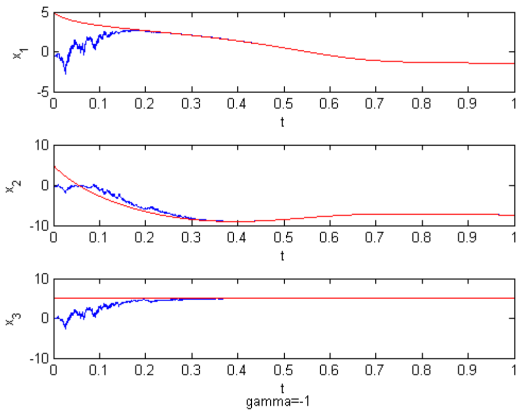
Usually, the proof for the synchronization in a dynamical system can be transformed as a stability problem on the corresponding error dynamics. Here, we also consider stability problem on the error dynamics of system (3). To this end, we need to complete the following steps for validations:

1. The error dynamics cannot approach zero almost surely before it explodes;

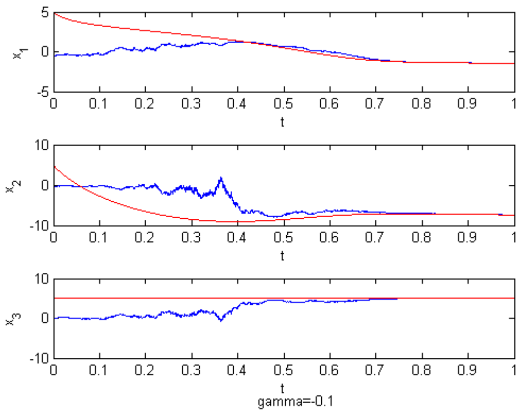
2. The error dynamics does not explode in any finite time almost surely;

3. There is a semi-martingale decomposition for some certain form of the error dynamics. Based on this decomposition as well as on a proper decomposition for the whole event set, we finally could get the convergence of the error dynamics in the sense of probability one.

We will present the detailed proof at the conference and seek somewhere else for its publication.



(a)



(b)

Figure 3: Synchronized sample paths of the noise-perturbed adaptive Hindmarsh-Rose neuron models when $\gamma = -1$ (a) and $\gamma = -0.1$ (b).

Acknowledgments

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