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Potential Games Based Coverage Control with Voronoi Partition

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Abstract—This paper presents sensor coverage control to cover a whole mission space and to maximize a sensing performance to detect targets. Suppose that each point in the mission space is covered by the nearest sensor. Then, a sensing area of each sensor is represented as a Voronoi partition. We introduce an objective function which represents the sensing performance based on the Voronoi partition and formulate the sensor coverage problem as an optimization problem. By introducing a barycentric coordinate over the mission space, we show that the sensor coverage problem can be transformed into a potential game. In potential games, local maximizers of a potential function are stable equilibrium points of the corresponding replicator dynamics. We propose distributed sensor coverage control based on the replicator dynamics to find the local maximizers of the objective function. Moreover, by simulation, we investigate the relation between a value function and stable equilibrium points of the replicator dynamics.

1. Introduction

Because of recent development of sensor network technologies and electronic devices, a group of autonomous sensors with computing, communication, and mobility capabilities is expected to perform a variety of distributed sensing tasks such as surveillance and environmental monitoring [1]. In such a sensor network, each sensor communicates with its neighbor sensors locally and decides its optimal placement based on the local information. So, the coverage control requires a decentralized control method for the optimal placement of a large number of sensors to achieve a high sensing performance [2, 3]. We consider a sensor coverage problem which places mobile sensors in the mission space to maximize an objective function. To solve such a maximization problem, Martínez *et al.* applied motion coordination such as swarming behaviors in biological groups [3].

On the other hand, the concept of potential games was introduced by Monderer and Shapley [4]. In potential games, all information about payoffs that is relevant to agents' incentives can be captured in a scalar-valued function which is called a potential function. Therefore, the profitable strategy revisions increase a value of the potential function. Marden *et al.* proposed a control method based on potential games to maximize an objective function [5]. Hayashi *et al.* extended it to a power-aware sensor coverage problem [6].

In this paper, we consider a sensor coverage problem to cover the whole mission space and to maximize the sensing performance to detect targets in the mission space. We assume that each point in the mission space is covered by the nearest sensor. Then, a sensing area of each sensor is represented as a Voronoi partition. The objective function to maximize the sensing performance is considered as a potential function in a potential game. Then, we show that the sensor coverage problem can be represented as a potential game by introducing a barycentric coordinate over the mission space. Sandholm showed that all local maximizers of a potential function in potential games are stable equilibrium points of the corresponding replicator dynamics [7]. Based on the result, we use replicator dynamics to find a suboptimal position of each sensor.

The remainder of this paper is organized as follows. In section 2, we review the problem setting of the sensor coverage problem. We introduce a barycentric coordinate over the mission space and rewrite it as an optimization problem based on the barycentric coordinate in section 3. Section 4 shows that the sensor coverage problem can be transformed into a potential game where the barycentric coordinate corresponds to a mixed strategy. We also propose distributed optimal sensor coverage control based on replicator dynamics. In section 5, we show some simulations. Finally, we conclude this paper in section 6.

2. Sensor Coverage Problem

In this paper, we consider a sensor coverage problem to cover a whole mission space and maximize sensing performance. We model the mission space of n mobile sensors as a convex polytope $Q \subset \mathbf{R}^2$ with m vertices $v_1 = (v_{1_1}, v_{1_2})^T, v_2 = (v_{2_1}, v_{2_2})^T, \ldots,$ and $v_m = (v_{m_1}, v_{m_2})^T$. Shown in Fig. 1 is an example of the convex polytope Q with 5 vertices. Let $I = \{1, \ldots, n\}$ be a set of n mobile sensors. Suppose that a position of each sensor i is represented as $r^i = (r^i_1, r^i_2)^T \in Q$. A value function ϕ: $Q \to [0, \infty)$ is integrable and represents the relative importance of each point in the mission space Q. We assume that all sensors have the same performance. For sensor i at the point $r^i \in Q$, its ability to detect a target originating at a point $q = (q_1, q_2)^T \in Q$ degrades with a distance $\|q - r^i\|$. This ability is measured by a performance function $h : [0, \infty) \to \mathbf{R}$ which is assumed to be nonincreas-

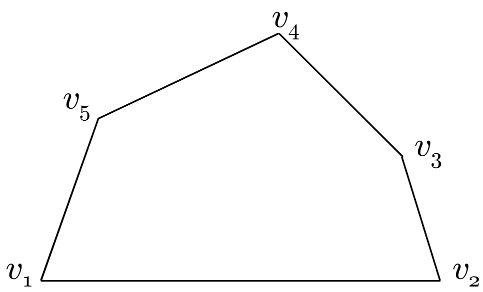

Figure 1: Example of a convex mission space Q where each v_i is a vertex ($i = 1, \ldots, 5$).

ing and piecewise-differentiable. With the value function ϕ and the performance function h, we introduce an objective function H by

$$H(r) := \int_Q \max_{i \in I} h(\|q - r^i\|)\phi(q)dq, \tag{1}$$

where $r = (r^1, \ldots, r^n)$.

We assume that each point in the mission space is covered by the nearest sensor. Then, a sensing area of each sensor is represented as a Voronoi partition. Given $Q \subset \mathbf{R}^2$ and $r = (r^1, \ldots, r^n)$ of n distinct points, the Voronoi partition of Q generated by r is the collection of sets $\{v_1(r), \ldots, v_n(r)\}$ defined by $v_i(r) := \{q \in Q | \; \|q - r^i\| \leq \|q - r^j\| \text{ for all } i \neq j, \; j \in I\}$ [3]. We refer to $v_i(r)$ as the Voronoi cell of r^i. Then, since the function $h(\|q - r^i\|)$ is monotonically non-increasing, the objective function H is rewritten as follows:

$$H(r) = \sum_{i=1}^{n} \int_{v_i(r)} h(\|q - r^i\|)\phi(q)dq, \tag{2}$$

where v_i represents a sensing area of sensor i for all $i \in I$.

3. Optimization Problem with a Barycentric Coordinate

All points in a convex polytope are represented by a barycentric coordinate [8]. In this section, we propose a representation of a position of sensor i based on the barycentric coordinate. Let $x^i = (x^i_1, \ldots, x^i_m)^T$ be the barycentric coordinate of sensor i, where $x^i_k \geq 0$ and $\sum_{k=1}^m x^i_k = 1$ for all $i \in I$ and $k \in \{1, \ldots, m\}$. All sensors are placed in the mission space Q. The position r^i is represented by the barycentric coordinate $x^i = (x^i_1, \ldots, x^i_m)^T$ as follows:

$$r^i(x^i) = \begin{bmatrix} v_{1_1} & \ldots & v_{m_1} \\ v_{1_2} & \ldots & v_{m_2} \end{bmatrix} \begin{bmatrix} x^i_1 \\ \vdots \\ x^i_m \end{bmatrix} \quad \text{for all } i \in I, \tag{3}$$

where

$$\begin{aligned} &\sum_{k=1}^{m} x^i_k = 1 && \text{for all } i \in I, \\ &x^i_k \geq 0 && \text{for all } i \in I \text{ and } k \in \{1, \ldots, m\}. \end{aligned}$$

Thus, the objective function (2) is a function of the barycentric coordinate as follows:

$$H(x) := H(r(x)) = \sum_{i=1}^{n} \int_{v_i(r(x))} h(\|q - r^i(x^i)\|)\phi(q)dq, \tag{4}$$

where $x = (x^1, \ldots, x^n)$ is the barycentric coordinate of n sensors. Therefore, we can formulate the sensor coverage problem as the following optimization problem :

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & H(x) \\ \text{subject to} \quad & \sum_{k=1}^{m} x^i_k = 1 \quad \text{for all } i \in I, \\ & x^i_k \geq 0 \quad \text{for all } i \in I \text{ and } k \in \{1, \ldots, m\}. \end{aligned}$$

For all sets $v \subseteq Q$, let M_v and C_v be as follows:

$$M_v := \int_v \phi(q)dq, \tag{5}$$

$$C_v := \frac{1}{M_v} \int_v q\phi(q)dq. \tag{6}$$

C_v is called a centroid of a set v. If a combination of sensor positions r satisfies Eq.(7), r is a local maximizer of objective function (2).

$$r^i = C_{v_i} \quad \text{for all } i \in I. \tag{7}$$

4. Potential Game for Sensor Coverage Problem

In potential games, local maximizers of a potential function are stable equilibrium points of the corresponding replicator dynamics. Therefore, we formulate the sensor coverage problem as a potential game, and search local maximizers of the objective function H.

We consider sensor i as population i in a potential game and a pure strategy $k \in S = \{1, \ldots, m\}$ is to locate a sensor at a vertex v_k of the mission space. Suppose that S is the common set of pure strategies of all populations. Then, the barycentric coordinate x^i of sensor i can be considered as a population state of population i. Note that $\sum_{k=1}^m x^i_k = 1$ for all $i \in I$. Moreover, we consider the objective function H as the potential function in the potential game. Therefore, introducing a payoff function $F^i_k(x)$ defined by

$$F^i_k(x) = \frac{\partial H}{\partial x^i_k}(x) \quad \text{for all } i \in I \text{ and } k \in S, \tag{8}$$

we can transform the sensor coverage problem into an n population potential game, which has m vertices of the mission space Q as the pure strategies, the barycentric coordinate x^i as the population state for population i, and the objective function $H(x)$ as the potential function.

For simplicity, we assume that $h(d) = -d^2$. Let

$$J_{r,v} := \int_v \|q - r\|^2 \phi(q) dq. \quad (9)$$

Then, we have the following equation:

$$J_{r,v} = J_{C_v,v} + M_v \|r - C_v\|^2, \quad (10)$$

and the objective function (4) is rewritten as follows:

$$H(x) = -\sum_{i=1}^{n} J_{C_{v_i},v_i} - \sum_{i=1}^{n} M_{v_i} \left\| r_i - C_{v_i} \right\|^2. \quad (11)$$

Thus, the payoff function $F_k^i(x)$ is given by

$$\begin{aligned} F_k^i(x) &= \frac{\partial H}{\partial x_k^i}(x) \\ &= \frac{\partial}{\partial x_k^i}\left(-\sum_{j=1}^{n} M_{v_j} \left\| \sum_{l=1}^{m} x_l^j v_l - C_{v_j} \right\|^2\right) \\ &= -2M_{v_i}\left\{ v_{k_1}\left(\sum_{l=1}^{m} x_l^i v_{l_1} - C_{v_{i1}}\right) + v_{k_2}\left(\sum_{l=1}^{m} x_l^i v_{l_2} - C_{v_{i2}}\right)\right\}. \end{aligned} \quad (12)$$

We introduce replicator dynamics to search the local maximizers of the potential function. Suppose that the increase rate of agents with strategy $k \in S$ is proportional to the difference between the payoff F_k^i and an average payoff of population i. Then, replicator dynamics is given as follows [9]:

$$\dot{x}_k^i = x_k^i\left(F_k^i(x) - \sum_{l=1}^{m} x_l^i F_l^i(x)\right). \quad (13)$$

By substituting the payoff function (12) for Eq. (13), we have

$$\begin{aligned} \dot{x}_k^i = -2M_{v_i} x_k^i &\left[\left\{ v_{k_1}\left(\sum_{l=1}^{m} x_l^i v_{l_1} - C_{v_{i1}}\right) + v_{k_2}\left(\sum_{l=1}^{m} x_l^i v_{l_2} - C_{v_{i2}}\right)\right\}\right. \\ &\left. - \sum_{l=1}^{m} x_l^i \left\{ v_{l_1}\left(\sum_{l=1}^{m} x_l^i v_{l_1} - C_{v_{i1}}\right) + v_{l_2}\left(\sum_{l=1}^{m} x_l^i v_{l_2} - C_{v_{i2}}\right)\right\}\right]. \end{aligned} \quad (14)$$

The stable equilibrium points of replicator dynamics (14) are local maximizers of the potential function (11). Thus, we can search the optimal position of each sensor to maximize the objective function using Eq. (13).

5. Simulation

We consider that 5 sensors cover a square mission space $Q = [0,\ 100] \times [0,\ 100]$ with the following value function $\phi(q)$:

$$\phi(q) = \exp(-((q_1 - 80)^2 + (q_2 - 70)^2)/A), \quad (15)$$

where A is a parameter. Note that $\phi(q)$ is a one-hump function whose top is (80, 70). The smaller A is, the steeper its top is. As A goes to the infinity, ϕ converges to $\phi(q) = 1$. Shown in Figs. 2(a) and 2(b) are stable equilibrium points of the replicator dynamics when $\phi(q) = 1$. For $A = 12000$, these equilibrium points are moved as shown in Figs. 2(c) and 2(d). For $A = 10000$, however, the two equilibrium points coincide and we observe one Voronoi partition as shown in Fig. 2(e). In other words, a pitchfork bifurcation occurs. For any A less than 10000, we observe a unique stable equilibrium point which is a globally optimal sensor location. Shown in Fig. 2(f) is the stable equilibrium point for $A = 4000$. Moreover, shown in Fig. 3 is a bifurcation diagram for the two equilibrium points where the vertical and the horizontal axes are q_1 and A, respectively. As A is decreased, the two equilibrium points are closer and finally coincide. Shown in Fig. 4 is the relation between the parameter A and values of the objective function at the equilibrium points. As A is decreased, the value function is steeper and the sensors move to its top to make the value of the objective function higher. Therefore, the value of the objective function increases. From this simulation, it is shown that, if the value function is enough steep, the replicator dynamics has the unique stable equilibrium point which corresponds to the optimal sensor location. Such a bifurcation phenomenon is useful to obtain a globally optimal sensor location for a specified parameter A^*. First, we set A to be small enough to have the unique stable equilibrium point and obtain it using the replicator dynamics. Next, we increase A and set several initial points around it. If their trajectories converge to different equilibrium points, we select one which optimizes the objective function. Then, we repeat this procedure until A becomes the specified value A^*, for which we obtain the optimal sensor location. It is future work to evolve this idea.

6. Conclusion

We discussed an application of a potential game to a sensor coverage problem using a barycentric coordinate. We showed that the optimal position of each sensor to maximize the objective function can be obtained by replicator dynamics. It is our future work to investigate global bifurcation properties of sensor positions obtained by the replicator dynamics. It is also our future work to discuss improvements of our approach to search for the global optimal sensor location.

Acknowledgments

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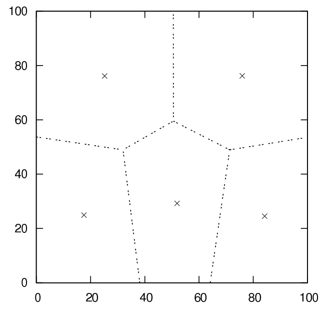
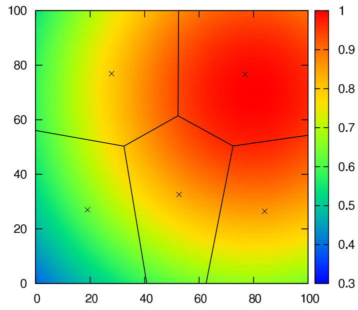
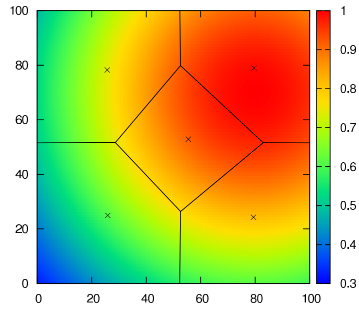


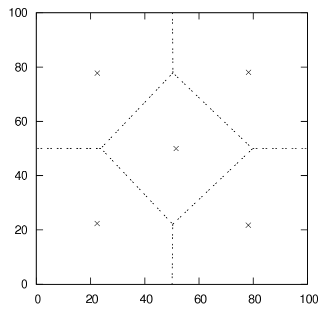
(a) $A = \infty$.



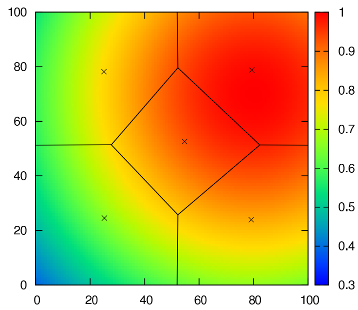
(c) $A = 12000$.



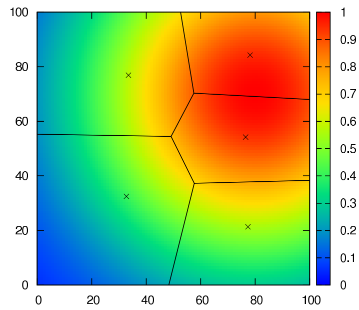
(e) $A = 10000$.



(b) $A = \infty$.



(d) $A = 12000$.



(f) $A = 4000$.

Figure 2: Stable equilibrium points: For $A = \infty$ (resp. 12000), there are two equilibrium points (a) and (b) (resp. (c) and (d)) and, for $A = 10000$ (resp. 4000), there is a unique one (e) (resp. (f)).

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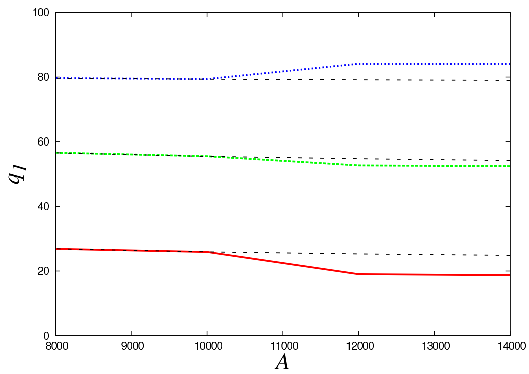

Figure 3: Bifurcation diagram.

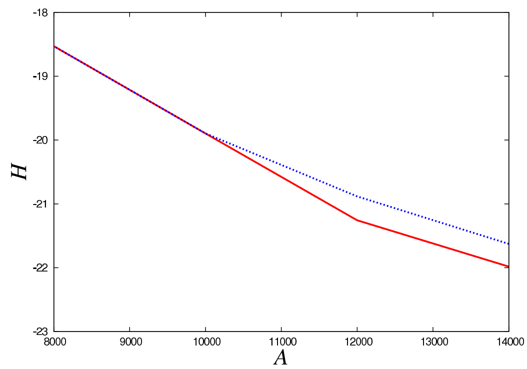

Figure 4: Relation between the parameter A and the objective function H.