# $M$-ary Modulation Scheme Using Separation of Chaotic Dynamics for Noncoherent Chaos-Based Communications

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**Abstract**—This paper proposes a new $M$-ary modulation scheme for noncoherent chaos-based communications. A chaotic sequence is successive based on the chaotic dynamics. Without the successive sequence based on the chaotic dynamics, general noncoherent systems are very difficult to demodulate the data. However, we consider that the systematic separation and the reconstruction of the chaotic dynamics can be applied as additional information. Namely, $M$-ary data symbols can be expressed by separating the chaotic dynamics purposely. We carry out computer simulations of the proposed scheme and confirm that a saving of over 3–4 dB is realized in transmitter energy by increasing $M$.

## 1. Introduction

Chaotic sequences obtained from a certain class of difference equations are nonperiodic and sensitive to initial conditions, and it is difficult to predict their future behavior from past observations. Many researchers have focused on these characteristics of chaos and have developed various digital communication systems using chaos [1]– [6]. Especially, demodulation techniques using incoming signals only or chaotic systems are called noncoherent detection techniques.

In recent years, chaos has also attracted attention in the field of intelligent transportation systems (ITS), as a UWB radar for ranging systems and a traffic flow forecasting [7]–[9]. For instance, Dake, et al. focused on the chaotic dynamics which means the specific rules for generating the chaotic sequence and proposed the UWB chaotic radar for removing a noise which is called a clutter in the radar [8]. As results, they confirmed that the performance of their proposed clutter reduction method is better than that of the conventional method. Furthermore, in our previous research, we proposed the error-correcting method using the chaotic dynamics [11]. As results, we observed the better performance of the proposed error-correcting method. Thus, we consider that the utilization of characteristics of chaos is important to improve the performance of chaos-based communication systems and ITS applications.

In this study, we focus on the chaotic dynamics again and propose a new $M$-ary modulation scheme for noncoherent chaos-based communication systems. The $M$-ary modulation scheme can transmit $b$ coded information bits at a time by using $M = 2^b$ distinct waveforms, i.e., one waveform for each of the $2^b$ expresses $b$-bit sequences [10]. The chaotic signal samples are successive based on the chaotic dynamics. General noncoherent systems have performed modulation and demodulation using the chaotic sequence generated according to the chaotic dynamics. Especially, in the case of the optimal receiver and the suboptimal receiver [2] [3] [12], to demodulate the information without the successive sequence based on the chaotic dynamics is very difficult. We consider that the $M$-ary information can be expressed by separating the chaotic dynamics of the successive sequence purposely. A transmitter separates the sequence based on the chaotic dynamics according to a specific rule, and a receiver reconstructs the original chaotic sequence. Namely, the separation and reconstruction of the chaotic dynamics can be applied for noncoherent chaos-based communication systems as additional information. In this paper, we describe a concept and a operation of the proposed $M$-ary modulation scheme. Moreover, we carry out computer simulations and evaluate the performance of the proposed scheme.

## 2. Concept of Proposed $M$-ary Modulation Scheme

In this section, we explain a concept of the proposed $M$-ary modulation scheme. Our proposed $M$-ary modulation scheme applies the chaotic dynamics and its separation. Figure 1(a) shows a general noncoherent detection method, such as the optimal receiver and the suboptimal receiver. We take the optimal receiver for instance. The optimal receiver calculates the probability density function (PDF) between the chaotic sequence which modulated the data using chaos and the chaotic maps used on the transmitting side and outputs the data by choosing the larger probability.

Here, let us consider the sequence which rearranged the order of the chaotic sequence at random, as shown in Fig. 1(b). In this figure, each signal sample is not successive depending on its anteroposterior samples. In this case, since the chaotic dynamics of this sequence was lost, the detection of the data is impossible even if it is the sequence without the noise. Namely, for detecting the data using the optimal receiver, the successive chaotic sequence based on
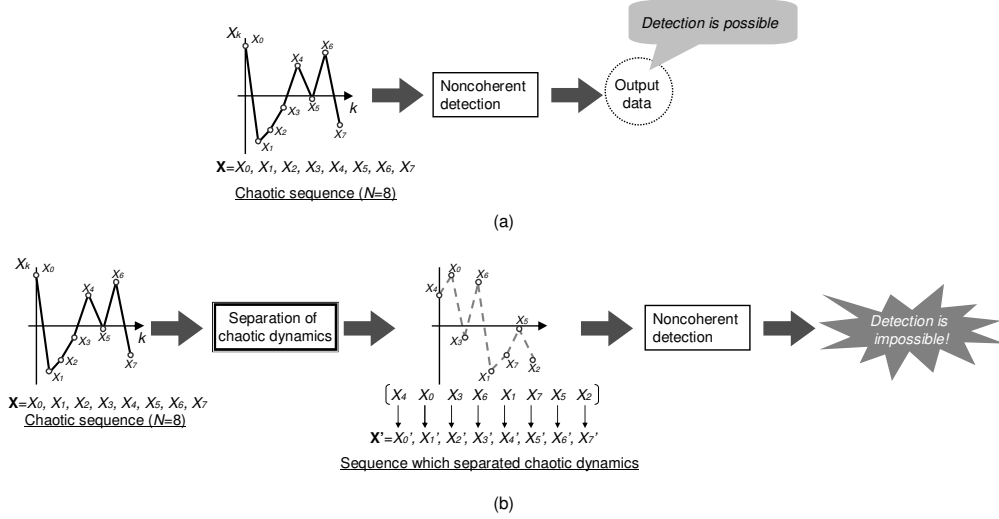
Figure 1: Concept of proposed $M$-ary modulation scheme.

the chaotic dynamics is essential.

However, we consider that the loss of the chaotic dynamics can be applied as additional information for noncoherent chaos-based communications, i.e., it is $M$-ary modulation scheme using the separation of the chaotic dynamics which is proposed in this paper. In advance, we provide $M$ interleavers which separate the chaotic dynamics, and $M$ deinterleavers which reconstructs the original chaotic sequence. Since the arbitrary symbol is assigned to each pair of the interleaver and the deinterleaver from $M$-ary symbols, it is possible to design the $M$-ary modulation scheme.

These are the concept of the proposed $M$-ary modulation scheme. In the next section, we describe the operation of the proposed $M$-ary modulation scheme in detail.

### 3. Operation of Proposed $M$-ary Modulation Scheme

Figure 2 shows a block diagram of the proposed $M$-ary modulation scheme. This system consists of a transmitter, a channel and a receiver. In this scheme, the transmitter and the receiver have $M$ interleavers and $M$ deinterleavers, respectively, for separating and reconstructing the chaotic dynamics. In advance, the arbitrary symbol is assigned to each pair of these from $M$-ary symbols. The operations of modulation and demodulation are described in detail below.

#### 3.1. Transmitter

In the transmitter, a chaotic sequence is generated by a chaotic map. In this study, the transmitter uses a skew tent map, a simple chaotic map, which is described by Eq. (1),

$$
x_{k+1} = \begin{cases} \dfrac{2x_k + 1 - a}{1 + a} & (-1 \le x_k \le a) \\[2ex] \dfrac{-2x_k + 1 + a}{1 - a} & (a < x_k \le 1) \,, \end{cases} \tag{1}
$$

where $a$ denotes the position of the peak of the skew tent map, $k$ denotes the number of iterations. To perform the proposed $M$-ary modulation, the chaotic dynamics of the

chaotic sequence is separated by the $m$-th interleaver according to the input data $m$ $(m : 0, 1, \cdots, M-1)$. To transmit each input data $m$, $N$ chaotic signal samples are generated. Therefore, the transmitted signal block is denoted by a vector $\mathbf{S}$, which is described by Eq. (2),

$$
\begin{aligned}
\mathbf{S} &= x_{0,m}, x_{1,m}, \cdots, x_{k,m}, \cdots, x_{N-1,m} \\
&= S_0, S_1, \cdots, S_k, \cdots, S_{N-1} \,,
\end{aligned} \tag{2}
$$

where $x_{k,m}$ means the separated signal samples by the $m$-th interleaver.

The transmitted signal arrives at the receiver through the AWGN channel. The AWGN channel is the most commonly used basic channel model. Here, the noise signal is denoted by the noise vector $\mathbf{n} = (n_0,\ n_1,\ \cdots,\ n_{N-1})$. Thus, the received signal block is given by $\mathbf{R} = (R_0,\ R_1,\ \cdots, R_k,\ \cdots,\ R_{N-1}) = \mathbf{S} + \mathbf{n}$.

#### 3.2. Receiver

The receiver recovers the transmitted signal from the received signal and demodulates the information data. Since we consider a noncoherent detection system, the initial value is not known on the receiving side. However, the receiver has the chaotic map used for the modulation at the transmitter in its memory.

We describe the operation of the proposed $M$-ary demodulation. First, the receiver feeds the received signal into $M$ deinterleavers and rearranges the order of the sequence. Since the interleaver and the deinterleaver corresponding to each data symbol are only 1 pair between the transmitter and the receiver, the only one of $M$ interleavers can reconstruct the original chaotic sequence. The received signal rearranged by $m$-th deinterleaver is denoted by $\mathbf{R_m} = (R_{0,m},\ R_{1,m},\ \cdots,\ R_{k,m},\ \cdots,\ R_{N-1,m})$,

Next, the receiver measures the chaotic dynamics using the rearranged received signal. In this study, we calculate shortest distances between the rearranged received signal and the chaotic map for the measurement of the chaotic dynamics. The calculation method is based on our suboptimal receiver in our previous study [12].
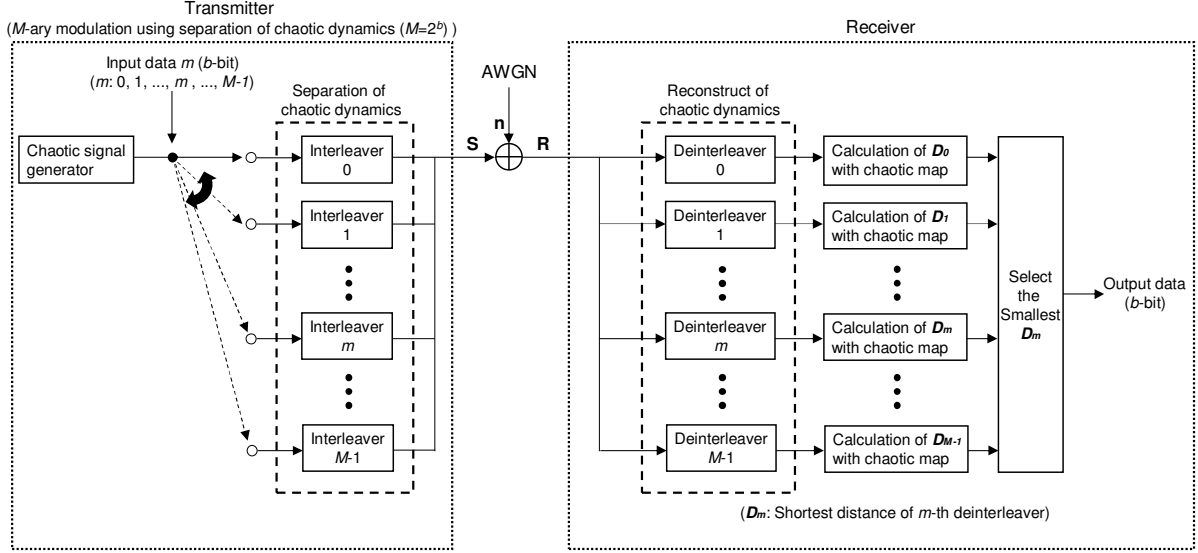
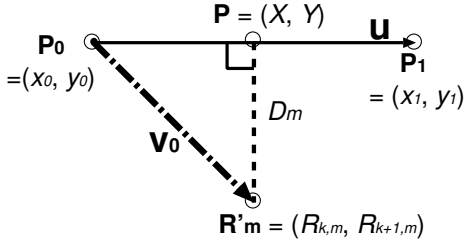Figure 2: Block diagram of proposed $M$-ary modulation scheme using separation of chaotic dynamics.



Figure 3: Calculation of shortest distance.

The receiver calculates the shortest distance between the received signal and the maps in the $N_d$-dimensional space using $N_d$ successive received signal samples ($N_d$ : $2, 3, \cdots$). As an example, we explain the case of $N_d = 2$. In this case, we consider two successive signal samples $\mathbf{R'_m} = (R_{k,m}, R_{k+1,m})$ as coordinate of chaotic map. To decide which map is closer to the point $\mathbf{R'_m}$ the shortest distance between the point and the map has to be calculated. Therefore, the receiver can calculate the shortest distance using the scalar product of the vector.

Any two points $\mathbf{P_0} = (x_0, y_0)$ and $\mathbf{P_1} = (x_1, y_1)$ are chosen from each straight line in the $N_d$-dimensional space, as shown in Fig. 3. Using Fig. 3, the receiver can calculate the point $\mathbf{P} = (X, Y)$ closest to $\mathbf{R}$ and the shortest distance of $m$-th deinterleaver $D_m$ using the following equations.

$$\mathbf{P} = (X, Y) = (\mathbf{u} \cdot \mathbf{v_0}) \mathbf{u} + \mathbf{P_0} \tag{3}$$

$$
\begin{aligned}
D_m &= ||\mathbf{P} - \mathbf{R'_m}|| \\
&= \sqrt{(X - R_{k,m})^2 + (Y - R_{k+1,m})^2}
\end{aligned} \tag{4}
$$

where

$$\text{unit vector } \mathbf{u} = \frac{\mathbf{P_1} - \mathbf{P_0}}{||\mathbf{P_1} - \mathbf{P_0}||} \tag{5}$$

$$\mathbf{v_0} = \mathbf{R'_m} - \mathbf{P_0} . \tag{6}$$

In the case of 2-dimensional space, there are two straight lines in the space. Therefore, the minimum value from two distances is chosen as $D_m$. The receiver performs these operations until the last sample (i.e., $R_{N-1,m}$) is included, and calculates the summation $\sum D_m$.

Finally, by comparing each $\sum D_m$, the receiver selects the smallest one. Therefore, the decoded data which is assigned $m$-th interleaver and deinterleaver can be outputted.

## 4. Simulation Results and Discussion

To evaluate the proposed $M$-ary modulation scheme, we carry out computer simulations under the following conditions. On the transmitting side, 100,000 bits are transmitted with different initial values. We assume $N = 32$ and $64$, and the parameter of the skew tent map is fixed as $a = 0.05$. In addition, we assume $M = 4, 8, 16, 32$ and $64$, i.e., $b = 2, 3, 4, 5$ and $6$. On the receiving side, we use 4-dimensional space ($N_d = 4$) for calculation of $D_m$. Based on these conditions, we change the number of $M$ and calculate the average of the bit error rate (BER) for each $M$-ary.

Figures 4(a) and (b) show the BER versus $E_b/N_0$ (SNR per bit) for the proposed method with $N = 32$ and $64$, respectively. Also, Figure 4(c) shows the BER performance with the same SNR per bit. To observe the effectiveness of the proposed method, Figs. 4(a)–(c) also show the performance of the suboptimal receiver as the binary system.

From Fig. 4(a) and (b), we can observe that the proposed scheme can reduce the SNR per bit by increasing the number of $M$. To achieve BER = $10^{-4}$, the required SNRs per bit of $N = 32$ and $64$ are about 18 dB for the binary system, but if $M$ is increased to 64 ($b$ =6 bits), the required SNRs per bit of $N = 32$ and $64$ are about 15 dB and 14 dB, respectively. Namely, a saving of over 3–4 dB is realized in transmitter energy required to achieve BER= $10^{-4}$ by increasing to $M = 64$.

Next, we compare the BER performances of the proposed scheme and the binary system with same SNR per bit. As shown in Fig. 4(c), the BER performance of the proposed scheme shows gain over the binary system. About 2dB gain in error performance is obtained by using the propose scheme. We consider that the separation of the chaotic

dynamics by the interleaver works to the reconstruction of the original chaotic sequence effectively. Therefore, it can be said that the proposed method is very effective as the $M$-ary modulation scheme. However, when the SNR decreases from 10 dB, the BER of the binary system is slightly better than that of the proposed scheme. As the reason for this, we consider that the successive sequence based on the chaotic dynamics decrease by the influence of noise. Also, the selection for deciding the data might become difficult by increasing $M$. For solving this problem, we would redesign the interleaver and the deinterleaver. In this study, the separation of the chaotic dynamics by the interleaver is performed at random. In other words, we have to optimize a parameter which decides the separation of the chaotic dynamics. Thus, optimizing a parameter of the proposed scheme is our future work.

## 5. Conclusions

In this study, we have proposed the $M$-ary modulation scheme using the separation of the chaotic dynamics. As results of computer simulations, a saving of over 3–4 dB is realized in transmitter energy required to achieve BER= $10^{-4}$ by increasing to $M = 64$. Therefore, we have confirmed that the separation and the reconstruction of the chaotic dynamics are very effective for the noncoherent chaos-based communications. As a future work, we will develop the ITS application based on our proposed method.

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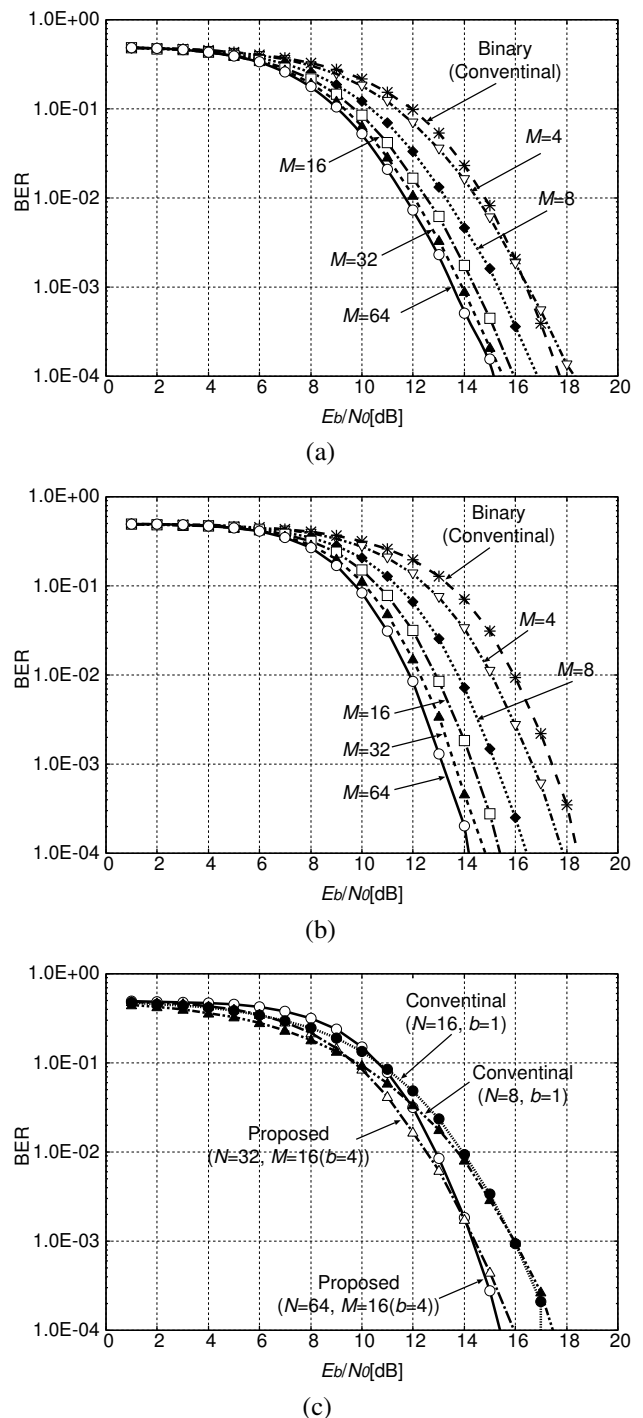
(a)



(b)



(c)

Figure 4: BER performances: (a) $N = 32$, (b) $N = 64$, (c) Comparison with same SNR per bit.