Transcript-based coupling directionality indicators and applications

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Abstract—We compare the information transfer measured by the symbolic entropy transfer (STE) and a new directionality indicator based on the mutual information of transcripts (TMI), using brain recording data, specifically local field potentials of the visual cortex and thalamus during spontaneous activity. The comparison shows the adequacy of TMI, the interesting point being that TMI has one dimension less than STE.

1. Introduction

The concept of transcript can be defined in any symbolic representation of time series whose symbols build an algebraic group. In particular, the symbolic representation in ordinal time series analysis is implemented with permutations of L elements (called ordinal patterns of length L), the algebraic representation group thus being the symmetric group of order L. Based on ordinal patterns and their corresponding transcripts, one can then define some information directionality indicators via conditional mutual informations with a varying number of conditioning variables. These indicators have the advantage of containing one conditioning variable less than their standard counterparts, e.g., symbolic transfer entropy and momentary sorting information transfer. This dimensional reduction can make a difference in time series analysis of real world data just because they are usually short in supply.

Transcripts were introduced in [6] for characterizing the synchronization of two coupled, chaotic oscillators. In [1] they were used to define two complexity indices for coupled time series. Their basic properties were studied in [7] and [4]. The dimensional reduction of conditional mutual informations was proved in [8] (see below) and generalized in [4]. These results were communicated in NOLTA 2012 [2] and NOLTA 2013 [3] as they were obtained.

The present communication is a follow-up of [2] and [3]. Its scope is to discuss the application of transcript-based mutual information as an information directionality indicator to real time series. Specifically, the data are local field potentials measured at the visual cortex and thalamus during spontaneous activity.

2. Ordinal patterns and transcripts

Suppose that $\{x_t\}_{t=t_0}^{\infty}$ is a sequence whose elements (entries, symbols,...) x_t belong to a set (state space, alphabet,...) endowed with a total ordering '$<$'. In practice $\{x_t\}$ is obtained by sampling an analog signal. Let $T \geq 1$ be a *delay time*. We say that a length-L, time delay block (vector, window,...) $\mathbf{v}_{T,L}(x_t) = (x_t, x_{t+T}, ..., x_{t+(L-1)T})$ defines the *ordinal* (L-)*pattern* $\pi = \langle \pi_0, ..., \pi_{L-1} \rangle$ if

$$x_{t+\pi_0 T} < x_{t+\pi_1 T} < ... < x_{t+\pi_{L-1} T}, \qquad (1)$$

where in case $x_i = x_j$, we agree to set $x_i < x_j$ if, say, $i < j$. In nonlinear time series analysis, L is called the *embedding dimension*.

Alternatively we also say that the block $\mathbf{v}_{T,L}(x_t)$ is of type π, or that π is realized by $\mathbf{v}_{T,L}(x_t)$, and write $\pi = o(\mathbf{v}_{T,L}(x_t))$. Therefore, an ordinal L-pattern (or ordinal patterns of length L) is nothing else but a permutation of the integer numbers 0, 1,..., $L-1$ showing the ranking (according to their size) of the elements $x_t, x_{t+T}, ..., x_{t+(L-1)T}$, where t is arbitrary and $L \geq 2$. Specifically, $\pi = \langle \pi_0, ..., \pi_{L-1} \rangle$ may be identified with the permutation $i \mapsto \pi_i$, $0 \leq i \leq L-1$.

The set of ordinal L-patterns will be denoted by $\mathcal{S}_L$. This set can be promoted to a group of order $L!$, called the *symmetric group of degree* L, if equipped with the product of permutations,

$$\begin{aligned} \pi\sigma &= \langle \pi_0, ..., \pi_{L-1} \rangle \langle \sigma_0, ..., \sigma_{L-1} \rangle \\ &= \langle \sigma_{\pi_0}, ..., \sigma_{\pi_{L-1}} \rangle \end{aligned} \qquad (2)$$

the inverse element being given by

$$\pi^{-1} = o(\pi_0, ..., \pi_{L-1}), \qquad (3)$$

and the unity by the identity permutation,

$$id = \langle 0, 1, ..., L-1 \rangle . \qquad (4)$$

One way of exploiting the group-theoretical structure (2)-(4) of the ordinal patterns is the following. Given $\alpha, \beta \in \mathcal{S}_L$ there always exists a unique $\tau = \tau(\alpha, \beta) \in \mathcal{S}_L$, called *transcript* from the *source pattern* α to the *target pattern* β, such that

$$\tau\alpha = \beta, \tag{5}$$

where, according to (2), $\tau\alpha = \left\langle \alpha_{\tau_0}, \alpha_{\tau_1}, ..., \alpha_{\tau_{L-1}} \right\rangle$. When the source and target patterns are important for the discussion, we generally write $\tau_{\alpha,\beta}$. It follows from (5) that $\tau_{\beta,\alpha} = (\tau_{\alpha,\beta})^{-1}$.

As the source pattern α and the target pattern β vary over $\mathcal{S}_L$, their transcript varies according to $\tau(\alpha, \beta) = \beta \circ \alpha^{-1}$. Note that given $\tau \in \mathcal{S}_L$ there exist $L!$ pairs $(\alpha, \beta) \in \mathcal{S}_L \times \mathcal{S}_L$ such that τ is the transcript from α to β.

All this can be generalized to $N \geq 2$ coupled time series. In this case we write α_1,..., α_N for the random ordinal L-patterns obtained from the time series $\{x_t^1\}$, ..., $\{x_t^N\}$, respectively, and $\tau_{1,2}$, ..., $\tau_{N-1,N}$ for the corresponding transcripts τ_{α_1,α_2},..., $\tau_{\alpha_{N-1},\alpha_N}$. The *coupling complexity index* among the random ordinal L-patterns α_1,..., α_N, denoted by $C(\alpha_1, ..., \alpha_N)$, is defined as [1, 7]

$$\begin{aligned} C(\alpha_1, ..., \alpha_N) &= \min_{1 \leq n \leq N} H(\alpha_n) - H(\alpha_1, ..., \alpha_N) \\ &\quad + H(\tau_{1,2}, ..., \tau_{N-1,N}), \end{aligned} \tag{6}$$

where $H(X_1, ..., X_n)$ is the (Shannon) entropy of the random variables X_1, ..., X_n. It can be proved [7, 4] that

$$C(\alpha_1, ..., \alpha_N) = \min_{1 \leq n \leq N} I(\alpha_n; \tau_{1,2}, ..., \tau_{N-1,N}), \tag{7}$$

where

$$I(X_1; X_2) = H(X_1) + H(X_2) - H(X_1, X_2) \tag{8}$$

is the mutual information between the random variables X_1 and X_2. Note that $C(\alpha_1, ..., \alpha_N) \geq 0$, and that it depends on L. It can be proved that $C(\alpha_1, ..., \alpha_N)$ is invariant under permutations of its arguments. For this and other properties of the coupling complexity index, see [7, 4].

3. Symbolic transfer entropy and transcript mutual information

The following theorem was proved in [8].

Theorem 1. Let $\alpha_1, \alpha_2, \beta$ be random ordinal L-patterns. If (i)

$$\min\{H(\alpha_1), H(\alpha_2)\} \geq H(\beta) \tag{9}$$

and (ii)

$$C(\alpha_1, \alpha_2, \beta) = 0, \tag{10}$$

then

$$I(\alpha_1; \alpha_2 \,|\beta) = I(\tau_{\alpha_1,\beta}; \tau_{\alpha_2,\beta}). \tag{11}$$

Note that (11) equates a conditional mutual information (with three variables and $(L!)^3$ possible values) to an unconditioned mutual information (with two variables and $(L!)^2$ possible values) thanks to the use of transcripts. This dimensional reduction can make a difference in symbolic time series analysis if the data sequences are short, as often happens in practice. The Eq. (11) was generalized to multi-information functions (Studeny 1999) conditioned on an arbitrary number of random ordinal patterns in [4].

The symbolic transfer entropy is a conditional mutual information of the form considered in Theorem 1. Perhaps for its simplicity, symbolic transfer entropy is one of the most popular information directionality indices used with ordinal symbolic representations. Let ξ and η be two $\mathcal{S}_L$-valued random variables obtained from two coupled time series $\{x_t\}_{t=t_0}^{\infty}$ and $\{y_t\}_{t=t_0}^{\infty}$, respectively, and let ξ_Λ the $\mathcal{S}_L$-valued random variable obtained from $\{x_{t+\Lambda}\}_{t=t_0}^{\infty}$, where $\Lambda \geq 1$. The *symbolic transfer entropy from System 2 to System 1* is defined [10] as

$$T_{2\to1}^S := I(\xi_\Lambda; \eta \,|\xi). \tag{12}$$

$T_{2\to1}^S > 0$ for some Λ if System 2 drives System 1. Replace α_1 by ξ_Λ, α_2 by η, and β by ξ in Theorems 1 and 2 to obtain the following result.

Corollary 1. [8] If $H(\xi) \leq H(\eta)$ and

$$C(\xi_\Lambda, \eta, \xi) = 0 \tag{13}$$

then

$$T_{2\to1}^S = I(\tau_{\xi_\Lambda,\xi}; \tau_{\eta,\xi}). \tag{14}$$

Analogously, the *symbolic transfer entropy from System 1 to System 2,* is defined as

$$T_{1\to2}^S = I(\eta_\Lambda; \xi \,|\eta).$$

In this case, $T_{1\to2}^S > 0$ for some Λ if System 1 drives System 2. Theorems 1 and 2 yield this time the following result.

Corollary 2. [8] If $H(\xi) \geq H(\eta)$ and

$$C(\eta_\Lambda, \xi, \eta) = 0 \tag{15}$$

then

$$T_{1\to2}^S = I(\tau_{\eta_\Lambda,\eta}; \tau_{\xi,\eta}). \tag{16}$$

To detect the net information flow between the Systems 1 and 2 one can use, e.g., the difference

$$\Delta STE\ (2 \to 1) = T_{2\to1}^S - T_{1\to2}^S \tag{17}$$

so that $\Delta T_{2\to1}^S > 0$ indicates a net information transfer from System 2 to System 1, weil $\Delta T_{2\to1}^S < 0$ indicates the contrary.

In sum, Corollaries 1 and 2 spell out that the TMI

$$\begin{array}{ll} I(\tau_{\xi_\Lambda,\xi};\tau_{\eta,\xi}) & \text{if } H(\xi)\leq H(\eta) \\ I(\tau_{\eta_\Lambda,\eta};\tau_{\xi,\eta}) & \text{if } H(\xi)\geq H(\eta) \end{array} \quad (18)$$

may be used as information directionality indicators if the condition (13) in the first case, and the condition (15) in the second case, is satisfied (at least, approximately). In particular, if $H(\xi)\simeq H(\eta)$ holds then one can use, analogously to (17),

$$\Delta TMI(2\to 1) := I(\tau_{\xi_\Lambda,\xi};\tau_{\eta,\xi}) - I(\tau_{\eta_\Lambda,\eta};\tau_{\xi,\eta}), \quad (19)$$

or

$$\frac{I(\tau_{\xi_\Lambda,\xi};\tau_{\eta,\xi}) - I(\tau_{\eta_\Lambda,\eta};\tau_{\xi,\eta})}{I(\tau_{\xi_\Lambda,\xi};\tau_{\eta,\xi}) + I(\tau_{\eta_\Lambda,\eta};\tau_{\xi,\eta})} \quad (20)$$

under the corresponding provisos on coupling complexity coefficients. One expects $H(\xi)\simeq H(\eta)$ at least if the coupling is strong enough, or if both systems have a similar entropy and their coupling is weak. As commented before, the lesser dimensionality of TMI as compared to STE might prevent undersampling in short data set and improves the statistical significance of the results in any case.

Numerical simulations show [8, 4] that the conditions on the coupling complexity indices can be generally achieved by taking the delay time T large enough. This being the case, the question arises whether TMI, in the appropriate form according to the entropy condition, may be used as information directionality indicator even if the pertinent condition on the coupling complexity coefficient(s) does not hold. This question was numerically tackled in [8], and the answer was positive. We address the same question with real data in the next section. Specifically, we study the local potentials of the cortex and thalamus during spontaneous activity. See [5] for an interesting application of transfer entropy to brain waves.

4. Applications to biomedical time series

In this section we apply the measurement of information flow directionality to data obtained during spontaneous oscillatory activity in the recurrently connected thalamocortical loop. Such recordings consist of simultaneous local field potentials obtained from the visual thalamus and different layers of the visual cortex. Thalamus and cortical networks can be considered as two independent oscillators reciprocally connected. There is a debate regarding the role of the thalamic activity on the initiation of slow oscillatory waves, a problem that we will approached with the use of information directionality indicators. Furthermore, laminar recordings should allow us to explore the dominant information flow during the different stages of cortical rhythmic activity and to compare these results to the underlying network connectivity. The data were obtained as in [9]. The time series are 3,968,400 points long, corresponding to 400 sec of recording at a sampling frequency of 9921 Hz.

As for the parameter chosen to compute the symbolic transfer entropy and the transcript mutual information, $L = 4$ and $T = 1050/9921 = 0.106$ sec (about 100 times the sampling time). All directionality indicators are given in units of bit (i.e., logarithms are taken to base 2).

To illustrate the results obtained, let System 1 (variable ξ) be the visual thalamus and let System 2 (variable η) consists of the infragranular layer V of the visual cortex. The entropy of these two systems is $H(\xi) = 4.27 \pm 0.11$ and $H(\eta) = 4.41 \pm 0.08$, therefore we may use the transcript-based indicator $\Delta TMI(2\to 1)$, Eq. (19), to measure the net information transfer from System 2 to System 1. Table I below lists the average over 5 registers of $\Delta TMI(2\to 1)$ for $\Lambda = 0.02, 0.04, 0, 06, 0.08, 0.10$ sec, i.e., for time shifts smaller than the delay time $T \simeq 0.10$ sec. As benchmark we take the symbolic directionality indicator $\Delta STE(2\to 1)$, Eq. (17) with the same time shifts. Table I lists as well the corresponding average of $\Delta STE(2\to 1)$ for comparison. The error margins are the standard deviation over 5 subjects.

Λ	$\Delta TMI(2\to 1)$	$\Delta STE(2\to 1)$
0.02	0.14 ± 0.02	0.07 ± 0.01
0.04	0.17 ± 0.02	0.06 ± 0.02
0.06	0.16 ± 0.03	0.04 ± 0.03
0.08	0.15 ± 0.04	0.02 ± 0.02
0.10	0.14 ± 0.05	0.02 ± 0.03

Table I

Comparison of $\Delta TMI(2\to 1)$ with $\Delta STE(2\to 1)$ in Table I shows a satisfactory coincidence of both indicators in the envisaged case of coupled systems with similar entropies. Note that the information direction has to do with the sign of the indicators, not with their magnitude. Bearing this in mind, the neurological implication from Table I with either indicator is that the layer V of the visual cortex leads the thalamus during spontaneous activity.

A similar coincidence was found with data from other layers of the visual cortex. This confirms the suitability of the new indicator (19) and (20) with real data as well.

Acknowledgments

J.M.A. was financially supported by the Spanish *Ministerio de Economía y Competitividad*, project MTM2012-31698. M.V.S.V. was financially supported by the Spanish *Ministerio de Economía y Competitividad*, project BFU2011-27094, and by the EU PF7 FET CORTICONIC, contract 600806.

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