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Solution-Space Reduction Method by Using Geographic Complex Network Model for Traveling Salesman Problem

Yusuke Kawamura†, Yutaka Shimada†, Takafumi Matsuura† and Tohru Ikeguchi‡

Graduate School of Science and Engineering, Saitama University
255 Shimo-ohkubo, Saitama, 338-8570 Japan
Email: †{kawamura,sima,takafumi}@nls.ics.saitama-u.ac.jp, ‡tohru@mail.saitama-u.ac.jp

Abstract—The traveling salesman-problem (TSP) is one of the most typical $\mathcal{NP}$-hard combinatorial optimization problems. To construct a heuristic algorithm for solving TSPs, a nearest-neighbor method is often used to reduce a solution-space because it is very rare to connect two cities which are located in the far distance. On the other hand, to analyze the characteristics of real networks many geographical complex network models have already been proposed. The networks constructed by the geographical complex network model have similar properties with the optimal tours of TSPs: the edges which connect distant two vertices and intersection of edges are inhibited. In this paper, we propose a new reduction method for the solution-space of TSP using the geographical complex network models.

1. Introduction

Finding optimal solutions of combinatorial optimization problems such as scheduling problem, routing problem, drilling problem, computer wiring and VLSI design is one of the most important issues in science and engineering. The travelling salesman problem (TSP) is one of the most famous combinatorial optimization problems. If we develop an efficient algorithm for solving the TSP, we can apply it to reduce costs, time and human resources of many optimization problems.

The TSP is formulated by the following definition. A set of N cities $V_C = \{1, \ldots, N\}$ and the distance $d_{ij}\,(i, j \in V_C)$ between two cities i and j are given. Then, a solution of the TSP is a tour described by a permutation $\sigma = (\sigma(1), \ldots, \sigma(N))$, where $\sigma(k) = i$ indicates that city i is visited at the kth order in the tour. Thus, the aim of the TSP is to find a permutation σ which minimizes the following equation:

$$f(\sigma) = \sum_{i=1}^{N-1} d_{\sigma(i)\sigma(i+1)} + d_{\sigma(N)\sigma(1)}. \tag{1}$$

In this paper, we study the symmetric TSP which satisfies $d_{ij} = d_{ji}$ for all pairs of two cities i and j. For an N-city TSP, the possible number of all tours is $(N-1)!/2$. Thus, the number of tours increases exponentially with N. The TSP belongs to a class of $\mathcal{NP}$-hard, it is believed that no algorithms can find an optimal tour in polynomial time. Therefore, many approximation algorithms have been proposed to find near optimal solutions of TSPs.

To obtain the near optimal solutions in a practical computational time by an approximation algorithm, we reduce a solution-space using an intrinsic property of an optimal-tour of TSP: the most of edges in the optimal-tour (optimal-edges) connect two cities locally. In fact, it has already shown that a distribution of an order of neighbors in optimal tours decays exponentially [1].

One of the simplest methods for reducing a solution-space is the mth-Nearest-Neighbor (mNN). In the mNN, only edges that are connected with m nearest cities could be elements of the solution-space. However, if we want to include all the edges in the optimal tour by the mNN, we have to set the value of m large, because relatively long edges often exist in the optimal tour. Thus, the mNN includes many redundant edges.

On the other hand, many studies of the complex networks have revealed a fact that many real networks share some topological characteristics such as small-world [3] and scale-free [4] structures. Many real networks embedded in a metric space, for example, road traffic networks, railroad networks and airline networks are analyzed by a geographical complex network model [5]. These networks have the following intrinsic properties that are similar to an optimal tour of the TSP: it is very rare to connect and no intersection of edges are allowed.

Then we have already proposed a reduction method for the TSP using an idea of the geographical complex network models [6, 7]. In the method, to apply the geographical complex network models to the TSP, we have also introduce an order of neighbors instead of distance. Although the proposed method reduces a solution-space in a stochastic manner, the proposed method reduces the solution-space efficiently in some cases.

In this paper, we propose an application of the geographical complex network models to the TSP in respect to reduce a solution-space. Introducing an intrinsic property of the TSP, we systematically reduce the solution-space. As a result, we can construct smaller solution-spaces than the mNN that includes more optimal-edges.

2. The proposed method

We propose a reduction method for the solution-space of the TSP based on idea of a geographical threshold graph [8], which is one of the geographical complex network models. In the geographical threshold graph, a vertex in a set of N vertices $V_G = \{v_1, \ldots, v_N\}$ is distributed in the dth Euclidean space randomly with a uniform density ρ. A vertex $v_i\,(i = 1, \ldots, N)$ whose coordinates are denoted by $(x_{i1}, \ldots, x_{id})$ has a weight $w_i \geq 0$ which follows a density function $f(w)$. The connection rule of a pair of vertices v_i and $v_j\,(v_i, v_j \in V_G)$ is defined by the following inequality:

$$(w_i + w_j)\, h(d_{ij}) \geq \theta, \qquad (2)$$

where θ is a constant threshold and $h(d_{ij})$ is a decreasing function of $d_{ij}\,(> 0)$ which is the Euclidean distance between two vertices v_i and v_j.

To apply the geographical threshold graph to the TSP, we consider a network constructed by the geographical threshold graph as a solution-space of the TSP. Thus, each city corresponds to vertex and edges describe the route of neighbor between two cities. However, we cannot apply the generation rule of the geographical threshold graph directly. In some instances of the TSPs, some cities are isolated from the others. Thus, long edges are required to connect those cities, however, from Eq. (2), the edges are hardly connected when a summation of weight is small or distance of them is long. Therefore, this restriction causes a division of the solution-space.

To solve the division problem, we introduced a new criterion based on an order of neighbors [6, 7]. We focus on the order of neighbors of city j from city i and define it as the neighbor rank $n_{ij}\,(n_{ij} = 1, \ldots, N-1)$. For example, if city j is the nearest neighbor of city i, then $n_{ij} = 1$. We define the connection rule in the proposed method by the following inequality:

$$\frac{w_i + w_j}{{n_{ij}}^{\beta}} \geq \theta, \qquad (3)$$

where β is the scaling parameter of n_{ij} and w_i is the weight of city i. Note that we consider a solution-space as a directed network because the neighbor rank n_{ij} is not equal to n_{ji}.

Furthermore, to decide a weight of each city, we introduce local density information. If the neighbor rank n_{ij} is small or summation of the weights is large, city i and city j are connected. Thus, in the proposed method, it is very important to decide the value of w_i. In Refs [6, 7], we assigned random values w_i to each city. As a result, when a large value is assigned to a city which is far from the other cities, the proposed method obtains better solutions than the mNN. To assign the large value to the isolated cities, in this paper, to decide the weight, we use the number of cities in a circle (Fig. 1). A radius d_r of the circle is defined

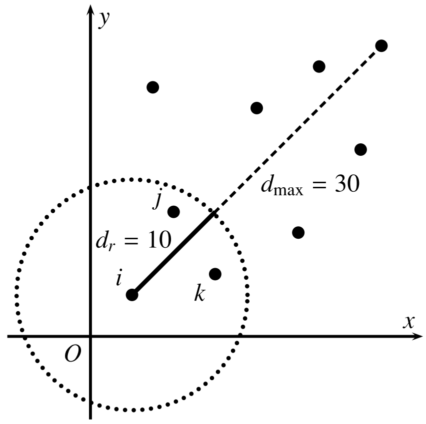

Figure 1: How to decide a weight of city i. In this example, the number of cities is $N = 9$. The length of dashed line and solid line correspond to $d_{\max} = 30$ and $d_r = d_{\max}/\sqrt{N} = 10$, respectively.

by the following equation:

$$d_r = \frac{d_{\max}}{\sqrt{N}}, \qquad (4)$$

where $d_{\max}$ is the longest distance among all pairs of cities. The weight for city i is decided by the following equation:

$$w_i = \frac{1}{c_i}, \qquad (5)$$

where c_i is the number of cities in the circle centered at city i with radius d_r. Then, a weight of each city is assigned by the following procedure.

1. The maximum length $d_{\max}$ is calculated from all pairs of cities.
2. The radius d_r is calculated by Eq. (4).
3. The number of cities c_i in a circle centered at city i is calculated.
4. The value of w_i is assigned by Eq. (5).

Figure 1 shows an example of $d_{\max}$, d_r and a circle. In Fig. 1, three cities i, j and k are included in the circle. Thus, the value of a weight of city i is $w_i = 1/3$.

Let us consider a set of N cities V_C as in Sec. 1. A solution-space is generated by the following procedure.

1. Assign a weight $w_i\,(i = 1, \ldots, N)$ to each city.
2. Calculate the distance d_{ij} for all pairs of city i and city $j\,(i, j \in V_C, i \neq j)$.
3. Calculate the neighbor rank n_{ij}.
4. Connect two cities i and j according to Eq. (3).

3. Computational experiments

To compare the performance of the proposed method with that of the mNN, we generate solution-space by the

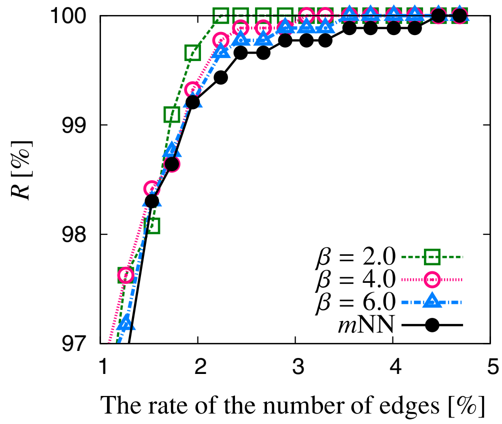

Figure 2: The rate of the number of optimal-edges included in the solution-space (R [%]). The vertical axis is R and the horizontal axis is the rate of the number of edges in the solution-space to the number of edges in complete directed graph $N(N-1)$ [%].

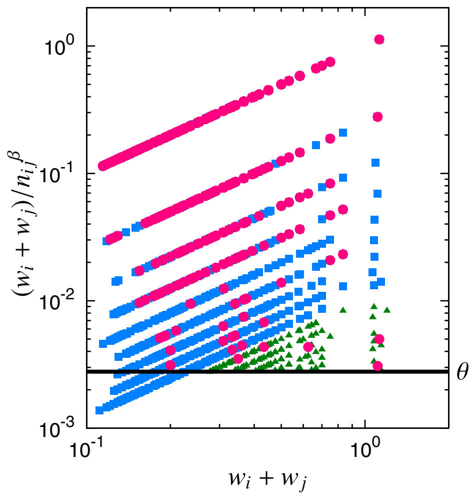

Figure 3: The relation between $w_i + w_j$ and the left-hand side of Eq. (3) (in this example, $m = 9$ and $\beta = 2.0$). Circles, squares and triangles correspond to optimal-edges, edges selected by the mNN and edges selected only by the proposed method, respectively. Solid line shows the threshold θ of the proposed method.

mNN and by the proposed method, adjusting the threshold θ in Eq. (3) to set the same size of the solution-space. Then, we compare the performance of the proposed method and the mNN.

We evaluated the performance of the proposed method and the mNN with the following measure R: the rate of the number of optimal-edges included in the solution-space. The measure R is defined by the following equation:

$$R = \frac{1}{N} \sum_{i=1}^{N} \frac{o_i}{2} \times 100 \text{ [\%]}, \quad (6)$$

where o_i ($o_i = 0, 1, 2$) is the number of optimal-edges from city i included in the reduced solution-space, and N is the number of cities. If the value of R equals to 100 [%], a solution-space includes all optimal-edges. To solve efficiently, the value of R is required to be high in a small solution-space.

We use pcb442 in the TSPLIB [2] for this simulation. The number of neighbors m for the mNN is varied from 1 to 20. The parameter β in Eq. (3) is set to 2.0, 4.0 and 6.0.

4. Results

We compare the value of R by the proposed method with that by the mNN. Figure 2 shows the relationships between the size of a solution-space and R. From Fig. 2, the proposed method includes more optimal-edges than the mNN in many cases. In particular, the proposed method with $\beta = 2.0$ includes all optimal-edges only a half size of solution-space by the size of the mNN. Figure 2 also shows that the value of R of the proposed method with $\beta = 6.0$ is less than that with $\beta = 2.0$ and 4.0. This reason is that large β strongly inhibits to connect edges from city i to city j. Therefore, the solution-space of the proposed method with large β becomes similar to that of the mNN.

To analyze the difference of the solution-space between the proposed method and the mNN, we examine the relation between the summation of weights, $w_i + w_j$, and the left-hand side of Eq. (3). Figure 3 shows the relation between the summation of weights and the left hand side of Eq. (3). In Fig. 3, each point on the mth top line corresponds to the edges whose neighbor rank is $n_{ij} = m$. For example, each point on the top line correspond to the edge which connects its nearest city. Thus, the solution-space by the mNN corresponds to squares on the top mth line, while all edges in the solution-space by the proposed method are over the threshold θ. From Fig. 3, some optimal-edges are included only in the solution-space reduced by the proposed method. This reason is that large weights are assigned to two cities connected by an optimal-edge even though its neighbor rank is large.

We investigate the optimal-edges included in the solution-space by the proposed method and the mNN. Figure 4 shows edges included in the solution-space by the proposed method and the mNN. In Figs. 4 (a) and (c), the mNN cannot include some optimal-edges which connect the isolated cities, while the proposed method includes these edges as shown in Figs. 4 (b) and (d). In the mNN, the value of m must be set to large enough to connect such edges. On the other hand, in the proposed method, although the neighbor rank is large, the isolated cities are easily connected because such cities have a large weight. Therefore, the proposed method includes all optimal-edges by a smaller solution-space than the mNN.

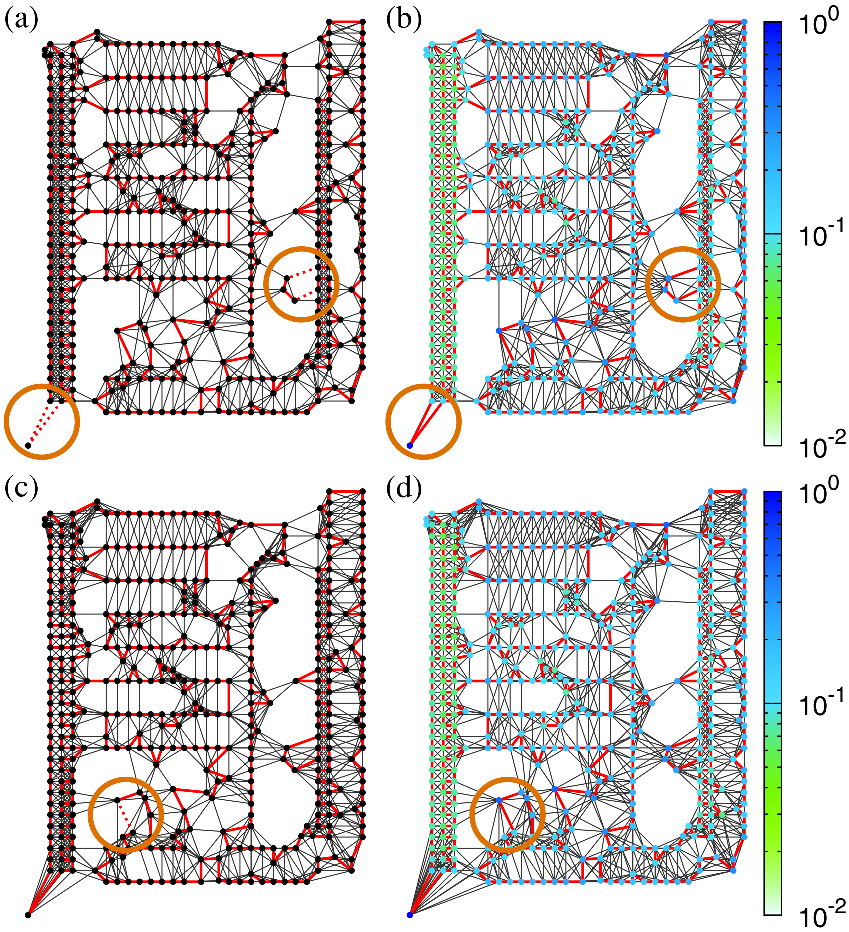

Figure 4: The solution-space obtained by the mNN and the proposed method (in this example, $m = 9$ and $\beta = 2.0$). The edges from city i to city j where $i < j$ connected by the mNN and the proposed method are shown in (a) and (b), respectively. The edges from city i to city j where $i > j$ connected by the mNN and the proposed method are shown in (c) and (d), respectively. Black edges show the solution-space. Red solid edges show the optimal-edges included in the solution-space. Red dashed edges indicated by orange circles in (a) and (c) show the optimal-edges which are not included in the solution-space by the mNN. Color bars in (b) and (d) indicate that the weight of each city by the proposed method. Orange circles in (b) and (d) indicate that the optimal-edges can be included in the proposed method.

5. Conclusion

In this paper, we propose a new reduction method for a solution-space of the TSP with an idea of the geographical threshold graph, the order of neighbors, and local information for a deterministic weight assignment. As a result, the proposed method includes optimal-edges more efficiently than the mNN.

One of the important future works is how to find the best size for local density and the parameter of the connection rule for the proposed method. It is also important to apply approximation algorithms to the reduced solution-space and evaluate its performance in the solution-space.

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