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Dynamical Noise injection to Chaotic Dynamics for Solving Combinatorial Optimization Problems

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Abstract—To solve the quadratic assignment problems (QAPs), two types of chaotic search methods have already been proposed. In one method, mutual connection chaotic neural network (CNN), Hopfield-type CNN method is used, and a firing pattern of the CNN represents a solution of the QAP. For another method, execution of local search algorithm is control by the chaotic dynamics. In both methods, chaotic dynamics works to avoid local minima. To improve performances of the these methods, we have already proposed new methods which combine chaotic dynamics and dynamical noise. As a result, when small amount of dynamical noise is added to the CNN, the solving performance is improved. However, we have not clarified yet why the small amount of dynamical noise is effective to find good solutions and how to change the searching states of the chaotic neural network by the dynamical noise. In this paper, to clarify the reason, we analyze the internal states of the neurons.

1. Introduction

In the real world, various combinatorial optimization problems exist, for example, VLSI design, scheduling problem, routing problem, facility layout problem, and so on. It is important to obtain optimal solutions of these problems because operation costs can be reduced. These kind of problems can be formulated as a quadratic assignment problem (QAP) [1]. The QAP is described as follows: when two $N \times N$ matrices, a distance matrix D and a flow matrix C are given, find a permutation $\boldsymbol{p}$ which minimizes a value of the following objective function $F(\boldsymbol{p})$:

$$F(\boldsymbol{p}) = \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} c_{p(i)p(j)}, \quad (1)$$

where d_{ij} is the (i, j)th element of D, $p(i)$ is the ith element of $\boldsymbol{p}$, $c_{p(i)p(j)}$ is the $(p(i), p(j))$th element of C, and N is a size of the problem. The QAP belongs to a class of NP-hard. Thus, it is required to develop effective approximate algorithms for finding near optimal solutions in a reasonable time frame.

As an approximate algorithm, a method which uses the mutual connection neural networks, or the Hopfield-Tank neural network (HNN), has already been proposed [2]. In this method, a firing pattern of HNN represents a solution of the QAP. If we decide good synaptic weights of HNN for solving the QAP, we can obtain a good solution by descent down-hill dynamics of HNN. However, this method cannot always get good performance because the states of the HNN get stuck at local minima.

To avoid local minima, a method which injects chaotic dynamical noise into HNN for solving combinatorial optimization problems has been proposed [10, 11]. Using fluctuation of chaotic time series as dynamical noise, the state can escape from local minima. As another method for avoiding local minima, a method which uses chaotic neural network (CNN) [3] has also been proposed [4, 5]. In the method, chaotic dynamics of CNN works to avoid the local minima effectively.

To realize more effective algorithm, we have already proposed a new method which uses both chaotic dynamics and dynamical noise for avoiding local minima [7, 8]. As a result, when the small amount of noise is added to CNN, the proposed method shows good performance. However, as the amount of noise increases, the performance of the proposed method becomes worse gradually. Namely, the small amount of noise leads to effective search.

As another approach for solving QAP, 2-opt algorithm driven chaotic dynamics has been proposed [6]. This method shows higher performance than the method which used Hopfield-type CNN. Alternatively we combined this algorithm with dynamical noise. As a resut [9], when small amount dynamical noise is added to the CNN. this method also shows good performance,

Although, the methods which use chaotic dynamics and dynamical noise shows good performance [9, 4, 5], we have not clarified yet why the small amount of dynamical noise is effective to find good solutions and how to change the searching states of the CNN by the dynamical noise. If the reasons are identified, the algorithm can be improved further. Thus, it is important to analyze influence of the dynamical noise to the chaotic dynamics. In this paper, we investigated how to change value of internal state of chaotic neurons depending on the amount of dynamical noise. From a result, temporal average and variance of the internal states of the takes almost same value when we added small amount of dynamical noise in the Hopfield-type CNN method. However, when the large amount of dynamical noise is added to the CNN, the temporal average and variance take quite difference value. On the other hand, in the method which 2-opt algorithm driven by chaotic dynamics, the temporal average and the variance of the neurons increase monotonously as the amount of noise increases.

2. Method using both chaotic dynamics and dynamical noise

2.1. Hopfield-type Chaotic Neural Network with dynamical noise

As an approximate algorithm for solving the QAP, a method which uses mutual connection chaotic neural network (CNN) [3] has already been proposed [4, 5]. The CNN model is constructed by chaotic neurons [3]. This neural network model can qualitatively reproduce a chaotic dynamics observed in real neural membrane. To solve an N-size QAP, $N \times N$ chaotic neurons are prepared in the method. The (i, m)th neuron corresponds an assignment of the ith facility and the mth city.

An internal state of the (i, m)th chaotic neuron of CNN is defined as follows:

$$\begin{aligned} y_{im}(t+1) &= k y_{im}(t) + \sum_{j=1}^{N}\sum_{n=1}^{N} w_{im;jn} f(y_{jn}(t)) \\ &\quad -\alpha f(y_{im}(t)) + \theta_{im}(1-k), \end{aligned} \tag{2}$$

where k is a decay parameter, α is a strength parameter of a refractory effect, and f is an output function. Synaptic weights between the (i, m)th neuron and the (j, n)th neuron $w_{im;jn}$ and thresholds of the (i, m)th neuron θ_{im} are defined as follows:

$$\begin{aligned} w_{im;jn} &= -2\{A(1-\delta_{mn})\delta_{ij} + B\delta_{mn}(1-\delta_{ij}) \\ &\quad + \frac{d_{ij}c_{mn}}{q}\}, \end{aligned} \tag{3}$$

$$\theta_{im} = -(A+B), \tag{4}$$

where A and B are positive constants, δ_{ij} is Kronecker's delta, and q is a normalization parameter. As an output function, a sigmoidal function is used:

$$f(y) = \frac{1}{1+\exp(-\frac{y}{\epsilon})}, \tag{5}$$

where ϵ is a gradient parameter of the sigmoidal function. In the method, the chaotic dynamics works to avoid the local minimum problem.

To improve the performance of the method, we have already proposed a new method which injects dynamical noise into the CNN [7, 8]. Thus, a term of the dynamical noise is added to Eq.2. An internal state of the (i, m)th neuron with dynamical noise is defined as follows:

$$\begin{aligned} y_{im}(t+1) &= k y_{im}(t) + \sum_{j=1}^{N}\sum_{n=1}^{N} w_{im;jn} f(y_{jn}(t)) - \alpha f(y_{im}(t)) \\ &\quad +\theta_{im}(1-k) + \lambda z_{im}(t), \end{aligned} \tag{6}$$

where λ is a weight of dynamical noise and $z_{im}(t)$ is a sequence of dynamical noise added to the internal state of the (i, m)th neuron at time t. In the method, both the chaotic dynamics and the dynamical noise are used to avoid local minima.

A single iteration is defined as an update of all neurons asynchronously. Then, the CNN generates solutions for updating each neuron asynchronously. However, we cannot always obtain feasible solutions from outputs of the neurons because an output of the chaotic neuron takes an analog value. Then, we use the firing decision method [4] which can always generate a feasible solution for QAP. The procedure of the method is described as follows:

1. Choose an index (i, m) whose internal state y_{im} takes the maximum value among all the neurons. Then, set the (i, m)th neuron as to firing state, and let $x_{im} = 1$.
2. Set other neurons in the ith row and the mth column to a resting state, and let $x_{ik} = 0 (k \neq m)$ and $x_{ml} = 0 (l \neq i)$. Then, exclude neurons which have already been selected in Steps 1 and 2.
3. Repeat Steps 1 and 2 until all states of neurons are decided.

2.2. 2-opt Algorithm Driven by Chaotic Dynamics with Dynamical Noise

As another approach for solving QAP, a method in which chaotic dynamics drives the 2-opt algorithm has been proposed [6]. Although the 2-opt algorithm is one of the simplest local search methods, this algorithm does not obtain good solutions because of local minimum problem. To avoid local minima, 2-opt algorithm driven by chaotic dynamics has been proposed [6]. As a result, this method shows better performance than the method which uses mutual connected chaotic neural network.

Then, to improve the performance of this method, we have already proposed a method which combined 2-opt algorithm driven by chaotic dynamics and the dynamical noise [9]. The dynamics of the (i, j)th chaotic neuron is described as follows:

$$\xi_{ij}(t+1) = \beta\Delta_{ij}(t), \tag{7}$$

$$\zeta_{ij}(t+1) = k\zeta_{ij}(t) - \alpha x_{ij}(t) + (1-k)\theta, \tag{8}$$

where $\xi_{ij}(t)$ is a gain effect of the (i, j)th neuron, $\Delta_{ij}(t)$ is a difference between value of the current objective function and that of a new objective function when the ith and the jth elements in permutation $\boldsymbol{p}$ are exchanged by the 2-opt algorithm. $\zeta_{ij}(t)$ is a refractory effect of the (i, j)th neuron and β is a scaling parameter. The output of the (i, j)th neuron at time $t + 1$, $x_{ij}(t + 1)$, is calculated as:

$$x_{ij}(t + 1) = f(\xi_{ij}(t + 1) + \zeta_{ij}(t + 1) + \gamma z_{ij}(t + 1)), \quad (9)$$

where $f(y) = 1/(1 + e^{-y/\epsilon})$, $z_{ij}(t)$ is dynamical noise, and γ is a weight of the dynamical noise. If $x_{ij}(t+1) \geq 1/2$, the 2-opt algorithm which exchanges the ith and the jth elements is applied. Each neuron is updated asynchronously.

3. Experimental results

3.1. Performance with Respect to Weight of Noise

To evaluate the performance of the proposed methods [7, 8], we use the benchmark problem from QAPLIB[13]. Parameters of the Hopfield-type CNN method (Eq.(6)) are decided as $A = 0.34$, $B = 0.34$, $k = 0.87$, $\alpha = 1.01$, $\epsilon = 0.02$, $q = 1100$(Had20) and $q = 100000$(Tai20a). Parameters of the 2opt driven chaotic dynamical method (Eq.(8)) are decided asf $k = 0.5$, $\alpha = 1.0$, $\theta = 1.0$, $\beta = 0.08$(Had20) and $\beta = 0.0002$(Tai20a). The weight of the dynamical noise is set to several values. Then, we use white Gaussian noise whose average is zero and variance is unity. The proposed method is applied for 2,000 iterations.

Figure 1 shows results of the proposed methods. In Fig.1, the results are expressed by percentages of average gaps between obtained solutions and the optimal solutions for 30 trials. From Fig.1(a), if an amount of dynamical noise is small (weight of noise λ or γ takes small value), the proposed method shows higher performance. However, it is almost same performance for Had20, when we use 2-opt algorithm driven by chaotic dynamics, because it has already obtained good solutions (Fig.1(b)).

3.2. Temporal Average and Variance of Internal States of the Neural Network

To analyze the influence of the dynamical noise to the CNN, we investigate the value of the internal state of the chaotic neurons. We calculate the temporal average and the temporal variance of the chaotic neuron. The temporal average $\bar{y}_{im}$ and temporal variance σ_{im} of the internal states of the (i, m)th neuron are calculated as follows:

$$\bar{y}_{im} = \frac{1}{T}\sum_{t=1}^{T} y_{im}(t), \quad (10)$$

$$\sigma_{im} = \frac{1}{T}\sum_{t=1}^{T}(y_{im}(t) - \bar{y}_{im}). \quad (11)$$

Figure 2 shows relationships between average of $\bar{y}_{im}$ and σ_{im} and the weight of dynamical noise λ for Hopfield-type CNN with dynamical noise method. Figure 3 shows relationships between average of $\bar{y}_{im}$ and σ_{im} and the weight of dynamical noise γ for the method in which 2-opt algorithm is driven by chaotic dynamics with dynamical noise.

From Fig. 2, if an amount of dynamical noise is large ($\lambda > 0.007$), the temporal average becomes small and the temporal variance becomes larger value. Then, the solving performance becomes worse sharply for large amount of noise (Fig. 1). It is consider that the dynamics of chaotic search is broken by large amount of noise. However, if small amount of dynamics noise ($\lambda < 0.007$) is added to the CNN, both of temporal average and variance slightly increase, and the performances are better than original CNN ($\lambda = 0.0$). For the 2-opt algorithm driven by chaotic dynamics with dynamical noise, both of temporal average and temporal variance monotonically increase as the amount of noise increases (Fig. 3). The effects of dynamical noise to the internal states of the CNN are different from two methods.

4. Conclusions

In this paper, to analyze the influence of the dynamical noise added to the CNN, we examine the temporal average and variance of the internal states of the CNN. As a result, when we add an appropriate amount of dynamical noise to the CNN, the temporal average and temporal variance takes almost same value in the Hopfield-type CNN method. However, in the case of large amount of dynamical noise, the temporal average and the temporal variance take quite different from value of the CNN without noise. On the other hand, when we add the dynamical noise to the method in which chaotic dynamics drives the 2-opt algorithm, the temporal average and the variance of the neurons monotonically increase as the amount of noise increases.

In the Hopfield-type CNN, a firing pattern of CNN represents a solution of the QAP. However, feasible solutions cannot be always obtained from outputs of the neurons because an output of the chaotic neuron takes an analog value. To generate a feasible solution, we use the firing decision method [4]. Then, in the feature work, it needs to investigate relationships between solution decided by the firing decision method [4] and amount of the noise.

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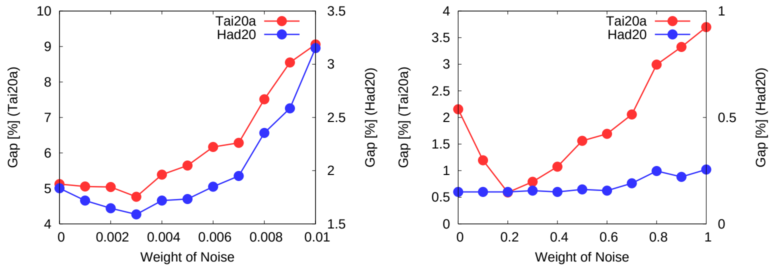
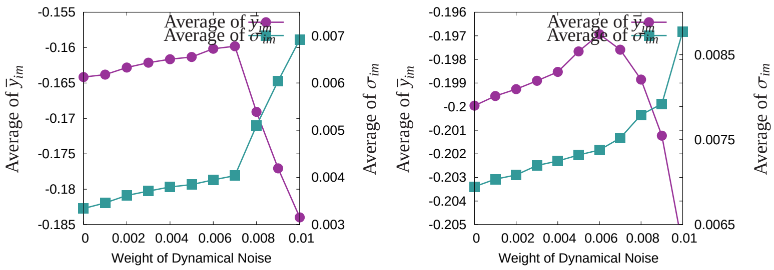


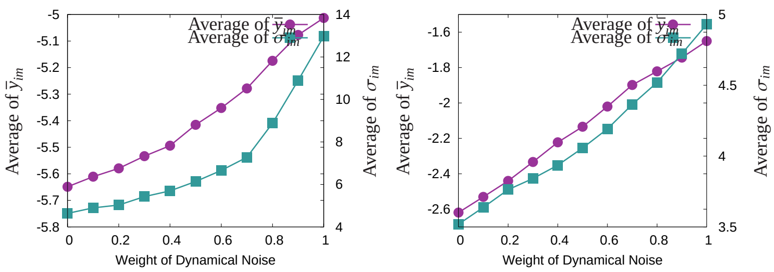
(a) Hopfield-type CNN (b) 2-opt driven by chaotic dynamics

Figure 1: Percentages of average gaps between obtained solutions and the optimal solutions for 30 trials.



(a) Had20 (b) Tai20a

Figure 2: Average of $\bar{y}_{im}$ and average of σ_{im} when we change the weight of dynamical noise γ by using Hopfield-type CNN with dynamical noise method.



(a) Had20 (b) Tai20a

Figure 3: Average of $\bar{y}_{im}$ and average of σ_{im} when we change the weight of dynamical noise γ by using 2-opt algorithm driven by chaotic dynamics with dynamical noise.

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