Wave and Particle Models of Single-Photoelectron Device

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Abstract—A one-dimensional model of an quantum electromagnetic wave detector is analyzed. The detector based on quantum photoelectronics has multiple barriers between its wave-receiving section and photoelectron-outputting section to acquire sensitiveness to frequency of the wave. Regarding an electron in the detector both as quantum wave and as a probabilistic particle, we describe the behavior of the electron by the Schrödinger and the Langevin equations. Solving the two equations, we demonstrated that the detector possessed the sensitiveness and that the wave and particle nature of the electron were equivalent.

1. Introduction

Nanometer scale devices such as quantum dots can detect not only light waves but also terahertz electromagnetic waves [1, 2]. The detectors absorbing the wave emits so-called photoelectrons. Circuits consisting of single-electron tunneling (SET) devices [3] can process streams of photoelectrons [4, 5]. In the near future, very sensitive sensing and very low power communication will be achieved with terahertz detectors and SET signal processors.

Electrons have wave-particle duality. Electrons excited by electromagnetic waves in the detectors are described by wave functions obtained by solving the Schrödinger equation. On the other hand, SET circuits processing the photoelectron streams regard the electrons as particles with probabilistic behavior. In circuit simulation, conversion from the spatially spread waves to sample particles will be high in computational complexity if the quantum front-end circuits are many-body or high dimensional quantum systems [6]. This problem will be solved if the photoelectrons in the detectors can be described as particles. In addition, the description helps to make lumped parameter device models suitable for conventional circuit simulators.

In this paper, we use Nelson's stochastic quantization theory [7] to describe electrons in the detectors as particles with probabilistic behavior. The detectors have to be sensitive to electromagnetic waves of specified frequency. The detectors analyzed in this paper have multiple barriers between their wave-receiving section, or aperture section, and photoelectron-outputting section to attain the frequency-depending sensitiveness, which is the main difference from the detector analyzed in [6]

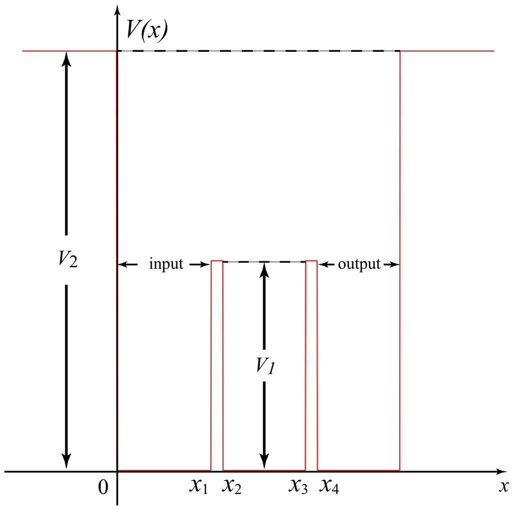

Figure 1: Potential of a model of electromagnetic wave detectors.

In this paper, the Planck constant $\hbar$ is normalized to 1.0. The energy of a photon of frequency $\omega_p = 2\pi \times 10$THz is also normalized to 1.0. Thus, unit energy $\hbar\omega_p$ is approximately 10^{-20} Joule or 0.04 eV. Because of the energy normalization, unit time corresponds to 0.1psec. We also normalize electron mass m to 1.0 and the velocity of electron with kinetic energy $\hbar\omega_p$ to 1.0. Then, unit velocity and unit length correspond respectively to 10^5m/sec and 10nm.

2. Structure of the Detector

Figure 1 shows a one-dimensional model of the detector with two barriers in its inside. The detector consists of three sections, aperture section where electrons are exposed and excited by electromagnetic waves, transfer section, and output section from which photoelectrons exit. In the transfer section, an electron takes discrete states of energy $E_{T,i}$, $i = 1, 2, \cdots$. Then, electrons of specific kinetic energy pass through the section. An electron in the aperture section also takes discrete states of energy $E_{A,j}$, $j = 1, 2, \cdots$. When an electron is excited by electromagnetic wave from ground level $E_{A,0}$ to an higher level $E_{A,j}$ which is almost equal to $E_{T,i}$, it penetrates the transfer section. Thus, the detector is sensitive to electromagnetic waves of frequency ω such that $\hbar\omega = E_{T,i} - E_{A,0}$ ($\hbar = h/2\pi$, h : Plank constant).

3. Description of the Detector

We analyze motion of an electron in the detector. A one-dimensional potential in Fig. 1 is expressed by

$$V(x) = \begin{cases} V_2 & \text{for } x < 0 \\ 0 & \text{for } 0 \le x < x_1 \\ V_1 & \text{for } x_1 \le x < x_2 \\ 0 & \text{for } x_2 \le x < x_3 \\ V_1 & \text{for } x_3 \le x < x_4 \\ 0 & \text{for } x_4 \le x < x_5 \\ V_2 & \text{for } x_5 \le x \end{cases} \tag{1}$$

In a later numerical example, the parameters of the potential are $V_1 = 30$, $V_2 = 10$, $x_1 = 4$ or 8, $x_2 - x_1 = x_4 - x_3 = 0.5$, and $x_5 - x_4 = 4$.

The behavior of an electron in the potential is governed by a Schrödinger equation possessing the following Hamiltonian H or H_0 depending on the presence or absence of electromagnetic waves:

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(x) \tag{2}$$

$$H = H_0 + \mu(x)E_{ext}\cos\omega t,\ \mu(x) = \begin{cases} -x & \text{for } -a \le x < a \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In the absence of electromagnetic waves, the solution of the Schrödinger equation is expressed by using the eigenenergies $E_{0,n}$ and the eigenfunctions $\phi_{0,n}(x)$ of Hamiltonian operator H_0 as

$$\psi_0(x,t) = \sum_{n=0}^{\infty} c_{0,n}\phi_{0,n}(x)\exp\left(-i\frac{E_{0,n}}{\hbar}t\right) \tag{4}$$

We express the wave function in the presence of electromagnetic waves by using the same eigenenergies and eigenfunctions as

$$\psi(x,t) = \sum_{n=0}^{\infty} c_n(t)\phi_{0,n}(x)\exp\left(-i\frac{E_{0,n}}{\hbar}t\right) \tag{5}$$

Let coefficients $c_n(t)$ in Eq. (5) be expanded into the following power series of ϵ:

$$c_n(t) = c_{0,n} + \epsilon c_{n,1}(t) + \epsilon^2 c_{n,2}(t) + \cdots \tag{6}$$

The first and the higher-order time-varying terms are determined by the perturbation method [8]. The zeroth-order constant term $c_{0,n}$ is the coefficient for the wave functions $\psi_0(x,t)$ given by Eq. (4).

Consider a probabilistic lumped parameter system described by the following nonlinear Langevin equation:

$$\frac{dx(t)}{dt} = b(x,t) + \sqrt{\frac{\nu}{2}}\Gamma(t) \tag{7}$$

where $\Gamma(t)$ is a white Gaussian noise with the following correlation property:

$$< \Gamma(t)\Gamma(t') >= \delta(t-t') \tag{8}$$

Let $\rho(x,t)$ be the probability distribution of x. Nelson's stochastic quantization theory [7] asserts that if

$$b(x,t) = \mathscr{R}[\chi(x,t)] + \mathscr{I}[\chi(x,t)], \tag{9}$$
$$\chi(x,t) = \nu\nabla\ln\psi(x,t),\quad \nu = \frac{\hbar}{m}$$

then,

$$\rho(x,t) = |\psi(x,t)|^2 \tag{10}$$

where $\mathscr{R}[\chi]$ and $\mathscr{I}[\chi]$ are real and imaginary parts of χ respectively.

4. Numerical Experiments

Assuming that $\psi(x,0) = \psi_0(x,0)$ or $c_n(0) = c_{0,n}$, we determine the constant term from the initial distribution of wave function, that is, we determine it by the following equations:

$$c_{0,n} = \int_{x=-\infty}^{\infty} \phi_0(x)\phi_{0,n}^*(x)dx, \tag{11}$$
$$\phi_0(x) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\phi_0(p)\exp\left(i\frac{p}{\hbar}x\right)dp,$$
$$\phi_0(p) = \frac{1}{(2\pi\sigma_p^2)^{1/4}}\exp\left(-i\frac{p}{\hbar}x_0 - \frac{(p-p_0)^2}{4\sigma_p^2}\right)$$

The above function $\phi_0(p)$ implies that momentum p of the electron initially distributes in the Gaussian form.

We set the expectation of initial location x_0 of the electron to $x_0 = 0$. Expectation p_0 and variance σ_p^2 of initial momentum are determined so that the electron initially takes ground state and exists in aperture section at high probability. Electromagnetic excitation waves of frequency $\omega = \omega_{a,1} = 7.21$ and $\omega_{a,2} = 8.14$ are applied when $x_1 = x_a = 4$. When $x_1 = x_b = 8$, the frequency is $\omega = \omega_{b,1} = 6.46$ and $\omega_{b,2} = 8.21$. Now, we can let wave function $\psi(x,t)$ evolve.

5. Result

Figure 2 shows the wave function at $t = 150$. As the wave function is determined, we can derive a nonlinear Langevin equation from Eq. (9). Samples of numerical solutions of the Langevin equation are shown in Fig. 3. In Fig. 4, transmission rate of an electron through the transfer section is plotted against the kinetic energy of the electron. Table 1 shows probability that an electron exists in the output section for $x_4 \le x \le x_5$. Figures 3 and 4 imply that the probability is higher when $\omega = \omega_{a,1}$, $\omega_{b,1}$ than when $\omega = \omega_{a,2}$, $\omega_{b,2}$. This is quantitatively shown in Tab. 1. These are because electrons excited by electromagnetic wave of frequency $\omega = \omega_{a,1}$ and $\omega_{b,1}$ obtain energy at which the transmission rate is maximum. We also see from the table that the probabilities obtained from the evolving wave function and the sample electron trajectories are almost the same.

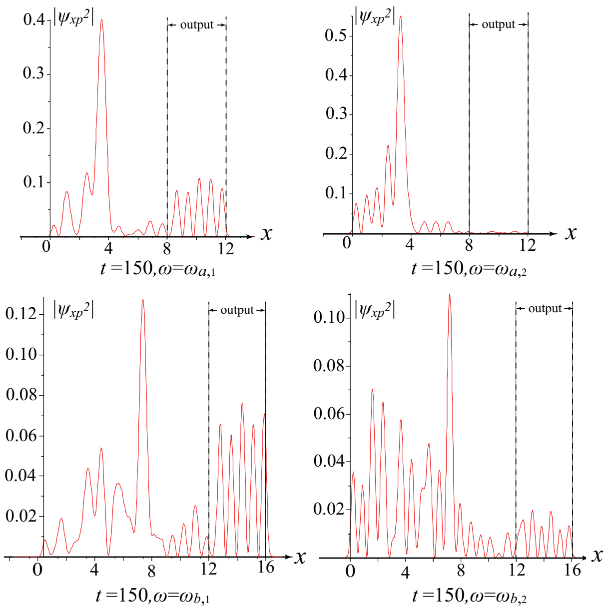

Figure 2: Marginal probability distribution of the x-directional location of an electron.

Table 1: Probability that an electron exists in the output section.

Excitation frequency ω	Estimation by wave functions	Estimation by electron trajectories
$\omega_{a,1}$ =7.21	0.17	0.19
$\omega_{a,2}$ =8.14	0.01	0.02
$\omega_{b,1}$ =6.46	0.13	0.14
$\omega_{b,2}$ =8.21	0.03	0.00

6. Conclusions

A one-dimensional model of an electromagnetic wave detector was analyzed. The quantum photoelectronic detector has multiple barriers between their aperture section and photoelectron-outputting section to acquire sensitiveness to frequency of the wave. Regarding an electron in the detector both as quantum wave and as a probabilistic particle, we described the behavior of the electron by the Schrödinger and the Langevin equations. Solving the two equations, we demonstrated that the detector possessed the sensitiveness and that the wave and particle nature of the electron were equivalent.

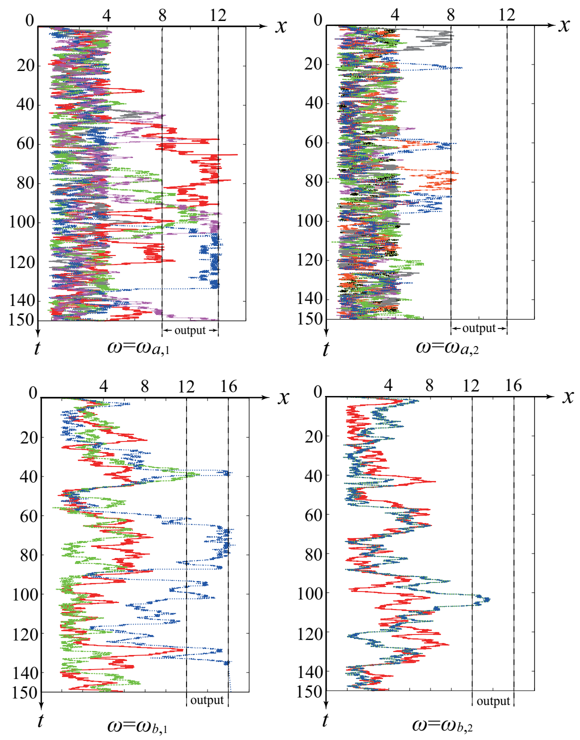

Figure 3: Sample trajectories of electrons in the detector model.

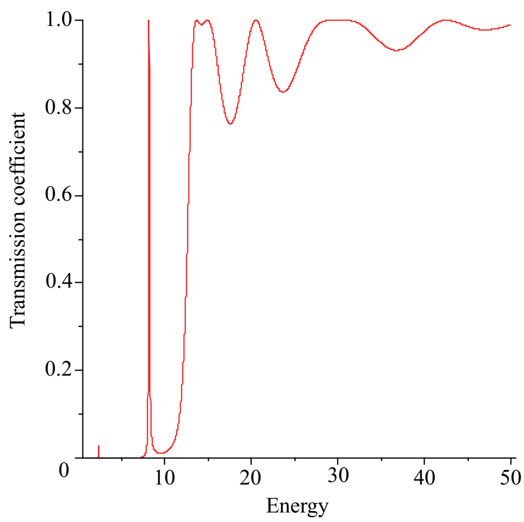

Figure 4: Transmission rate of an electron through the transfer section.

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