2009 International Symposium on Nonlinear Theory and its Applications
NOLTA'09, Sapporo, Japan, October 18-21, 2009

Bistable Micro-circuit For The Combined Electric Field And Magnetic Field Sensor

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Abstract—Bistable systems are prevalently found in many sensor systems. Recently, we have explored (unidirectionally) coupled overdamped bistable systems that admit self-sustained oscillations when the coupling parameter is swept through the critical points of bifurcations. Complex behaviors emerge from these systems when an external signal (AC, DC) is applied uniformly to all the elements in the array. The results are being used to develop extremely sensitive magnetic sensors capable of resolving field changes as low as 150pT. Extending on this work, we now explore the underlying dynamics of a coupled bistable system realized in microelectronic circuits which belong to the same class of dynamics as the afore-mentioned system. The emergent behavior is being applied to develop an extremely low current sensing system, which forms the basis for the combined electric and magnetic field sensors on the same substrate.

1. Introduction

Well-designed coupling schemes, together with an appropriate choice of initial conditions, can induce oscillations (i.e. periodic transitions between stable attractors) in overdamped dynamical systems when a control parameter exceeds a threshold value [1, 2, 3]. These dynamical elements would not display oscillatory behavior when isolated (i.e. uncoupled), unless they were appropriately driven. We have demonstrated this behavior in a specific prototype system, three unidirectionally coupled ferromagnetic cores which is the basis of a coupled core fluxgate magnetomete subject to a "residence times" readout scheme [1] that is used to detect magnetic flux signals. Our analysis showed that N (taken to be odd, although the oscillatory behavior can also be seen for N large and even) unidirectionally coupled elements with cyclic boundary conditions would, in fact, oscillate when a control parameter – in this case the coupling strength – exceeded a critical value.

Typically, the oscillations emerge with an infinite-period through a heteroclinic-cycle bifurcation, i.e., a global bifurcation to a collection of solution trajectories that connects sequences of equilibria and/or periodic solutions. In the particular case of overdamped bistable systems, the cycle includes mainly saddle-node equilibria. As a control parameter (usually the coupling strength) approaches from above a critical value, the frequency of the oscillations decreases, approaching zero at the critical point. Past the critical value, the oscillations disappear and the system dynamics settles into an equilibrium.

In this article, we describe a device comprised of coupled bistable microelectronic elements; the object is to detect very low (taken to be dc) electric fields via their asymmetrizing effect on the coupled oscillator dynamics. In the following, we provide an overview of the circuit details and the associated dynamical equations. We then discuss the system response and explain how one can use it to quantify the target electric or magnetic field.

2. The Circuit and its Dynamics

The basic building block of each bistable element is a differential pair consisting of two transistors and a current source, as shown in figure 1. A single bistable element is schematized in figure 2. It consists of two differential pairs that employ NPN transistors with one of them being cross-coupled, two PMOS transistors and a pair of resistors. Figure 3 and 4 illustrate input signal circuitry and connectivity of the device to form the coupled bistable system, respectively. In each differential pair, the current source in Fig. 1 is replaced by a current mirror. The two PMOS transistors are used as the load of the two differential pairs, and the two resistors are used for both the system dynamics and common-mode feedback. Since the circuit is fully-differential (i.e. the output, V_{out+} and V_{out-}, are equal in magnitude but are out of phase by π), the common-mode voltages at V_{out+} and V_{out-} need to be "tracked" to take account of mismatches in the manufacture of the device; this is done by the resistor pairs which "track" the voltages at these two nodes and can be adjusted to make sure that they are the same.

A standard nodal analysis yields, after some calculations, the equation for the time rate of change of the differential output of the i^{th} element:

$$C_L \dot{V}_i = -gV_i + I_s \tanh[c_s V_i] + I_c \tanh[c_c V_{i-1}] - \varepsilon, \quad (1)$$

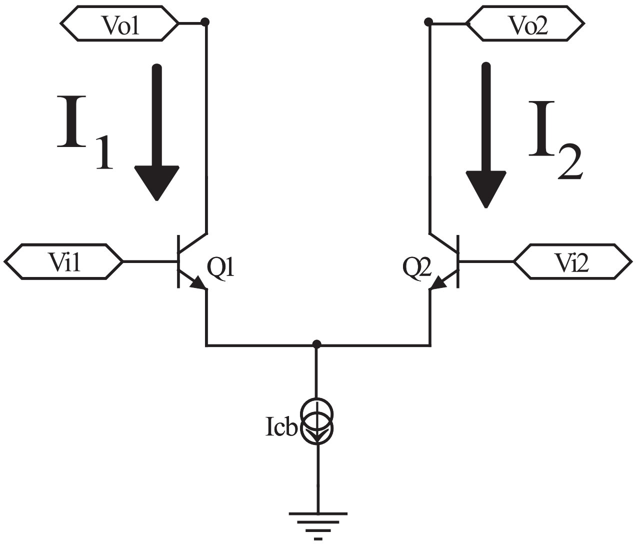

Figure 1: The basic building block of a bistable circuit element.

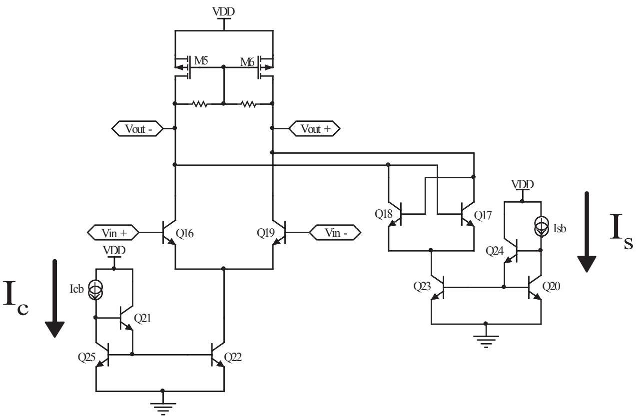

Figure 2: A single (bistable) element containing a cross coupled pair, input signal circuitry, and coupling components.

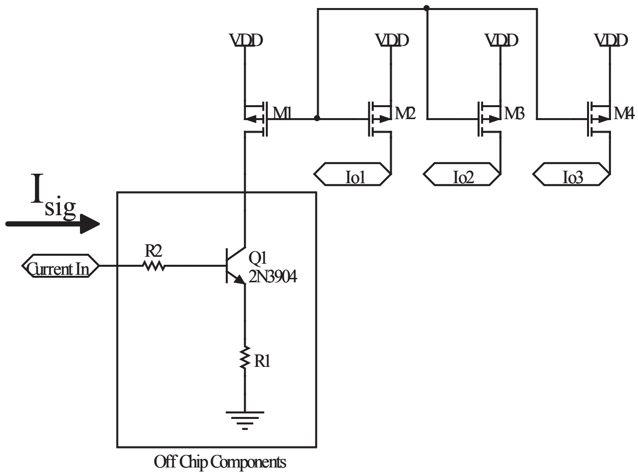

Figure 3: The input current mirror provides the same current to each of the bistable elements of the coupled system shown below. ϵ in Eq. (1) is defined as $\epsilon = \beta I_{sig}$, where β is the current gain of the NPN transistor.

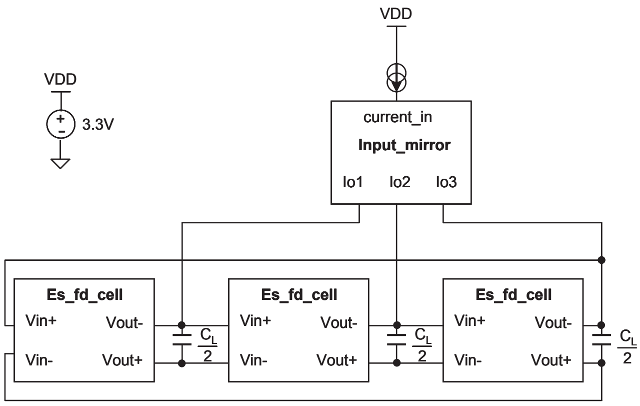

Figure 4: Top-level schematic of a circuit realization of a coupled system with three units coupled unidirectionally.

where the constants in the dynamics correspond to circuit element values that have been defined in [4]. One can, readily, write down a scaled version of (1) for $N = 3$:

$$\begin{aligned}
\dot{x}_1 &= -x_1 + I_s \tanh[dx_1] - I_c \tanh[dx_3] - \varepsilon \qquad (2)\\
\dot{x}_2 &= -x_2 + I_s \tanh[dx_2] - I_c \tanh[dx_1] - \varepsilon\\
\dot{x}_3 &= -x_3 + I_s \tanh[dx_3] - I_c \tanh[dx_2] - \varepsilon
\end{aligned}$$

with the change of variables $x_i = gV_i$ and scaling time by $\tau = g/C_L$, so that differentiation in (2) is with respect to τ. Note $d = c/g$. In the absence of coupling, each element describes a particle in a bistable potential that has been asymmetrized through the addition of the target signal ε. Note that the signal ε is taken to be far smaller than the energy barrier height for the potential function corresponding to any (uncoupled) element. Then, one can compute the condition for bistability as $\frac{I_s c}{g} > 1$.

Figure 5 shows the effects of increasing the target dc signal continuously from zero. In particular, we note that the "Residence Times Difference (RTD)" increases with increasing ε. Here, the residence time is defined via two successive crossings of the t-axis by the waveform; the difference in these two crossing times yields the residence time in one state. The successive crossing time, yields the residence time in the other stable state. The RTD is the difference in these two residence times and it is clearly zero unless the dc asymmetry is present.

The unique character of the oscillations permits one to, analytically, compute the period as well as the RTD. Specifically, we take advantage of the observafor $\varepsilon = 0$ we obtain $t_1 = t_2$ and the RTD vanishes, as expected.

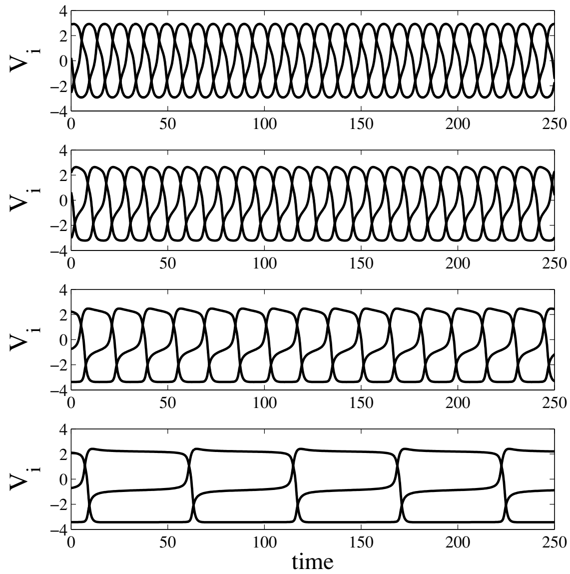

Figure 5: Time series of oscillatory behavior in a coupled ($N = 3$) coupled system. As ε increases from the top panel to the bottom one, the frequency of the oscillations decreases correspondingly. $g = 1, c = 1, I_s = 2, I_c = 1$.

tion that the elements evolve singly as they switch states; specifically, while one of the elements switches from the positive to the negative stable state, the other two elements remain close to their steady state values for the duration of the switch. This is readily apparent in figure 5, and serves as a way to decouple the system (2) for the duration of a switching event. We can, then, readily compute [1, 4] the period of an individual element as well as the RTD, to a very good approximation as:

$$T = \frac{3\pi}{\sqrt{\alpha}} \left[\frac{1}{\sqrt{(I_{2cc} - I_c)\tanh[dx_{1m}]}} + \frac{1}{\sqrt{(I_c - I_{1cc})\tanh[dx_{1p}]}} \right] \tag{3}$$

and

$$RTD = \left| \frac{\pi}{\sqrt{\alpha}} \left[\frac{1}{\sqrt{(I_{2cc} - I_c)\tanh[dx_{1m}]}} - \frac{1}{\sqrt{(I_c - I_{1cc})\tanh[dx_{1p}]}} \right] \right| \tag{4}$$

with the characteristic scaling with respect to the square root of the "bifurcation distances" $|I_{2cc} - I_c|$, $|I_{1cc} - I_c|$ readily apparent. We note that the signal (ε) induced asymmetry enters the above expressions through the critical values of the coupling coefficients;

3. Experiments

The prototype system is based on a device made following the design provided by this manuscript. The load capacitance, C_L, is set to 66nF, $I_c = 200\mu$A, $I_s = 300\mu$A and $R = 500\Omega$. The supplied voltage is 3.3V to power the microcircuit. A source measure unit (SMU) is used to supply the current I_{sig} which inputs into the bistable microcircuit to imitate the sensing current that would have appeared from the collecting apparatus via a conducting plate for an electric field sensing or some magnetoresistance material either the GMR or AMR type for sensing the magnetic field.

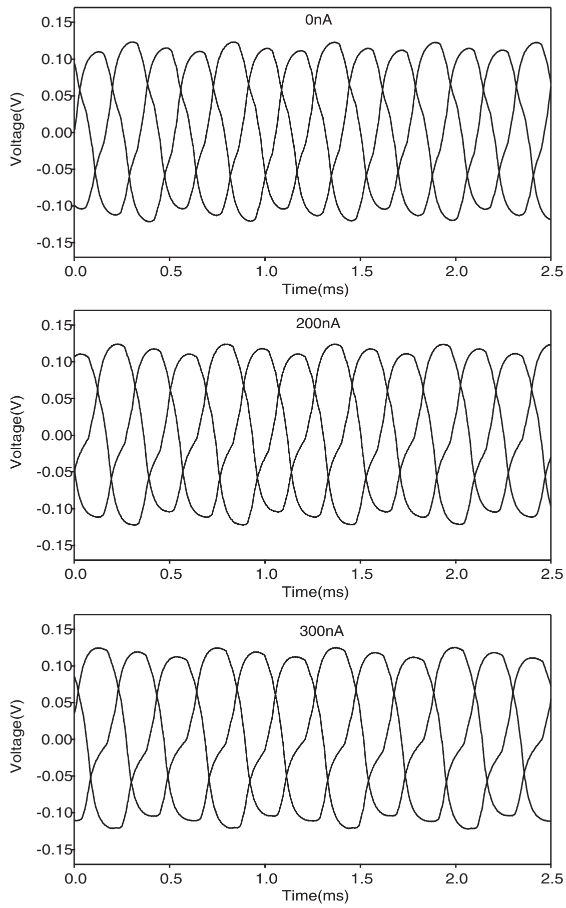

Figure 6: Time series from the experimental system. Each graph shows the three waveforms from the three oscillators used in the coupled network. The first graph (top) shows the oscillations with zero applied current. The second graph is for applied current $I_{sig} = 200$nA. The third graph (bottom) is for the applied current $I_{sig} = 300$nA.

Figure 6 shows the time series from the experiment with various applied currents I_{sig} similar to what has been seen from the model and the SPICE simulation of the design circuit. As seen in the plots, there are some mismatches in the waveforms among the three outputs of the oscillators. These mismatches can be attributed to the circuit components, mainly the transistors that make up the OTA's. Future designs will attempt to minimize these component mismatch issues but they are not trivial. Improvement in these mismatches would improve the sensitivity of the device even further.

Figure 7 shows the response characteristics of the oscillation frequency and the measured residence time difference (RTD) to the input signal I_{sig}, where β is determined to be, approximately, 150. In the experiment, the injected current I_{sig} cannot exceed 530nA to avoid overloading the microcircuit. Steps are taken to increase injected current range to push the system closer toward the bifurcation or to set the I_c parameter closer toward the bifurcation so that the RTD response curve is much closer to what is seen in both the SPICE simulation and the model, but for an ultra-sensitive sensor, this is not the goal. The goal is to discern a very small current which would translate into detecting very small change in electric field or the magnetic field.

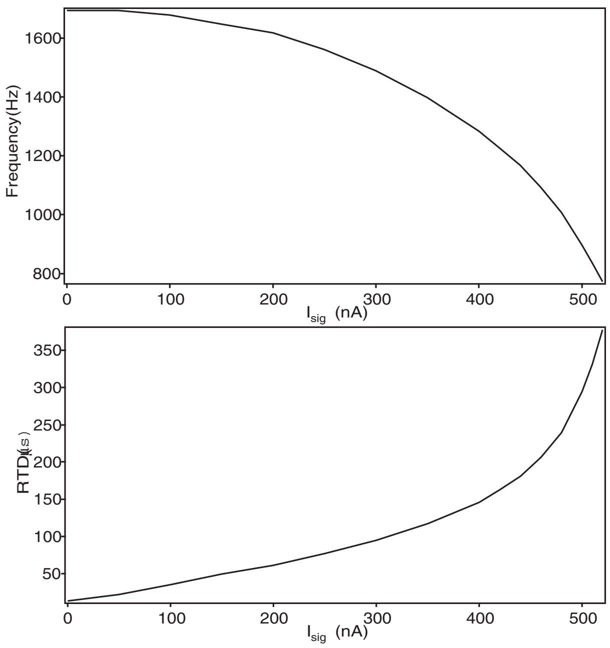

Figure 7: Frequency and RTD responses of the experimental coupled electric-field sensor system subject to an injected current I_{sig}.

4. Conclusion

The microelectronic circuit presented here, exploits the coupling induced oscillatory behavior to detect very weak external (dc) electric fields, via their asymmetrizing effect on each of the potential energy functions that underpin the dynamics of each element when uncoupled. Chip-sized detectors that follow the dynamics described in this article are, currently, under construction and testing. They already display the capacity to detect minute electrical current in the pA range. Combining the sensitivity of this microcircuit to the collecting apparati forms the basis for sensing both magnetic and electic field on the same substrate. With a simple connection to an AMR material, the apparatus can detect small magnetic field in the range of approximately 1-4nT. With the right shaping of the material, one can achieve even lower sensitivity in the pT range. This is the subject of future work.

5. Acknowledgements

The authors acknowledge supports from the Office of Naval Research (Code 30). A.P. was supported in part by National Science Foundation grants CMS-0625427 and CMS-0638814.

References

[1] V. In, A. Bulsara, A. Palacios, P. Longhini, A. Kho, J. Neff, "Coupling-Induced Oscillations in Overdamped Bistable Systems", *Phys. Rev. E* **68**, 045102(R) (2003). A. Bulsara, V. In, A. Kho, W. Rappel, J. Acebron, A. Palacios, P. Longhini, S. Baglio, B. Ando, "Emergent Oscillations in Unidirectionally Coupled Overdamped Bistable Systems", *Phys. Rev. E* **70**, 036103 (2004).

[2] V. In, A. Bulsara, A. Kho, P. Longhini, A. Palacios, "Complex Dynamics in Unidirectionally Coupled Overdamped Bistable Systems Subject to a Time-Periodic External Signal", *Phys. Rev. E***72**, 045104 (2005).

[3] V. In, A. Palacios, A. R. Bulsara, P. Longhini, A. Kho, J. D. Neff, S. Baglio and B. Ando, "Complex Behavior in Unidirectionally Coupled Overdamped Duffing Elements", *Phys. Rev. E* **73**, 066121 (2006).

[4] V. In, P. Longhini, N. Liu, A. Kho, P. Longhini, A. Palacios, A. Bulsara,"A Bistable Micro Electronic Circuit for Sensing Extremely Low Electric Field", *Jl. Appl. Phys.*, preprint (2009)