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Ghost stochastic resonance in an asynchronous sequential logic neuron model

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Abstract—A nonlinear system driven by multiple weak periodic input signals with different frequency components and a noise signal sometimes generates an output signal with frequency components that are not included in the input frequency components. Such a phenomenon is called a ghost stochastic resonance (GSR). In this paper, we generalize a digital spike neuron model that is described by an asynchronous cell automaton so that the model includes a leak current effect. It is shown that the generalized model can show a GSR whose characteristics is quite similar to that in one of the representative previous works.

1. Introduction

Stochastic resonance (SR) is a phenomenon wherein a weak periodic input signal including a particular non-zero level of noise signal improves the response of a nonlinear system [1]. Especially, we consider the case where multiple weak periodic input signals with different frequency components and a noise signal are input to a nonlinear dynamical system. Then, the system sometimes generates an output signal with frequency components that are not included the input frequency components. Such a phenomenon is called a ghost stochastic resonance (GSR) that has been observed in many nonlinear systems, e.g. , a model that fires a spike when an input signal exceeds an ignition threshold, a monostable Schmit trigger electronic circuit, and pulse-coupled excitable systems [2]-[5]. In this paper, we focus on a digital spike neuron (DSN) [6]-[7]. The DSN is a wired system of shift registers. The DSN has an internal clock and an input spike-train that may not synchronize, i.e., the DSN operates in an asynchronous mode. The DSN can exhibit various bifurcation phenomena and response characteristics to the input spike-train.

In this paper, we generalize the DSN so that the model includes a leak current effect. It is shown that the generalized model can show a GSR whose characteristics is quite similar to that in one of the representative previous works [2].

2. Degital Spike Neuron

In this section we present a generalized digital spike neuron (DSN). As shown Fig.1(a), the DSN consists of three parts: an M-bit one-hot-coded uni-directional register (rhythm register) ; an N-bit one-hot-coded uni-directional register (membrane register) ; and wires from a rhythm register to a membrane register. First, we explain dynamics of the rhythm register. Each bit of rhythm register has a binary state $\{0, 1\}$, and the rhythm register is one-hot-coded. The rhythm register commonly accept the following internal clock C(t),

$$C(t) = \begin{cases} 1 & \text{if t=}c_0, c_1, c_2, \ldots, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where we define an internal clock interval $\Delta_c \equiv c_{n+1} - c_n$. Then dynamics of the rhythm register is described as follows,

$$P(c_{n+1}) = \begin{cases} P(c_n) + 1 & \text{if } P(c_n) \neq M - 1, \\ 0 & \text{if } P(c_n) = M - 1, \end{cases} \quad n = 0, 1, 2, \ldots. \tag{2}$$

where $P(t) \in \{0, 1, \ldots, M-1\}$ is a hot bit of rhythm register (integer state) , and $P(0) = 0$ is initial $P(t)$. The state $P(t)$ oscillates periodically with period M.

Second, we explain the wires from rhythm register to a membrane register. We introduce a wiring function $A(i)$, that means which bit of membrane register is connected to i-th rhythm register by the wire. For example, when 5-th rhythm register and 4-th membrane register are connected by the wire, $A(4) = 3$ (because i starts from 0). And we define a parameter vector $A \equiv (A(0), A(1), \ldots, A(M-1))$ as the wiring pattern. In the case of Fig.1(a) the wiring pattern is $A = (0, 1, 2, 3, 3, 2, 1)$. Using the wiring function $A(i)$, a following signal $B(t)$ is defined as follows.

$$B(t) = A(P(t)). \tag{3}$$

We refer to $B(t)$ as a base signal. The base signal is a reset value when the DSN fires. In Fig.1(b) a base signal $B(t)$ is illustrated by white circles.

Third, we explain dynamics of the membrane register. Each bit of membrane register has a binary state $\{0, 1\}$, and the membrane register is one-hot-coded. The membrane register accepts the internal clock $C(t)$ and the input stimulation $S(t)$. $S(t)$ is described as follows.

$$S(t) = \begin{cases} 1 & \text{if t=}\tau_0, \tau_1, \tau_2, \ldots, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Accepting $C(t)$ and $S(t)$, dynamics of the membrane reg-

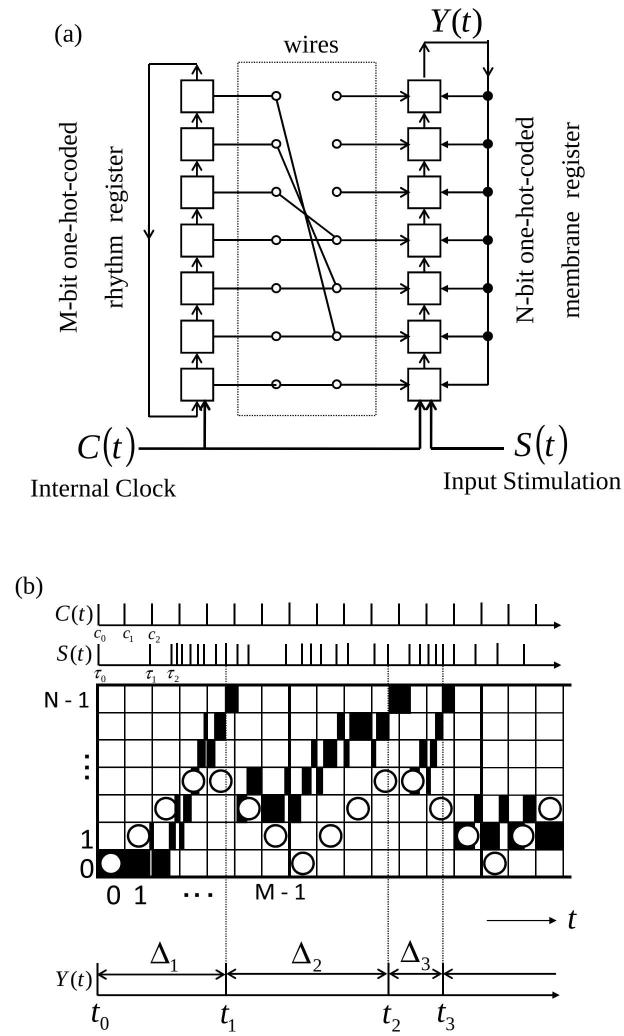

Figure 1: (a) Generalized DSN ($M = 7$, $N = 7$). (b) Basic dynamics of the generalized DSN.

ister is described as follows,

$$X(t^{+}) = \begin{cases} X(t)+1 & \text{if } S(t^{+}) = 1 \text{ and } X(t) \neq N-1, \\ B(t^{+}) & \text{if } S(t^{+}) = 1 \text{ and } X(t) = N-1, \\ X(t)-1 & \text{if } C(t^{+}) = 1 \text{ and } X(t) \neq 0, N-1, \\ X(t) & \text{if } C(t^{+}) = 1 \text{ and } X(t) = 0, N-1. \end{cases} \quad (5)$$

where $X(t) \in \{0, 1, \ldots, N-1\}$ is a hot bit of membrane register (integer state) , and $X(0) = 0$ is initial $X(t)$. In Fig.1(b) a typical waveform of the state $X(t)$ is illustrated by black boxes (which corresponds to a membrane potential). If a stimulation spike $S(t^{+}) = 1$ arrives, the black box is shifted upward ($X(t^{+}) = X(t)+1$), and, if an internal clock spike $C(t) = 1$ arrives, the black box is shifted downward ($X(t^{+}) = X(t)-1$). This downward shift corresponds to a leak current effect and it is a generalized feature of the DSN. If the black box reaches the top position (which corresponds to a firing threshold) at $t = \tau_n$, we define $t_m \equiv \tau_n$,

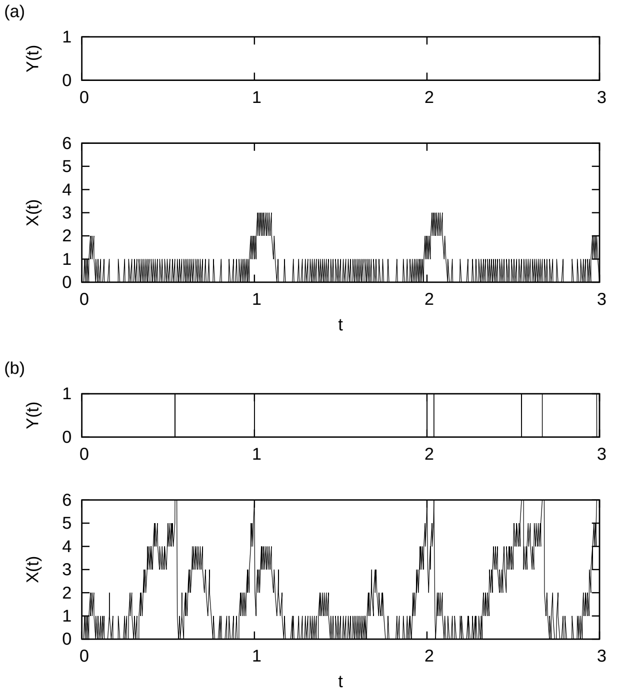

Figure 2: Time waveforms of the DSN. $M = 7$, $N = 7$, $A = (0, 1, 2, 3, 3, 2, 1)$. (a) The case without noise spike-train $N(t)$. (b) The case with noise spike-train $N(t)$.

and the black box at $t = \tau_{n+1}$ is reset to the position of the white circle ($B(\tau_{n+1})$). This reset action gives nonlinearity to the DSN. At $t = t_m$, the DSN outputs a spike $Y(t_m) = 1$. After the reset, the black box is shifted upward or downward. Repeating such *shift-and-fire* dynamics, the DSN generates the following output spike-train $Y(t)$ as shown in Fig.1(b),

$$Y(t) \equiv \begin{cases} 1 & \text{if } X(t) = N-1 \text{ and } C(t) \neq 1, \\ 0 & \text{if } \textit{otherwise}, \end{cases} \quad (6)$$
$$t = \tau_0, \tau_1, \tau_2, \cdots .$$

where we define an output spike interval $\Delta_n \equiv t_n - t_{n-1}$. The shift-and-reset dynamics corresponds to the integrate-and-fire dynamics of the neuron model.

3. Ghost Stochastic Resonance

We show that the generalized DSN can show a GSR and its characteristics is quite similar to that in an analog easy neuron model [2]. In this paper, an input stimulation $S(t)$ is a spike-train generated by logically summing two density-modulated spike-trains $F_1(t)$ and $F_2(t)$ and a noise spike-train $N(t)$ that are described as follows,

$F_1(t)$ = Spike-train whose average spike density is α_1 and instantaneous spike density is $\alpha_1(1+\beta_1\cos(2\pi f_1 t))$,

$$F_2(t) = \text{Spike-train whose average spike density is } \alpha_2 \text{ and instantaneous spike density is } \alpha_2(1+\beta_1\cos(2\pi f_2 t)),$$

$$N(t) = \text{Random spike-train whose average spike density is } p.$$

where α_1 and α_2 represent average spike densities of $F_1(t)$ and $F_2(t)$, and β_1 and β_2 represent degrees of modulations. So the input stimulation $S(t)$ is described as follows,

$$S(t) = F_1(t) + F_2(t) + N(t). \tag{7}$$

where "+" means the logical sum. Hence the input stimulation $S(t)$ has a multiple weak periodic signals with different frequency components f_1 and f_2 and a noise signal. We determine parameters of $S(t)$ so that the model doesn't generate output spikes without a noise spike-train $N(t)$ (Fig.2(a)) and the model generates output spikes (Fig.2(b)) with a noise spike-train $N(t)$. We define a noise intensity σ as follows.

$$\sigma \equiv \frac{\text{Average spike density of } N(t)}{\text{Average spike density of } S(t)}. \tag{8}$$

We define the n-th instantaneous output frequency $f_{out}(n)$ as follows.

$$f_{out}(n) = \frac{1}{\Delta_n}. \tag{9}$$

In order to make a histograms of the output frequency $f_{out}(n)$, we define the bins as follows.

$$I_i = \left(\frac{1}{(i_{max}-i)\delta}, \frac{1}{(i_{max}-i-1)\delta}\right], \quad I_0 = \left(0, \frac{1}{i_{max}\delta}\right], \quad i = 1,2,\ldots,i_{max}-1, \quad i_{max} = 499, \quad \delta = 0.01. \tag{10}$$

Then we define a histograms $h(i)$ as follows.

$$h(i) = \text{Number of output frequencies } f_{out}(n) \text{ that are included in the } i\text{-th bin } I_i. \tag{11}$$

Fig.3 shows typical histograms $h(i)$. In the case of Fig.3(a), the noise intensity is $\sigma = 0.1083$ and the histograms $h(i)$ almost has no peaks. In the case of Fig.3(b), the noise intensity is $\sigma = 0.1974$ and the histograms $h(i)$ has small peaks. In the case of Fig.3(c), the noise intensity is $\sigma = 0.2726$ and the histograms $h(i)$ has a large peak at $f_{out} \simeq 1$. Recall that the input stimulation $S(t)$ has the two frequency components $f_1 = 2$ and $f_2 = 3$ that do not appear in the histograms $h(i)$ of the output frequency f_{out}. In the case of Fig.3(c), the noise intensity is $\sigma = 0.3337$ and the histograms $h(i)$ has many peaks that are smaller than the peak in Fig.3(c). From the above results, we can say the DSN exhibits a GSR.

Next, we study characteristics of the GSR for the noise intensity σ. For this purpose, we define a k-local histogram $h_k(i)$ of the histogram $h(i)$ as follows.

$$h_k(i) \equiv \left(\sum_{j=i-k}^{i+k} h(j)\right) \Big/ \left(\sum_{j=1}^{i_{max-1}} h(j)\right). \tag{12}$$

Note that the argument i of the k-local histogram $h_k(i)$ is the index of the bin $\boldsymbol{I}_i$, i.e., i corresponds to the output frequency f_{out}. Fig.4 shows characteristics of the k-local histogram $h_k(i)$ for the noise intensity σ and the output frequency f_{out}. It can be seen that the output frequency $f_{out} = f_2 - f_1$ can be detected in the output frequencies f_{out} for a certain range $0.15 < \sigma < 0.25$ of the noise intensity. On the other hand, the output frequencies f_2 and f_1 cannot be detected in the output frequencies f_{out} for a wide range of the noise intensity σ. From the above results, we can again say the DSN exhibits a GSR. Fig.5 shows characteristics of the instantaneous output frequency $f_{out}(n)$ for the input frequency component f_1, where another input frequency component f_2 is given by $f_2 = f_1 + 1$. In this figure, the dotted line is a theoretically predicted characteristics of an instantaneous output frequency of an analog neuron model [2]. It can be seen that our DSN can reproduce the characteristics of the GSR of the analog neuron model.

4. Conclusions

In this paper, we have proposed a generalized DSN that is described by an asynchronous cell automaton and includes a leak current effect. It has been shown that the DSN can exhibit GSRs and their characteristics are quite similar to those of an analog neuron model. Future problems include the following ones: (a) more detailed analysis of the GSR, (b) FPGA implementation of the neuron, and (c) comparing with other discrete models [6]-[7]. The authors would like to thank Professor Toshimitsu Ushio of Osaka University for valuable discussions. This work is partially supported by the Center of Excellence for Founding Ambient Information Society Infrastructure, Osaka University, Japan and by KAKENHI (21700253).

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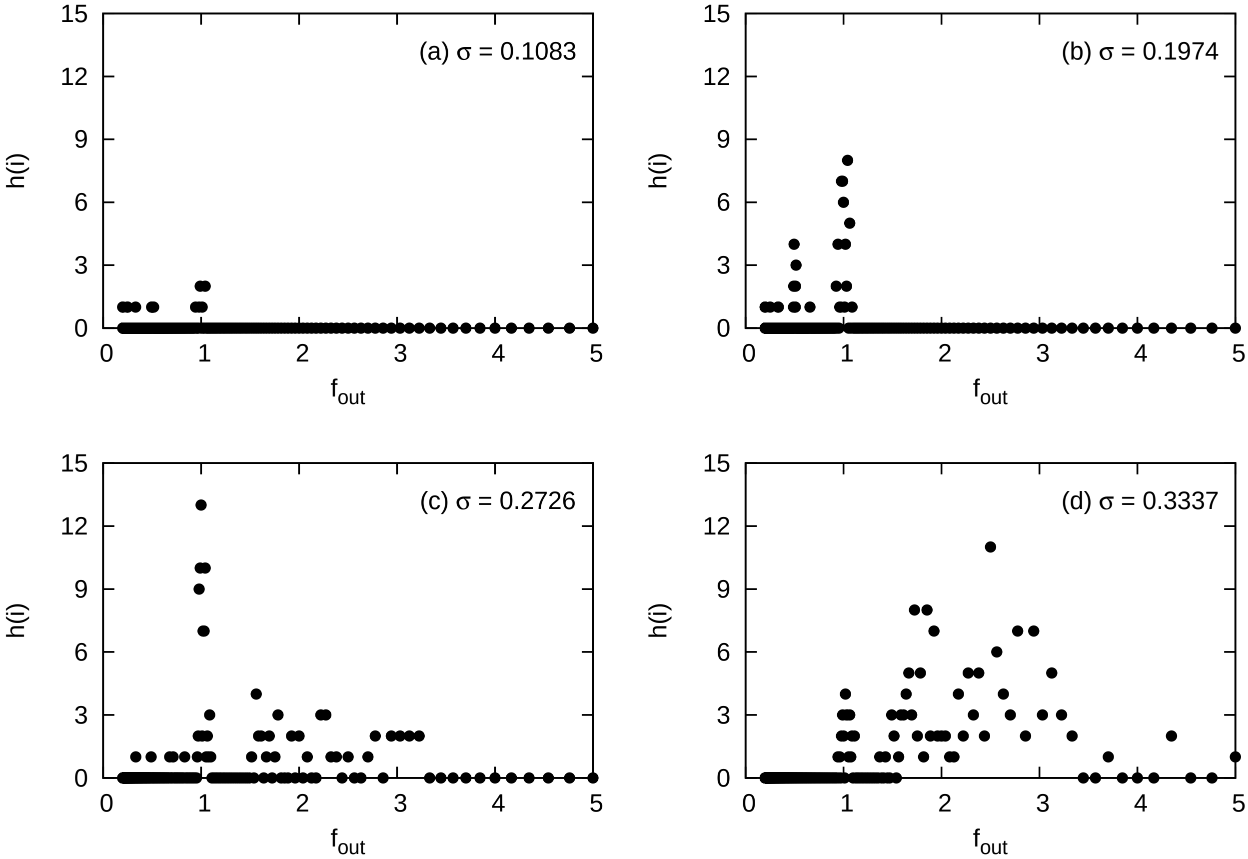

Figure 3: Typical histograms $h(i)$. σ =(a) 0.1083, (b) 0.1974, (c) 0.2726, (d) 0.3337, $\alpha_1 = 0.667$, $\alpha_2 = 0.667$, $\beta_1 = 0.9$, $\beta_2 = 0.9$, $f_1 = 2$, $f_2 = 3$, $M = 7$, $N = 7$, $A = (0, 1, 2, 3, 3, 2, 1)$, $\Delta_c = 0.01$, $t = 0$〜100.

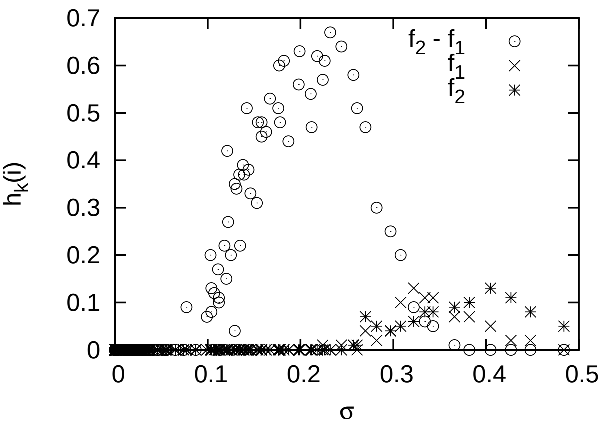

Figure 4: Characteristics of the k-local histogram $h_k(i)$ for the noise intensity σ and the output frequency f_{out}. $f_1 = 2$ ($h_4(50)$), $f_2 = 3$ ($h_4(100)$), $f_2 - f_1 = 1$ ($h_2(33)$), $\alpha_1 = 0.667$, $\alpha_2 = 0.667$, $\beta_1 = 0.9$, $\beta_2 = 0.9$, $M = 7$, $N = 7$, $A = (0, 1, 2, 3, 3, 2, 1)$, $\Delta_c = 0.01$, $t = 0$〜100.

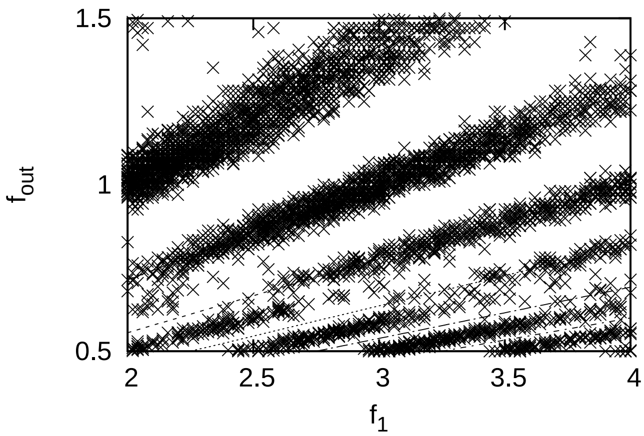

Figure 5: Characterisitcs of the instantaneous output frequency $f_{out}(n)$ for the input frequency component f_1. $f_2 = f_1 + 1$, $\sigma = 0.2726$, $M = 7$, $N = 7$, $A = (0, 1, 2, 3, 3, 2, 1)$, $\Delta_c = 0.01$, $t = 0$〜100.