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Canonical Particle Swarm Optimization System

Kenya Jin'no, Takuya Shindo

Electrical & Electronics Engineering,
Nippon Institute of Technology
Miyashiro, Minami-Saitama, Saitama, 345-8501, Japan
Email: jinno@nit.ac.jp

Abstract—A particle swarm optimization (PSO) system is one of the powerful systems for solving global optimization problems. The PSO algorithm can search an optimal value of a given evaluation function quickly compared with other proposed meta-heuristics algorithms. The conventional PSO system contains some random factors, therefore, the dynamics of the system can be regarded as stochastic dynamics. In order to analyze the dynamics rigorously, some papers pay attention to deterministic PSO systems which does not contain any stochastic factors. According to these results, the eigenvalues of the system impinge on the dynamics of the particles. Depending on the parameter, the searching ability of the deterministic PSO is decreased. In order to overcome this, we propose a canonical deterministic PSO which can control its eigenvalues easily, and can improve the searching ability. We will confirm relation between the eigenvalues and the searching ability of the optimal value from some numerical experiments.

1. Introduction

Searching for an optimal value of a given evaluation function of various problems is very important in engineering fields. In order to solve such optimization problems speedily, various heuristic optimization algorithms have been proposed. Particle swarm optimization (PSO), which was originally proposed by J. Kennedy and R. Eberhart [1],[2], is one such heuristic algorithm. The PSO algorithm is a useful tool for optimization problems[3]-[6].

The original PSO is described as

$$\boldsymbol{v}_j^{t+1} = w\boldsymbol{v}_j^t + c_1 r_1(\boldsymbol{pbest}_j^t - \boldsymbol{x}_j^t) + c_2 r_2(\boldsymbol{gbest}^t - \boldsymbol{x}_j^t) \quad (1)$$

$$\boldsymbol{x}_j^{t+1} = \boldsymbol{x}_j^t + \boldsymbol{v}_j^{t+1} \quad (2)$$

where $w \geq 0$ is an inertia weight coefficient, $c_1 \geq 0$, and $c_2 \geq 0$ are acceleration coefficients, and $r_1 \in [0, 1]$, and $r_2 \in [0, 1]$ are two separately generated uniformly distributed random numbers in the range $[0, 1]$. $\boldsymbol{x}_j^t \in \Re^N$ denotes the location of the j-th particle on the t-th iteration in the N-dimensional space, and $\boldsymbol{v}_j^t \in \Re^N$ denotes a velocity vector of the j-th particle on the t-th iteration. $\boldsymbol{pbest}_j^t \in \Re^N$ means the location that gives the best value of the evaluation function of the j-th particle on the t-th iteration. $\boldsymbol{gbest}^t \in \Re^N$ means the location which gives the best value of the evaluation function on the t-th iteration in the swarm.

The particles in the swarm fly through the N-dimensional space according with Eqs. (1) and (2). Each particle shares information of a current optimal value of the evaluation function and its corresponding location of the best particle. Also, each particle memorizes its record of the best evaluation value and its best location. On the basis of such information, the moving direction and velocity are calculated by Eq. (1). Namely, all particles will move toward a coordinate that gives the current best value of the evaluation function.

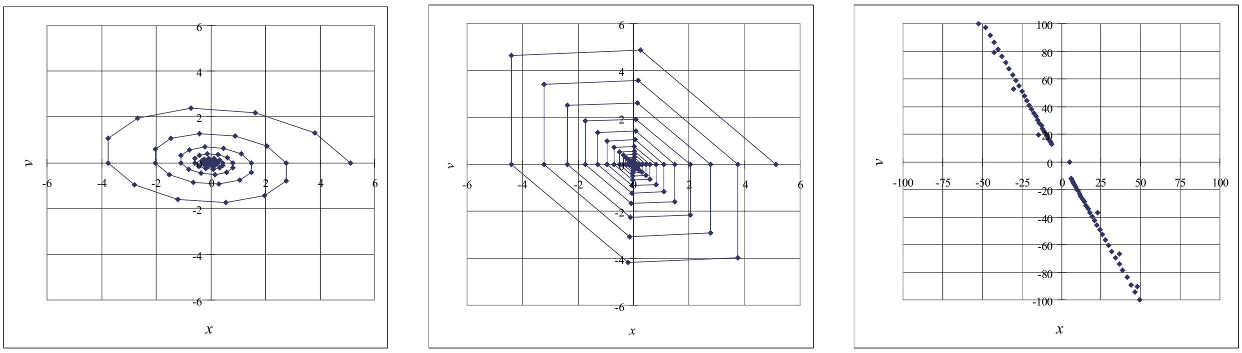
In such PSO system, the parameters play very important role to the searching ability. Therefore, many researchers study about adequate parameters selecting[7]. The searching ability of such PSO depends on the inertia weight coefficient, and the acceleration coefficients. Since the acceleration coefficients are multiplied by a random number, the system can be regarded as a stochastic system. The rigorous analysis of such stochastic system is difficult. In order to analyze the dynamics of such PSO, M. Clerc, and J. Kennedy proposed a simple deterministic PSO system, and analyzed its dynamics theoretically[8]. The simple deterministic PSO system does not contain stochastic factors, namely, the random coefficients have been omitted from the original PSO system. The analysis of such a deterministic PSO is very important for determining the effective parameters of the standard PSO[8]-[9]. We proposed a deterministic PSO and analyzed the searching ability of the optimal value of the given benchmark functions[10]. According to the results, the dynamics depends on the eigenvalues of the system[10]. The eigenvalues depend on the inertia weight coefficient, and the acceleration coefficients. In order to control the eigenvalues simply, we propose a canonical deterministic PSO.

2. Canonical deterministic PSO

The simplicity acceleration coefficients of the deterministic PSO system can be described as

$$\boldsymbol{p}_j^t = \gamma\boldsymbol{pbest}_j^t + (1-\gamma)\boldsymbol{gbest}^t \quad (3)$$

$$\gamma = \frac{c_1}{c_1 + c_2} \quad (4)$$



(a) $\Delta = 0.95$, $\theta = 30$ [deg] $w = 0.9025$, $c = 0.2571$

(b) $\Delta = 0.95$, $\theta = 60$ [deg] $w = 0.9025$, $c = 0.9525$

(c) $\Delta = 0.95$, $\theta = 180$ [deg] $w = 0.9025$, $c = 3.8025$

Figure 1: The trajectory of the deterministic PSO in the phase space $\boldsymbol{y} - \boldsymbol{v}$.

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where $\boldsymbol{p}_j^t$ can be regarded as a desired fixed point.

The parameter γ controls the mixture rate of the local best and the global best.

Since each dimension variable of the particle is independent, we can consider one dimensional case without loss of generality. Therefore, we consider one dimensional system hereafter. For one dimensional deterministic PSO can be transformed into the following matrix form:

$$\begin{bmatrix} v_j^{t+1} \\ y_j^{t+1} \end{bmatrix} = \begin{bmatrix} w & -c \\ w & 1-c \end{bmatrix} \begin{bmatrix} v_j^t \\ y_j^t \end{bmatrix} \tag{5}$$

where $y_j^t = x_j^t - p_j^t$ and $c = c_1 + c_2$.

Note that this system does not contain stochastic factors, therefore, this system can be regarded as a deterministic system.

The dynamics of the deterministic PSO is governed by the eigenvalues of the matrix in Eq. (5). The eigenvalue λ is given as

$$\lambda = \frac{1-c+w}{2} \pm \frac{\sqrt{(1-c+w)^2 - 4w}}{2}. \tag{6}$$

If the eigenvalue λ is complex conjugate number, the system exhibits remarkable searching ability. Since the system is discrete-time system, the damping factor Δ and the rotation angle θ on each iteration can be derived by its complex eigenvalues as

$$\Delta = \sqrt{w} \tag{7}$$

$$\theta = \arctan \frac{\sqrt{4w - (1-c+w)^2}}{1-c+w} \tag{8}$$

The trajectory in the phase space exhibits spiral motion as shown in Fig. 1 whose desired fixed point is the origin is assumed. In the cases of Fig. 1(c), the range of the variable x becomes wide, and the system can not search around the origin.

Based on the eigenvalue λ which is expressed in Eq. (6), we can calculate a translation matrix $\boldsymbol{P}$. By using this translation matrix $\boldsymbol{P}$, we derive the following matrix.

$$\begin{bmatrix} \delta & -\omega \\ \omega & \delta \end{bmatrix} = \boldsymbol{P}^{-1} \begin{bmatrix} w & -c \\ w & 1-c \end{bmatrix} \boldsymbol{P} \tag{9}$$

where,

$$\boldsymbol{P} = \sqrt{\frac{2}{\sqrt{4w-(1-c+w)^2}}} \begin{bmatrix} 1 & 0 \\ \frac{w+c-1}{2} & \frac{\sqrt{4w-(1-c+w)^2}}{2c} \end{bmatrix}$$

Here, we consider the following corrdinate transformation.

$$\begin{bmatrix} \tilde{v}_j^t \\ \tilde{y}_j^t \end{bmatrix} = \boldsymbol{P}^{-1} \begin{bmatrix} v_j^t \\ y_j^t \end{bmatrix} \tag{10}$$

By using this coordinate transformation of Eq. (10), we can derive the following canonical form:

$$\begin{bmatrix} \tilde{v}_j^{t+1} \\ \tilde{y}_j^{t+1} \end{bmatrix} = \begin{bmatrix} \delta & -\omega \\ \omega & \delta \end{bmatrix} \begin{bmatrix} \tilde{v}_j^t \\ \tilde{y}_j^t \end{bmatrix} \tag{11}$$

where $\tilde{y}_j^t = \tilde{x}_j^t - \tilde{p}_j^t$.

The new coordinate $(\tilde{\boldsymbol{y}}, \tilde{\boldsymbol{v}})$ can be regarded as a normalized coordinate. Each dimension component in $\tilde{\boldsymbol{v}}$ and $\tilde{\boldsymbol{y}}$ is independent, therefore The behavior of the system is governed by the eigenvalues of the canonical form of Eq. (11). The eigenvalues λ of the system is derived as

$$\lambda = \delta + j\omega \tag{12}$$

The system of (11) is a discrete-time system. For the system to become stable, the eigenvalues must exist within the unit circle on the complex plane. Therefore, the system is said to be stable if and only if the following condition is satisfied.

$$\delta^2 + \omega^2 < 1 \tag{13}$$

If the parameters satisfy above condition, the eigenvalues exist within the unit circle. The particle converges to a fixed point p_j. In generally, the fixed point p_j is varied with time steps. Therefore, the trajectory exhibits a complex motion.

The damping factor Δ is derived as

$$\Delta = \sqrt{\delta^2 + \omega^2} \quad (14)$$

Note that if the parameters satisfy the condition (13), the damping factor Δ satisfy the following.

$$|\Delta| < 1 \quad (15)$$

The rotational angle θ on each iteration is given as

$$\theta = \arctan \frac{\omega}{\delta} \quad (16)$$

By using the damping factor and the rotation angle, Eq. (11) can be transformed into the following.

$$\begin{bmatrix} v_j^{t+1} \\ y_j^{t+1} \end{bmatrix} = \Delta \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_j^t \\ y_j^t \end{bmatrix} \quad (17)$$

The trajectory of the canonical deterministic PSO as shown in Fig. 2 whose desired fixed point is the origin is assumed. Comparing the case of Fig. 1(c), the trajectory of Fig. 2(c) does not expand. We think this property gives an effective influence for searching. In the following section, we confirm this fact by some numerical experiments by using some benchmark functions.

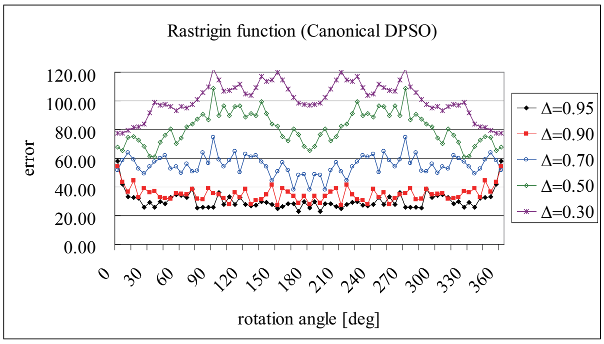
3. Simulation

In this section, we investigate the relation between the eigenvalues and the searching ability, we carry out some numerical simulations. We use two well-known benchmark problems. Each objective function consists of 10-dimensional variables. For each simulation, the population size of the swarm is 10.

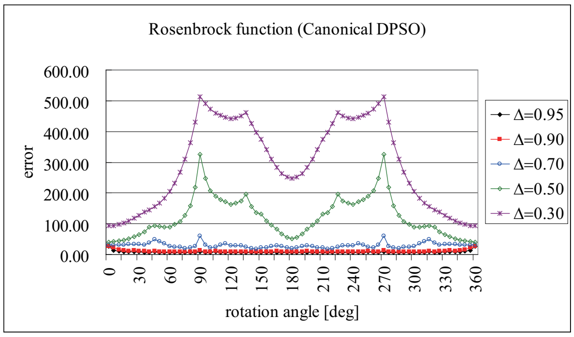
The parameter γ controls the mixture rate between the local best and the global best. $\gamma = 1.0$ denotes that the system uses only the local best information, and $\gamma = 0.0$ means that it uses only the global best information. The previous simulation results indicate the information of the global best is important[11]. We apply $\gamma = 0.0$ for the numerical simulation hereafter.

We confirm the relation between the rotation angle and the convergence property. First, we observe the convergence property when the rotation angle θ is varied from 0-degree to 180-degree in a period 10-degree. The simulation results are shown in Fig. 3 The vertical axis denotes the mean error from the optimal value with the searched value, the horizontal axis denotes the rotation angle θ. Each curve corresponds to each damping factor; $\Delta = 0.95, 0.90, 0.70, 0.50, 0.30$. Each data is the average of the experimental results with ten trials.

These results indicate that the characteristic of the convergence properties are depended on the damping factor. As the damping factor changes to large, the characteristic of the convergence property is changed. According to



(a) Rastrigin function



(b) Rosenbrock function

Figure 3: Canonical Deterministic PSO: The rotation angle - mean error characteristic (dimension:10, particles:10, trials:10)

the numerical simulation, when the damping factor sets as $\Delta = 0.95$, the canonical deterministic PSO exhibits the most effective performance.

The result of the deterministic PSO is shown in Fig. 4 to compare with the result of the canonical deterministic PSO. In the case of the deterministic PSO, the decline of the performance is observed around the 180 degree in the rotation angle comparing with the case of the canonical deterministic PSO. Therefore, we can say the proposed canonical deterministic PSO exhibits better performance.

4. Conclusions

In this article, we analyzed the convergence performance of the canonical deterministic PSO system. The canonical deterministic PSO system does not contain the stochastic factor. We confirm the relation between the eigenvalue and the convergence property by using the damping factor and the rotation angles. The results suggest these parameters have the optimal value. On the basis of this result, we will construct an effective stochastic PSO system.

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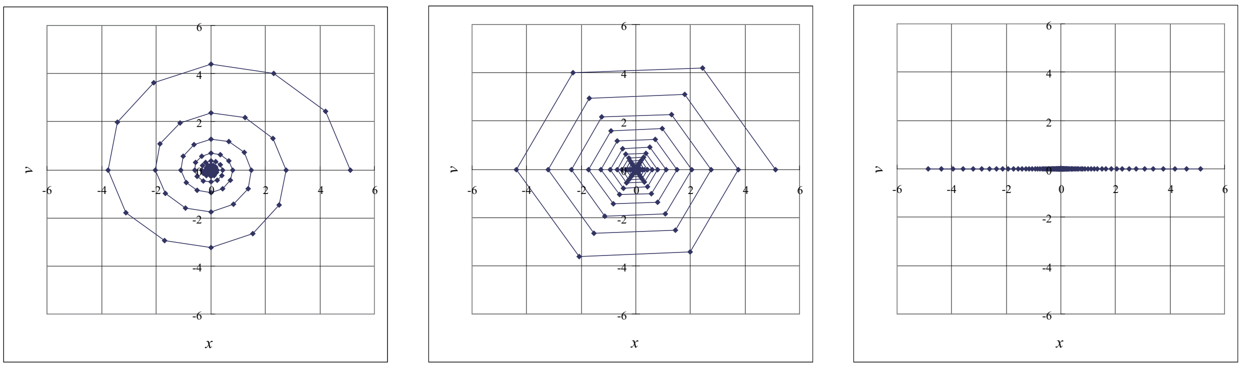

(a) $\Delta = 0.95, \theta = 30$ [deg] $\delta = 0.8227, \omega = 0.4750$

(b) $\Delta = 0.95, \theta = 60$ [deg] $\delta = 0.4750, \omega = 0.8227$

(c) $\Delta = 0.95, \theta = 180$ [deg] $\delta = -0.9500, \omega = 0.0000$

Figure 2: The trajectory of the canonical deterministic PSO in the phase space $\tilde{\boldsymbol{y}} - \tilde{\boldsymbol{v}}$.

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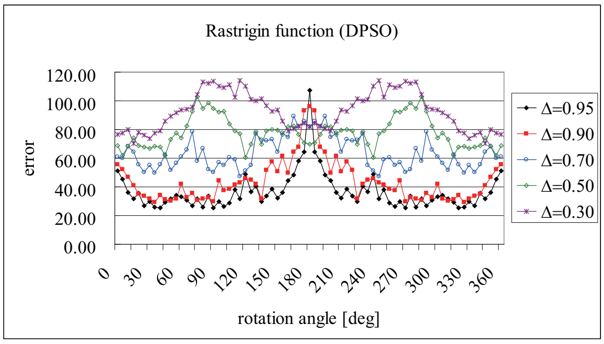
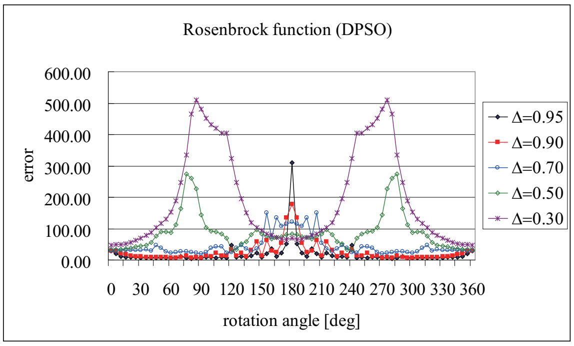


(a) Rastrigin function



(b) Rosenbrock function

Figure 4: Deterministic PSO: The rotation angle - mean error characteristic (dimension:10, particles:10, trials:10)