Call Blocking Probabilities in an Erlang Multirate Loss Model under a State-dependent Threshold Policy

I. D. Moscholios

Dept. of Informatics & Telecommunications, University of Peloponnese, 221 00 Tripolis, Greece.
E-mail: idm@uop.gr

Abstract– **We propose a state-dependent bandwidth sharing policy, based on the threshold (TH) policy, for a single link that behaves as a loss system and accommodates random traffic originated from different service-classes. If the number of in-service calls of a service-class exceeds a threshold (dedicated to the service-class), a new arriving call of the same service-class is accepted in the system with a predefined state-dependent probability. The proposed model has a Product Form Solution (PFS) for the steady state probabilities. Thanks to the PFS, a convolution algorithm is proposed for the accurate calculation of call blocking probabilities and link utilization.**

I. Introduction

Although link bandwidth capacity is increasing as broadband networks become economically viable, QoS assurance is not unnecessary. Instead, a QoS mechanism is essential to give access to the necessary bandwidth needed by the services. Considering call-level traffic in a single link which accommodates different service-classes with different QoS requirements, such a mechanism is a bandwidth sharing policy, since it affects call-level performance measures. The QoS assessment of service systems under a bandwidth sharing policy is accomplished through teletraffic models. The simplest bandwidth allocation policy is the Complete Sharing (CS) policy, where a new call is accepted in the system simply if the call's bandwidth is available [1]. The CS policy cannot guarantee a certain QoS to a service-class, while it is unfair to service-classes with higher bandwidth per call requirements, because it leads to higher Call Blocking Probabilities (CBP). This motivates research on other policies, such as the Bandwidth Reservation (BR) policy (see e.g., [2]-[13]) and the Threshold (TH) policy.

We concentrate on the TH policy, because it is broadly applicable in wired ([14]-[20]), wireless ([21]-[22]) and satellite networks ([23]-[24]) and is an attractive policy in contemporary access networks of a tree structure. The latter consists of a number of access links followed by a common link, like the Passive Optical Networks. Under the TH policy, in order for congestion to be avoided, the number of in-service calls of a specific service-class must not exceed a pre-defined threshold (dedicated to service-class), after the acceptance of a new call of this service-class. We propose a probabilistic TH policy (PrTH) for a single link that accommodates Poisson arriving calls of different service-classes. In this policy, call acceptance above a threshold is permitted with a predefined probability (dependent on both the service-class and system state), if bandwidth is available. The proposed loss system can be described analytically by a continuous time Markov chain which is reversible, resulting in a Product Form Solution (PFS) for the steady state distribution. Thanks to the existence of PFS, we determine accurately the CBP and the link utilization based on a convolution algorithm [25]. Compared to [14], where the ordinary TH policy is studied in a single link under random multirate traffic the differences are the following: a) the proposed PrTH policy covers the TH policy of [14], b) in the proposed PrTH policy, the calculation of the link occupancy distribution can be based only on a convolution algorithm. This is because the probabilities in the PrTH policy depend on the number of in-service calls in the link.

This paper is organized as follows: In Section II, we present the proposed policy (PrTH), show the PFS and provide a convolution algorithm for the calculation of CBP, mean number of calls of each service-class and link utilization. In Section III, we present analytical CBP results both for the proposed model and the models of [26] (CS policy), [14] (ordinary TH policy), for evaluation. We conclude in Section IV. In the Appendix, we provide a tutorial example in order to show the necessary CBP calculations when we apply the PFS or the convolution algorithm.

II. The Proposed Model

Consider a link of capacity C bandwidth units (b.u.) that accommodates Poisson arriving calls of K service-classes under the PrTH policy. A call of service class k (k =1,…,K) has an arrival rate λ_k and requests b_k b.u. If these b_k b.u. are not available in the link then the call is blocked and lost without further affecting the system; otherwise:

a) If the number n_k of in-service calls of service-class k in the steady state plus the new arriving call, does not exceed a predefined threshold n_k^*, i.e., $n_k + 1 \le n_k^*$, then the call is accepted in the link.

b) If $n_k + 1 > n_k^*$, the call is accepted in the system with probability $p_k(n_k)$ or blocked with probability 1- $p_k(n_k)$. The set of probabilities $p_k(n_k)$ constitutes the vector:

$$\boldsymbol{p_k} = (p_k(0), p_k(1)..., p_k(n_k^*),..., p_k(\lfloor C/b_k \rfloor -1), p_k(\lfloor C/b_k \rfloor)) \quad (1)$$

where $\lfloor C/b_k \rfloor$ is the maximum number of service-class k calls that the system can service.

In (1), we assume that:

a) $p_k(0) = p_k(1) = ...p_k(n_k^* - 1) = 1$, i.e., a service-class k call is always accepted if the threshold n_k^* is not exceeded,

2) the probabilities $p_k(n_k^*),..., p_k(\lfloor C/b_k \rfloor - 1)$ may be different. In the ordinary TH policy [14], these probabilities are zero. In the PrTH policy, they can be set either all positive, or zero after a certain number greater than n_k^*, and

3) $p_k(\lfloor C/b_k \rfloor) = 0$ obviously, due to lack of available link bandwidth.

An accepted call remains in the system for a generally distributed service time with mean μ_k^{-1}. The total offered traffic-load of service-class k calls is $a_k = \lambda_k \mu_k^{-1}$ (in erl).

Let the steady state vectors $\boldsymbol{n}=(n_1,...,n_{k-1},n_k,n_{k+1},...,n_K)$ and $\boldsymbol{n}_k^- = (n_1,...,n_{k-1},n_k - 1,n_{k+1},...,n_K)$, $\boldsymbol{n}_k^+ = (n_1,...,n_{k-1},n_k + 1,n_{k+1},...,n_K)$. The Global Balance (GB) equation for state $\boldsymbol{n}$ of the proposed multirate loss model, expressed as (*rate into state* $\boldsymbol{n}$) = (*rate out of state* $\boldsymbol{n}$), is expressed as:

$$\sum_{k=1}^{K} \lambda_k \delta_k^-(\boldsymbol{n}) p_k(n_k - 1) P(\boldsymbol{n}_k^-) + \sum_{k=1}^{K} (n_k + 1) \mu_k \delta_k^+(\boldsymbol{n}) P(\boldsymbol{n}_k^+) =$$
$$\sum_{k=1}^{K} \lambda_k \delta_k^+(\boldsymbol{n}) p_k(n_k) P(\boldsymbol{n}) + \sum_{k=1}^{K} n_k \mu_k \delta_k^-(\boldsymbol{n}) P(\boldsymbol{n}) \quad (2)$$

where:

$$\delta_k^+(\boldsymbol{n}) = \begin{cases} 1 & \text{if } \boldsymbol{n}_k^+ \in \boldsymbol{\Omega} \\ 0 & \text{otherwise} \end{cases}, \quad \delta_k^-(\boldsymbol{n}) = \begin{cases} 1 & \text{if } \boldsymbol{n}_k^- \in \boldsymbol{\Omega} \\ 0 & \text{otherwise} \end{cases},$$

$\boldsymbol{\Omega}$ is the state space of the system denoted by $\boldsymbol{\Omega}=\{\boldsymbol{n}: 0 \le \boldsymbol{nb} \le C, k=1,\dots,K\}$, $\boldsymbol{nb} = \sum_{k=1}^{K} n_k b_k$, $\boldsymbol{b} = (b_1,...,b_K)$

and $P(\boldsymbol{n}), P(\boldsymbol{n}_k^-), P(\boldsymbol{n}_k^+)$ are the probability distributions of states $\boldsymbol{n}, \boldsymbol{n}_k^-, \boldsymbol{n}_k^+$, respectively.

Since the corresponding Markov chain is reversible, Local Balance (LB) exists and the following LB equations are extracted as (*rate up* = *rate down*), for k $=1,\dots,K$ and $\boldsymbol{n} \in \boldsymbol{\Omega}$:

$$\lambda_k \delta_k^-(\boldsymbol{n}) p_k(n_k - 1) P(\boldsymbol{n}_k^-) = n_k \mu_k \delta_k^-(\boldsymbol{n}) P(\boldsymbol{n}) \quad (3)$$

$$\lambda_k \delta_k^+(\boldsymbol{n}) p_k(n_k) P(\boldsymbol{n}) = (n_k + 1) \mu_k \delta_k^+(\boldsymbol{n}) P(\boldsymbol{n}_k^+) \quad (4)$$

The system of LB equations (3) and (4) is satisfied by the following PFS:

$$P(\boldsymbol{n}) = G^{-1} \left(\prod_{k=1}^{K} \prod_{x=n_k^*}^{n_k - 1} p_k(x) \frac{a_k^{n_k}}{n_k!} \right) \quad (5)$$

where G is the normalization constant determined by:

$$G \equiv G(\boldsymbol{\Omega}) = \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}} \left(\prod_{k=1}^{K} \prod_{x=n_k^*}^{n_k - 1} p_k(x) \frac{a_k^{n_k}}{n_k!} \right).$$

To calculate the CBP B_k of service-class k, we define the state space $\boldsymbol{\Omega}_k = \{\boldsymbol{n}: 0 \le \boldsymbol{nb} \le C - b_k, k=1,\dots,K\}$ which denotes the set of states for which a new service-class k call will be definitely accepted in the system or accepted with a state-dependent probability. Thus:

$$B_k = 1 - \frac{G_k}{G} \quad (6)$$

where $G_k = \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}_k} p_k(n_k) P(\boldsymbol{n})$.

The computational complexity of (6) is in the order of $O(C^K)$ for the CS policy. To reduce it in the order of $O(KC^2)$, we exploit the PFS and the principle of independency among service-classes (i.e., multiplication of individual state probabilities), and present the following 3-step convolution algorithm for the calculation of CBP and link utilization, by modifying the convolution algorithm of [25]:

Let j be the occupied link bandwidth, $j = 0, 1,\dots,C$.

Step 1) Determine the occupancy distribution $q_k(j)$ of each service-class k ($k=1,\dots,K$), assuming that only service-class k exists in the link:

$$q_k(j) = \left\langle \begin{array}{ll} q_k(0) \dfrac{a_k^i}{i!} & , for\ 1 \le i \le n_k^*\ and\ j = ib_k \\ q_k(0) \dfrac{\prod_{x=n_k^*}^{i-1} p_k(x) a_k^i}{i!} & , for\ n_k^* < i \le \lfloor C/b_k \rfloor and\ j = ib_k \\ 0 & , otherwise \end{array} \right\rangle \quad (7)$$

Step 2) Determine the aggregated occupancy distribution $Q_{(-k)}$ based on the successive convolution of all service-classes apart from service-class k:

$$Q_{(-k)} = q_1 * q_2 * \dots * q_{k-1} * q_{k+1} * \dots * q_K \quad (8)$$

Note that the term "successive" means that initially q_1 and q_2 are convolved to give q_{12}, then q_{12} with q_3 are convolved to give q_{123}, and so on. The convolution operation between two service-classes k and r is determined as:

$$q_k * q_r = \left\{ q_k(0) q_r(0), \sum_{x=0}^{1} q_k(x) q_r(1-x), ..., \sum_{x=0}^{C} q_k(x) q_r(C-x) \right\} \quad (9)$$

Step 3) To calculate the CBP of service-class k, determine the convolution of $Q_{(-k)}$ (step 2) with q_k as follows:

$$Q_{(-k)} * q_k = \left\{ Q_{(-k)}(0) q_k(0), \sum_{x=0}^{1} Q_{(-k)}(x) q_k(1-x), ..., \sum_{x=0}^{C} Q_{(-k)}(x) q_k(C-x) \right\} \quad (10)$$

Normalizing the values of (10), we obtain the occupancy distribution $q(j)$, $j=0,1,\dots,C$ via the formulas:

$$q(0) = Q_{(-k)}(0) q_k(0) / G$$
$$q(j) = \left(\sum_{x=0}^{j} Q_{(-k)}(x) q_k(j-x) \right) / G,\ j = 1, ..., C \tag{11}$$

Having determined $q(j)$'s, we propose the following formula for the calculation of CBP of service-class k:

$$B_k = \sum_{j=C-b_k+1}^{C} q(j) + \sum_{x=n_k^* b_k}^{C-b_k} (1-p_k(x)) q_k(x) \sum_{y=x}^{C-b_k} Q_{(-k)}(C-b_k-y) \tag{12}$$

The first term of the right hand side of (12) refers to states j where there is no bandwidth available for service-class k calls. The second term refers to the case that there is available link bandwidth but nevertheless call blocking occurs; this happens in states $x = n_k^* b_k, ..., C-b_k$ (with probability 1-$p_k(x)$), when $n_k \geq n_k^*$.

The link utilization U (in b.u.) is given by the formula:

$$U = \sum_{j=1}^{C} j q(j) \tag{13}$$

where the values of $q(j)$ are given by (11).

III. Evaluation

In this section we present analytical results for an application example. Simulation results are not presented since the analytical results of the proposed model are accurate (due to the existence of PFS).

We consider a link of capacity C = 60 b.u., that accommodates calls of K = 2 service-classes, with the traffic characteristics shown in Table I:

Table I: Traffic parameters of both service-classes.

Service-class	Traffic-load (erl)	Bandwidth per call (b.u.)	Threshold
1st	$a_1 = 5.0$	$b_1 = 2$ b.u.	$n_1^* = 20$
2nd	$a_2 = 2.0$	$b_2 = 7$ b.u.	$n_2^* = 5$

We provide analytical results of CBP for the proposed model considering two scenarios: (a) Calls of the 1st service-class behave as in the TH policy of [14], i.e., $p_1(20) = p_1(21) = ... = p_1(30) = 0$, while calls of the 2nd service-class are accepted in the system with probability $p_2(5) = p_2(6) = p_2(7) = 0.5$, and $p_2(8) = 0$, (b) Calls of the 1st service-class are accepted with probability $p_1(20) = p_1(21) = ... = p_1(29) = 0.3$, and $p_1(30) = 0$, while calls of the 2nd service-class are accepted with probability $p_2(5) = p_2(6) = p_2(7) = 0.2$, and $p_2(8) = 0$. The results are compared with the CBP results under the CS and TH policy of [26] and [14], respectively (Figs. 1, 2). In the x-axis of Figs 1-2, the offered traffic-load of the 1st and 2nd service-class increases in steps of 0.5, 0.25 erl, respectively. So, point 1 refers to: (a_1, a_2) = (5.0, 2.0) while point 9 to: (α_1, α_2) = (9.0, 4.0).

Based on Figs. 1-2 we observe that: a) The PrTH policy clearly affects the blocking probabilities of both service-classes. Thus, it gives the opportunity for a fine call congestion control aiming at guaranteeing certain QoS to each service-class, in intermediate QoS levels between the CS policy and the pure TH policy. (b) The existing models fail to approximate the results obtained from the proposed model; this fact reveals the necessity of the new model.

V. Conclusion

We propose a teletraffic multirate loss model for a single link that accommodates random traffic under a state-dependent threshold-based bandwidth sharing policy. The link is analysed as a multirate loss system, via a reversible continuous-time Markov chain, which leads to a PFS for the steady state distribution. Based on the PFS, various performance measures can be accurately determined via a convolution algorithm. Comparison against other models under the CS or the TH policy, reveals the necessity of the new model.

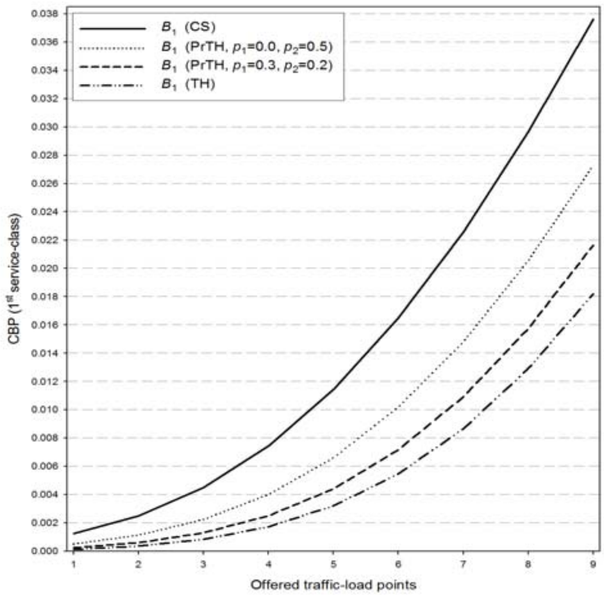

Fig. 1. CBP – 1st service-class.

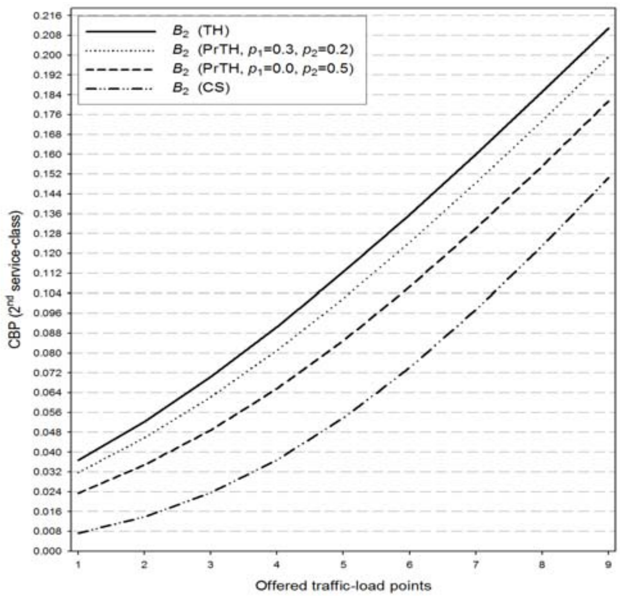

Fig. 2. CBP – 2nd service-class.

APPENDIX. TUTORIAL EXAMPLE

Consider a link of fixed capacity of C=4 b.u. that accommodates calls of K = 4 service-classes under the PrTH policy. An arriving call of service class k (k =1,…,4) follows a Poisson process with arrival rate λ_k and requests b_k b.u. We assume that $\boldsymbol{b}$ = (b_1, b_2, b_3, b_4) =(1, 2, 1, 2). Also, let: $a_1 = a_2 = a_3 = a_4 = 1.0$ erl .

Let $n_1^* = 2, n_2^* = 1, n_3^* = 3, n_4^* = 1$ be the corresponding threshold values for each service-class. The corresponding probability vectors are the following:

$$\boldsymbol{p}_1 = \left(p_1(0), p_1(1), p_1(2), p_1(3), p_1(4)\right) = \left(1, 1, 0.4, 0.4, 0\right)$$

$$\boldsymbol{p}_2 = \left(p_2(0), p_2(1), p_2(2)\right) = \left(1, 0.4, 0\right)$$

$$\boldsymbol{p}_3 = \left(p_3(0), p_3(1), p_3(2), p_3(3), p_3(4)\right) = \left(1, 1, 1, 0.95, 0\right)$$

$$\boldsymbol{p}_4 = \left(p_4(0), p_4(1), p_4(2)\right) = \left(1, 0.95, 0\right)$$

The state space $\boldsymbol{\Omega}$ of this small system consists of 30 states of the form $\boldsymbol{n}$ = (n_1, n_2, n_3, n_4) which are presented in the Table A.1, together with the corresponding values of the occupied bandwidth $j = \boldsymbol{nb}$ and the blocking states (columns 6-9) of each service-class. The symbol # refers to call blocking with a probability p due to the PrTH policy. The symbol * refers to call blocking due to lack of available link bandwidth.

Table A.1: State space (n_1, n_2, n_3, n_4), occupied bandwidth j and blocking states (B_1, B_2, B_3, B_4).

n_1	n_2	n_3	n_4	j	B_1	B_2	B_3	B_4
0	0	0	0	0				
0	0	0	1	2				#
0	0	0	2	4	*	*	*	*
0	0	1	0	1				
0	0	1	1	3		*		*
0	0	2	0	2				
0	0	2	1	4	*	*	*	*
0	0	3	0	3		*	#	*
0	0	4	0	4	*	*	*	*
0	1	0	0	2		#		
0	1	0	1	4	*	*	*	*
0	1	1	0	3		*		*
0	1	2	0	4	*	*	*	*
0	2	0	0	4	*	*	*	*
1	0	0	0	1				
1	0	0	1	3		*		*
1	0	1	0	2				
1	0	1	1	4	*	*	*	*
1	0	2	0	3		*		*
1	0	3	0	4	*	*	*	*
1	1	0	0	3		*		*
1	1	1	0	4	*	*	*	*
2	0	0	0	2	#			
2	0	0	1	4	*	*	*	*
2	0	1	0	3	#	*		*
2	0	2	0	4	*	*	*	*
2	1	0	0	4	*	*	*	*
3	0	0	0	3	#	*		*
3	0	1	0	4	*	*	*	*
4	0	0	0	4	*	*	*	*

In what follows we show the necessary CBP calculations when we consider: a) the PFS and b) the convolution algorithm.

Case a: CBP calculations based on the PFS

We initially define the state spaces:

$\boldsymbol{\Omega}_1$={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ C-b_1}={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ 3}={(0,0,0,0), (0,0,0,1), (0,0,1,0), (0,0,1,1), (0,0,2,0), (0,0,3,0), (0,1,0,0), (0,1,1,0), (1,0,0,0), (1,0,0,1), (1,0,1,0), (1,0,2,0), (1,1,0,0), (2,0,0,0), (2,0,1,0), (3,0,0,0)}

$\boldsymbol{\Omega}_2$={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ C-b_2}={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ 2}={(0,0,0,0), (0,0,0,1), (0,0,1,0), (0,0,2,0), (0,1,0,0), (1,0,0,0), (1,0,1,0), (2,0,0,0)}

$\boldsymbol{\Omega}_3$={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ C-b_3}={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ 3}={(0,0,0,0), (0,0,0,1), (0,0,1,0), (0,0,1,1), (0,0,2,0), (0,0,3,0), (0,1,0,0), (0,1,1,0), (1,0,0,0), (1,0,0,1), (1,0,1,0), (1,0,2,0), (1,1,0,0), (2,0,0,0), (2,0,1,0), (3,0,0,0)}

$\boldsymbol{\Omega}_4$={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ C-b_4}={$\boldsymbol{n}$:0 ≤ $\boldsymbol{nb}$ ≤ 2}={(0,0,0,0), (0,0,0,1), (0,0,1,0), (0,0,2,0), (0,1,0,0), (1,0,0,0), (1,0,1,0), (2,0,0,0)}

Then, based on the formulas, for k=1, …, 4:

$$B_k = 1 - \frac{G_k}{G}$$

where:

$$G_k = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_k} p_k(n_k) P(\boldsymbol{n}) \,,\; G \equiv G(\boldsymbol{\Omega}) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}} \left(\prod_{k=1}^{K} \prod_{x=n_k^*}^{n_k-1} p_k(x) \frac{a_k^{n_k}}{n_k\,!} \right)$$

and $P(\boldsymbol{n}) = G^{-1} \left(\prod_{k=1}^{K} \prod_{x=n_k^*}^{n_k-1} p_k(x) \frac{a_k^{n_k}}{n_k\,!} \right)$

we have, for $a_1 = a_2 = a_3 = a_4 = 1.0$ erl :

B_1 = 0.37122, B_2 = 0.65289, B_3 = 0.33696 and B_4 = 0.62306.

Note that the previous method is quite complex especially for systems of large capacity with many service-classes, since it requires enumeration and processing of the whole state space $\boldsymbol{\Omega}$.

Case b: CBP calculations based on the convolution algorithm

Step 1:

- Determination of $q_1(j)$, j = 1, ..., C

Since only the relative values of $q_1(j)$'s are important, we may choose $q_1(0)$ = 1 and calculate the remaining $q_1(j)$'s relative to $q_1(0)$:

$$j = 1 \rightarrow q_1(1) = \frac{q_1(0) \times a_1^1}{1!} \Rightarrow q_1(1) = 1$$

$$j = 2 \rightarrow q_1(2) = \frac{q_1(0) \times a_1^2}{2!} \Rightarrow q_1(2) = 0.5$$

$$j = 3 \rightarrow q_1(3) = \frac{q_1(0) \times p_1(2) \times a_1^3}{3!} \Rightarrow q_1(3) = 0.4/6$$

$$j = 4 \rightarrow q_1(4) = \frac{q_1(0) \times p_1(2) \times p_1(3) \times a_1^4}{4!} \Rightarrow$$

$$q_1(4) = 0.16/24$$

If necessary, we can avoid numerical problems (that may arise in large examples) by normalizing the values of

$q_1(j)$'s with $\sum_{j=0}^{\lfloor C/b_1 \rfloor} q_1(j)$. For this particular example we have $\sum_{j=0}^{\lfloor C/b_1 \rfloor} q_1(j) = 2.573333$ and the normalized values of $q_1(j)$'s are:

$$q_1(0) = q_1(1) = 0.388601, q_1(2) = 0.1943005,$$
$$q_1(3) = 0.02590673,\ q_1(4) = 0.002590673$$

- Determination of $q_2(j)$, $q_3(j)$ and $q_4(j)$

Similarly, the normalized values of $q_2(j)$'s, $q_3(j)$'s and $q_4(j)$'s are the following:

$$q_2(0) = q_2(2) = 0.4545454, q_2(4) = 0.090909$$
$$q_3(0) = q_3(1) = 0.369515, q_3(2) = 0.1847575,$$
$$q_3(3) = 0.0615858,\ q_3(4) = 0.0146266$$
$$q_4(0) = q_4(2) = 0.4040404,\ q_4(4) = 0.1919192$$

Step 2:

Determine all $Q_{(-k)}$ for k = 1, …, 4. Herein, we present only the calculations of $Q_{(-4)}$. In a similar way we can obtain $Q_{(-1)}$, $Q_{(-2)}$ and $Q_{(-3)}$.

- We start by convolving $q_1(j)$'s and $q_2(j)$'s in order to obtain $q_{12}(j)$'s.

$$j = 0 \to q_{12}(0) = q_1(0) * q_2(0) = 0.176636797$$
$$j = 1 \to q_{12}(1) = \sum_{x=0}^{1} q_1(x) * q_2(1-x) = 0.176636797$$
$$j = 2 \to q_{12}(2) = \sum_{x=0}^{2} q_1(x) * q_2(2-x) = 0.264955195$$
$$j = 3 \to q_{12}(3) = \sum_{x=0}^{3} q_1(x) * q_2(3-x) = 0.188412581$$
$$j = 4 \to q_{12}(4) = \sum_{x=0}^{4} q_1(x) * q_2(4-x) = 0.124823304$$

Since $\sum_{j=0}^{C} q_{12}(j) = 0.9314647$, the normalized values of $q_{12}(j)$'s are:

$$q_{12}(0) = q_{12}(1) = 0.18963338,\ q_{12}(2) = 0.28445006,$$
$$q_{12}(3) = 0.2022756,\ q_{12}(4) = 0.13400755$$

- We continue by convolving $q_{12}(j)$'s and $q_3(j)$'s in order to obtain $q_{123}(j) = Q_{(-4)}(j)$.

$$j = 0 \to q_{123}(0) = q_{12}(0) * q_3(0) = 0.070072378$$
$$j = 1 \to q_{123}(1) = \sum_{x=0}^{1} q_{12}(x) * q_3(1-x) = 0.140144756$$
$$j = 2 \to q_{123}(2) = \sum_{x=0}^{2} q_{12}(x) * q_3(2-x) = 0.21021713$$
$$j = 3 \to q_{123}(3) = \sum_{x=0}^{3} q_{12}(x) * q_3(3-x) = 0.226567343$$
$$j = 4 \to q_{123}(4) = \sum_{x=0}^{4} q_{12}(x) * q_3(4-x) = 0.191268361$$

Since $\sum_{j=0}^{C} q_{123}(j) = 0.83827$, the normalized values of $q_{123}(j)$'s i.e., $Q_{(-4)}(j)$'s are:

$$Q_{(-4)}(0) = q_{123}(0) = 0.08359165,$$
$$Q_{(-4)}(1) = q_{123}(1) = 0.1671833,$$
$$Q_{(-4)}(2) = q_{123}(2) = 0.25077496,$$
$$Q_{(-4)}(3) = q_{123}(3) = 0.2702796,$$
$$Q_{(-4)}(4) = q_{123}(4) = 0.22817$$

Step 3:

Determine $q(j)$'s and consequently the CBP of all service-classes. Herein, we determine $q(j)$'s and B_4 based on the normalized values of $Q_{(-4)}(j)$'s calculated in step 2.

- We start by convolving $q_{123}(j)$'s and $q_4(j)$'s in order to obtain $q(j)$'s.

$$j = 0 \to q(0) = q_{123}(0) * q_4(0) = 0.033774403$$
$$j = 1 \to q(1) = \sum_{x=0}^{1} q_{123}(x) * q_4(1-x) = 0.067548807$$
$$j = 2 \to q(2) = \sum_{x=0}^{2} q_{123}(x) * q_4(2-x) = 0.135097618$$
$$j = 3 \to q(3) = \sum_{x=0}^{3} q_{123}(x) * q_4(3-x) = 0.176752684$$
$$j = 4 \to q(4) = \sum_{x=0}^{4} q_{123}(x) * q_4(4-x) = 0.209555955$$

Since $\sum_{j=0}^{C} q(j) = 0.6227295$, the normalized values of $q(j)$'s are:

$$q(0) = 0.054236,\ q(1) = 0.1084721,\ q(2) = 0.216944,$$
$$q(3) = 0.283835,\ q(4) = 0.336512$$

Note that these values will also be obtained if we follow any another combination of $q_{xyz}(j)$'s and $q_w(j)$'s, e.g., $q_{234}(j)$'s and $q_1(j)$'s.

- Calculation of e.g., B_4 based on $q(j)$'s and $Q_{(-4)}(j)$'s.

$$B_4 = \sum_{j=C-b_4+1}^{C} q(j) + \sum_{x=n_4^* b_4}^{C-b_4} \left[(1 - p_4(x)) q_4(x) \sum_{y=x}^{C-b_4} Q_{(-4)}(C - b_4 - y) \right]$$
$$\Rightarrow B_4 = q(3) + q(4) + (1 - p_4(1)) q_4(2) Q_{(-4)}(0) = 0.62306$$

which is exactly the same value with that obtained by using the PFS.

In a similar way we can calculate: B_1 = 0.37122, B_2 = 0.65289, B_3 = 0.33696.

REFERENCES

1. M. Stasiak, M. Glabowski, A.Wisniewski, and P. Zwierzykowski, Modeling and Dimensioning of Mobile Networks, Wiley, 2011.
2. M. Stasiak, M. Glabowski, "A simple approximation of the link model with reservation by a one-dimensional Markov chain", Performance Evaluation, 41 (2-3), pp. 195-208, July 2000.

3. F. Cruz-Pérez, J. Vázquez-Ávila and L. Ortigoza-Guerrero, "Recurrent formulas for the multiple fractional channel reservation strategy in multi-service mobile cellular networks", IEEE Commun. Letters, 8 (10), pp. 629-631, Oct. 2004.
4. M. Glabowski, "Modelling of state-dependent multirate systems carrying BPP traffic", Annals of Telecommunications, 63 (7), pp. 393-407, August 2008.
5. M. Stasiak, P. Zwierzykowski, D. Parniewicz, "Modelling of the WCDMA Interface in the UMTS Network with Soft Handoff Mechanism", Proc. IEEE Globecom, Honolulu, Nov. 2009.
6. I. Moscholios, J. Vardakas, M. Logothetis and A. Boucouvalas, "A Batched Poisson Multirate Loss Model Supporting Elastic Traffic under the Bandwidth Reservation Policy", Proc. IEEE ICC, Kyoto, Japan, June 2011.
7. I. Moscholios, J. Vardakas, M. Logothetis and A. Boucouvalas, "QoS Guarantee in a Batched Poisson Multirate Loss Model Supporting Elastic and Adaptive Traffic", Proc. IEEE ICC, Ottawa, Canada, June 2012.
8. M. Glabowski, M. Sobieraj and M. Stasiak, "Modelling Limited-availability Systems with Multi-service Sources and Bandwidth Reservation", Proc. 8th AICT, Stuttgart, Germany, June 2012.
9. I. Moscholios, V. Vassilakis, M. Logothetis and M. Koukias, "QoS Equalization in a Multirate Loss Model of Elastic and Adaptive Traffic with Retrials", Proc. 5th EMERGING, Porto, Portugal, October 2013.
10. I. Moscholios, J. Vardakas, M. Logothetis and M. Koukias, "A Quasi-random Multirate Loss Model supporting Elastic and Adaptive Traffic under the Bandwidth Reservation Policy", Int. Journal on Advances in Networks and Services, 6 (3&4), pp. 163-174, December 2013.
11. L. Brewka, V. Iversen and G. Kardaras, "Integrated service resource reservation using queueing networks", IET Networks, 3 (1), pp. 16-21, March 2014.
12. F. Callegati, et al., "Trunk reservation for fair utilization in flexible optical networks", IEEE Commun. Letters, 18 (5), pp. 889-892, May 2014.
13. I. Moscholios, G. Kallos, M. Katsiva, V. Vassilakis and M. Logothetis, "Call Blocking Probabilities in a W-CDMA cell with interference cancellation and bandwidth reservation", Proc. IEICE ICTF, Poznan, Poland, May 2014.
14. D. Tsang and K. Ross, "Algorithms to determine exact blocking probabilities for multirate tree networks", IEEE Trans. Commun., 38 (8), pp. 1266-1271, August 1990.
15. J. Ni, D. Tsang, S. Tatikonda and B. Bensaou, "Optimal and Structured Call Admission Control Policies for Resource-Sharing Systems", IEEE Trans. Commun., 55 (1), pp. 158-170, Jan. 2007.
16. A. Al Daoud, M Alanyali and D. Starobinski, "On equilibrium analysis of acyclic multiclass loss networks under admission control", Operations Research Letters, 39 (6), pp. 406-410, November 2011.
17. I. Moscholios, M. Logothetis, J. Vardakas and A. Boucouvalas, "Performance Metrics of a Multirate Resource Sharing Teletraffic Model with Finite Sources under both the Threshold and Bandwidth Reservation Policies", IET Networks, 4 (3), pp. 195-208, May 2015.
18. I. Moscholios, M. Logothetis, A. Boucouvalas and V. Vassilakis, "An Erlang Multirate Loss Model Supporting Elastic Traffic under the Threshold Policy", Proc. of IEEE ICC 2015, London, U.K., 8-12 June 2015.
19. A. Ali, S. Wei and L. Qian, "Optimal admission and preemption control in finite source loss systems", Operations Research Letters, 43 (3), pp. 241-246, May 2015.
20. I. Moscholios, M. Logothetis, J. Vardakas and A. Boucouvalas, "Congestion Probabilities of Elastic and Adaptive Calls in Erlang-Engset Multirate Loss Models under the Threshold and Bandwidth Reservation Policies", Computer Networks, 92 (1), pp. 1-23, December 2015.
21. A. Al Daoud, M. Alanyali and D. Starobinski, "Pricing strategiesfor spectrum lease in secondary markets", IEEE/ACM Trans.Netw., 18 (2), pp. 462–475, Apr. 2010.
22. X. Y. Yu and H. B. Zhu, "An efficient method for loss performance modeling of hierarchical heterogeneous wireless networks", Int. Journal of Communication Systems, 27 (6), pp. 956-968, June 2014.
23. Z. Wang, P. T. Mathiopoulos and R. Schober, "Performance analysis and improvement methods for channel resource management strategies of LEO-MSS with multiparty traffic", IEEE Trans. Vehic. Tech., 57 (6), pp. 3832-3842, Nov. 2008.
24. Z. Wang, P. T. Mathiopoulos, and R. Schober, "Channel partitioning policies for multi-class traffic in LEO-MSS", IEEE Trans. Aerosp. Electron. Syst., 45 (4), pp. 1320 – 1334, Oct. 2009.
25. V. Iversen, "The exact evaluation of multi-service loss system with access control", Teleteknik (english edition), 31 (2), pp. 56–61, Aug. 1987.
26. J. Kaufman, "Blocking in a shared resource environment", IEEE Trans. Commun., 29 (10), pp. 1474-1481, October 1981.