Poznań University of Technology
Chair of Communication and Computer Networks

Application of non full-availability models in the analysis of multi-service network systems

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Plan of the presentation

- Introduction
- FAG and EIG models
- Multi service FAG model
- Multi service EIG model
- Multi service GEIG and GEIG(ml) models
- Application of multi-service EIG models
- Conclusion

Single-service Erlang's models

Agner Krarup Erlang
(1878-1929)

Full Availability Group (1917)*

Ideal Non-Full-Availability Group (1920)**

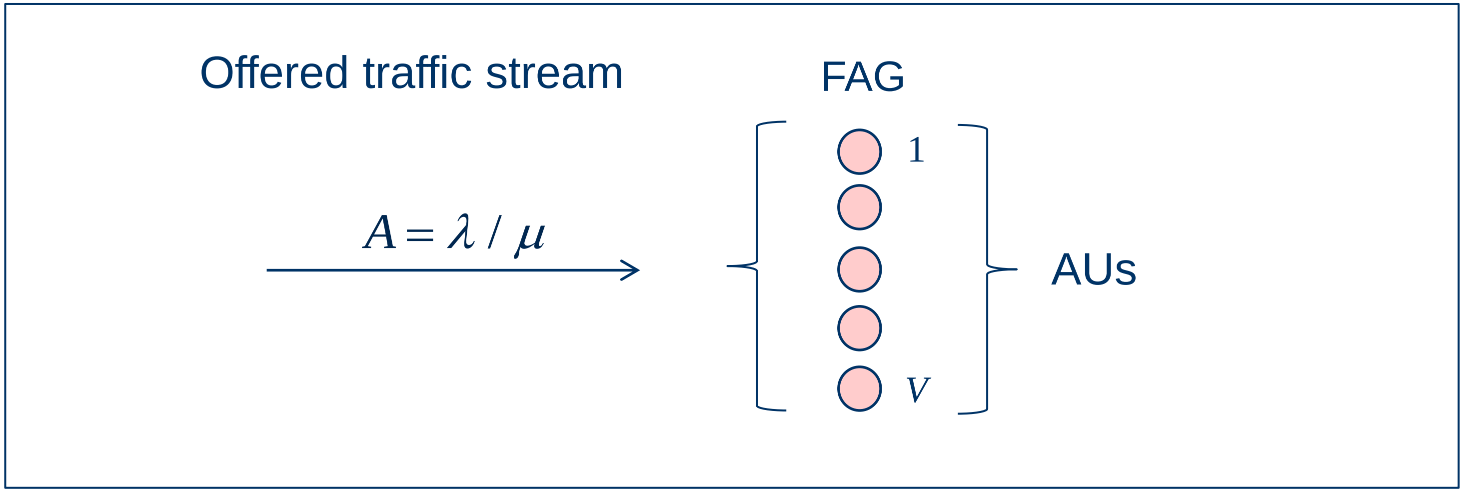
*Erlang A.K., Solution of some problems in the theory of probabilities of significance in automatic telephone exchanges, **Elektrotechnikeren**, vol. 13, 1917.

**Erlang A.K., The application of the theory of probabilities in telephone administration, Scandinavian H.C. Orsted Congress, Copenhagen, 1920.

**Brockmeyer E., Halstrom H., Jensen A., The life and works of A.K. Erlang, Acta Polytechnika Scandinavia, 6(287), 1960.

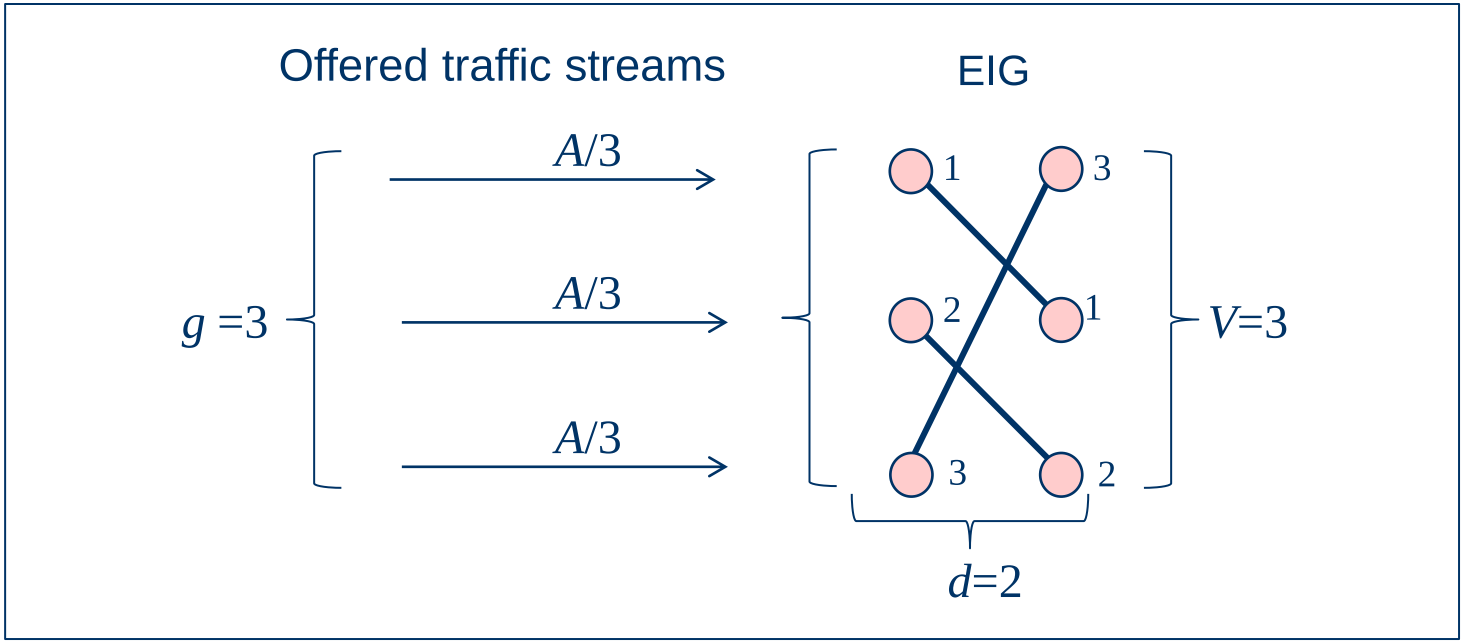
Full Availability Group (FAG)

- Capacity of the FAG: V AUs (Allocation Units);
- Arrival process is the Poisson process with parameter λ;
- Service time has exponential distribution with parameter μ;
- Rejected call is lost



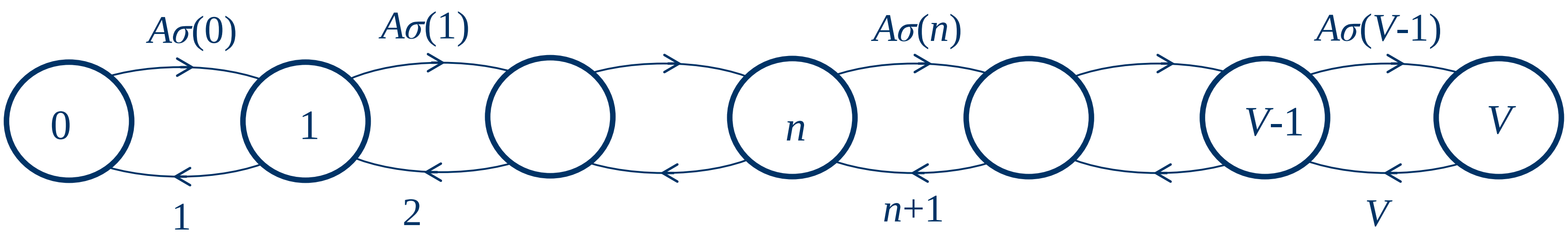
Erlang Ideal Grading (EIG)

- Structure: capacity V, availability d, input groups $g = \binom{V}{d}$;
- Arrival process is the Poisson process with parameter λ;
- Service time has exponential distribution with parameter μ;
- Rejected call is lost



Markov process

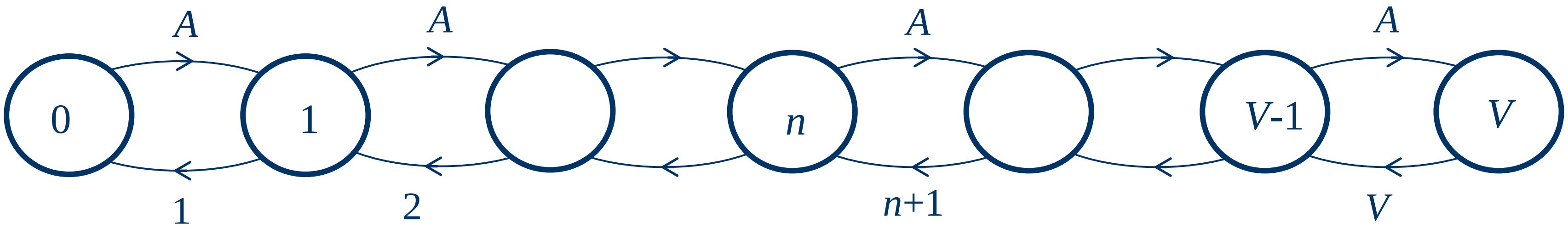
- Offered traffic intensity: $A = \lambda / \mu$;
- Conditional transition probability $\sigma(n)$;
- Number of serviced calls n.



FAG: $\sigma(n) = 1$ for $n \in \{0,\ldots,V\}$.

$$\text{EIG: } \sigma(n) = \begin{cases} 1 & \text{for } n \in \{0,\ldots,d-1\}, \\ 1 - \binom{n}{d} / \binom{V}{d} & \text{for } n \in \{d,\ldots,V\}. \end{cases}$$

FAG Markov process



$$\begin{cases} A\,[p_0]_V = 1\,[p_1]_V \\ \dots \\ A\,[p_{n-1}]_V = n\,[p_n]_V \quad \text{for} \quad 0 \le n \le V \\ \dots \\ A\,[p_{V-1}]_V = V\,[p_V]_V \\ \sum_{n=0}^{V} [p_n]_V = 1 \end{cases}$$

FAG Markov process

Occupancy distribution:

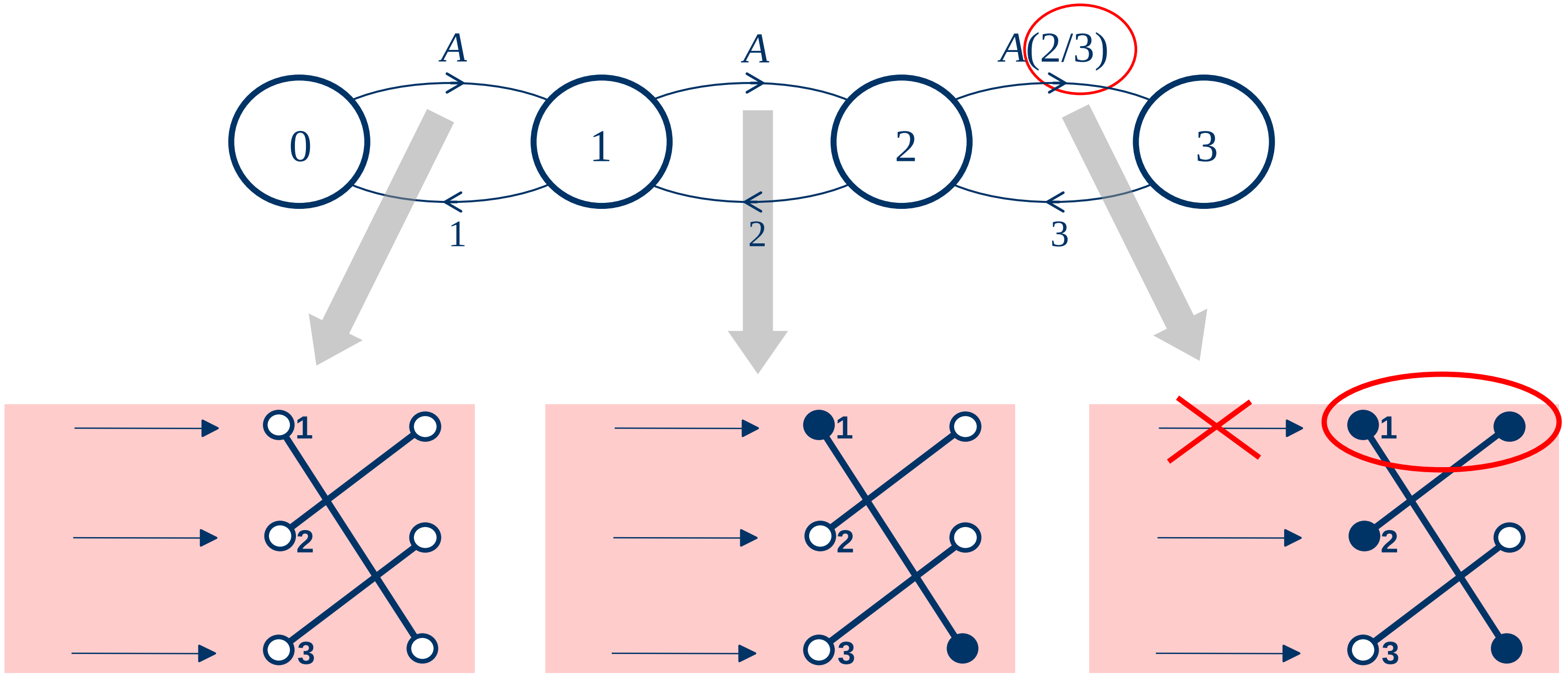
$$[p_n]_V = \frac{A^n}{n!} / \sum_{i=0}^{V} \frac{A^i}{i!}$$

Blocking probability (Erlang B formula):

$$E = E_V(A) = [p_V]_V = \frac{A^V}{V!} / \sum_{i=0}^{V} \frac{A^i}{i!}$$

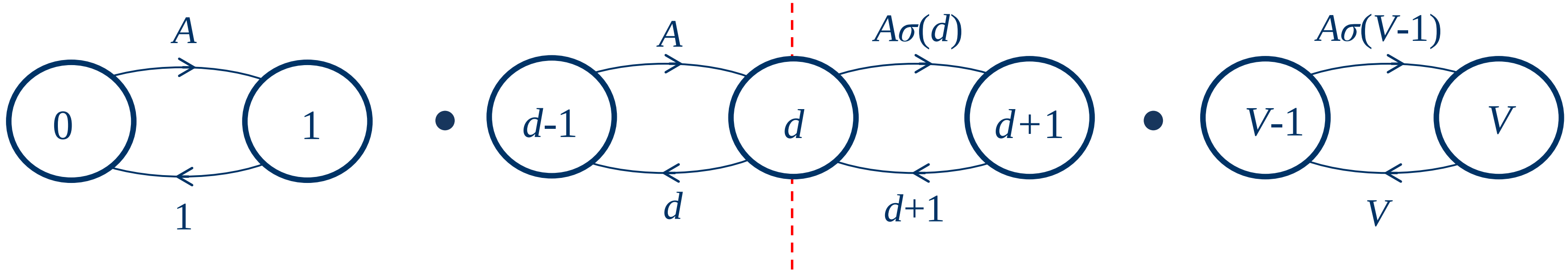
Blocking probability = f (offered traffic, capacity)

EIG Markov process



$$\sigma(n) = \begin{cases} 1 & \text{for} \quad n \in \{0,1\}, \\ 1 - \binom{n}{d} / \binom{V}{d} & \text{for} \quad n \in \{2,3\}. \end{cases}$$

EIG Markov process



$$
\begin{cases}
A\left[p_0\right]_V = 1\left[p_1\right]_V \\
\dots \\
A\left[p_{n-1}\right]_V = n\left[p_n\right]_V & \text{for} \quad n < d \\
A\,\sigma(n-1)\left[p_{n-1}\right]_V = n\left[p_n\right]_V & \text{for} \quad n \geq d \\
\dots \\
A\,\sigma(V-1)\left[p_{V-1}\right]_V = V\left[p_V\right]_V \\
\sum_{n=0}^{V}\left[p_n\right]_V = 1
\end{cases}
$$

EIG Markov process

Occupancy distribution:

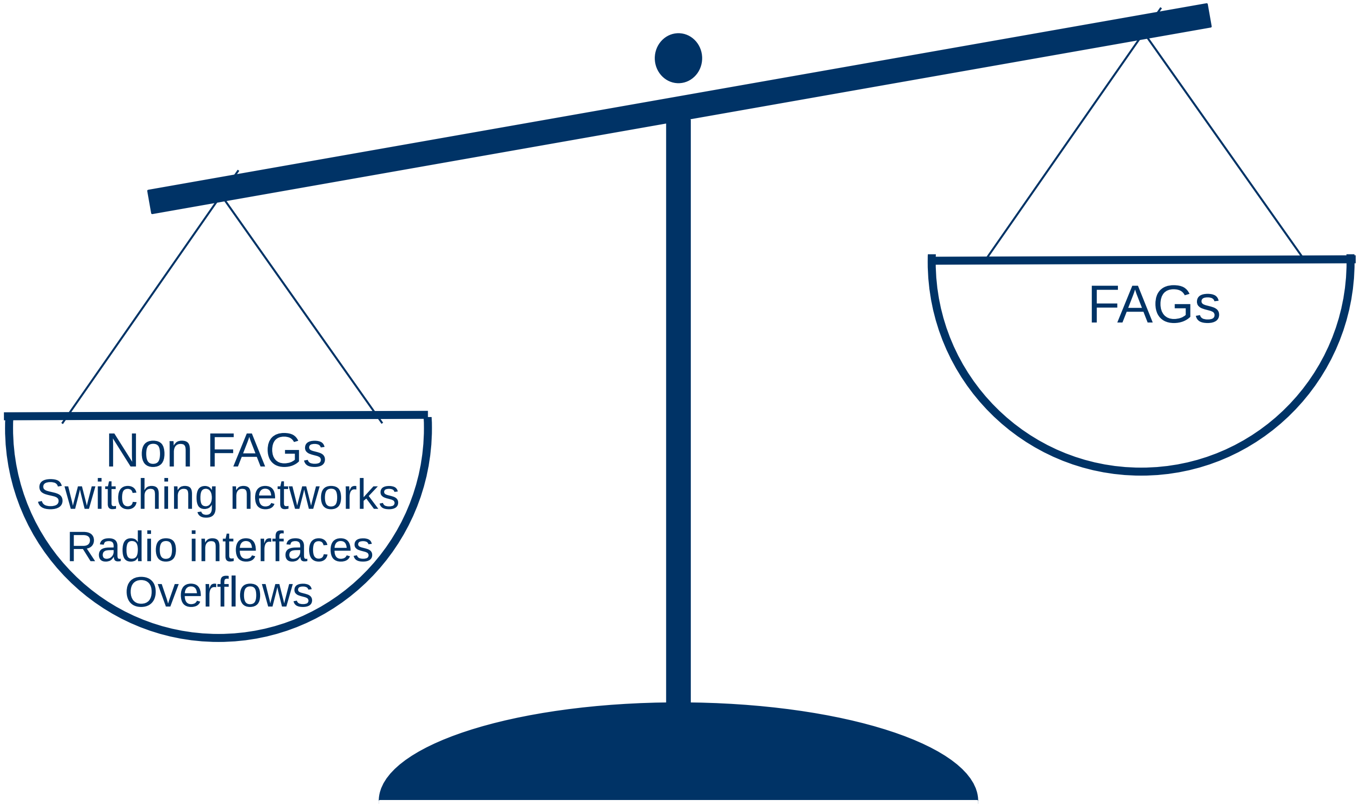
$$[p_n]_V = \frac{A^n}{n!}\prod_{k=d}^{n-1}\left[1-\frac{\binom{k}{d}}{\binom{V}{d}}\right] \Big/ \sum_{i=0}^{V}\frac{A^i}{i!}\prod_{k=d}^{i-1}\left[1-\frac{\binom{k}{d}}{\binom{V}{d}}\right]$$

Blocking probability (EIF formula): $E=\sum_{n=d}^{V}(1-\sigma(n))[p_n]_V$

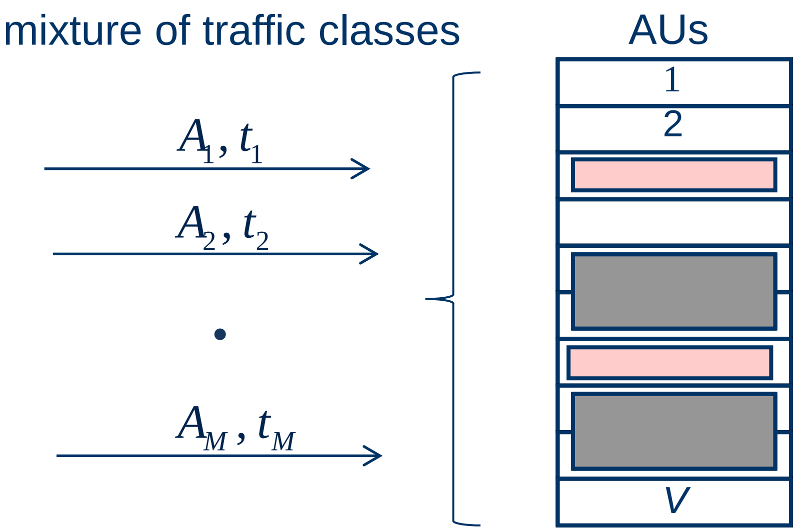
$$E=\mathrm{EIF}(A,V,d)=\sum_{n=d}^{V}\frac{\binom{n}{d}}{\binom{V}{d}}\frac{A^n}{n!}\prod_{k=d}^{n-1}\left[1-\frac{\binom{k}{d}}{\binom{V}{d}}\right] \Big/ \sum_{i=0}^{V}\frac{A^i}{i!}\prod_{k=d}^{i-1}\left[1-\frac{\binom{k}{d}}{\binom{V}{d}}\right]$$

Single and multi-service models

Single and multi-service systems



Multi-service system



V – capacity of the system expressed in AUs,

A_i – offered traffic of class i,

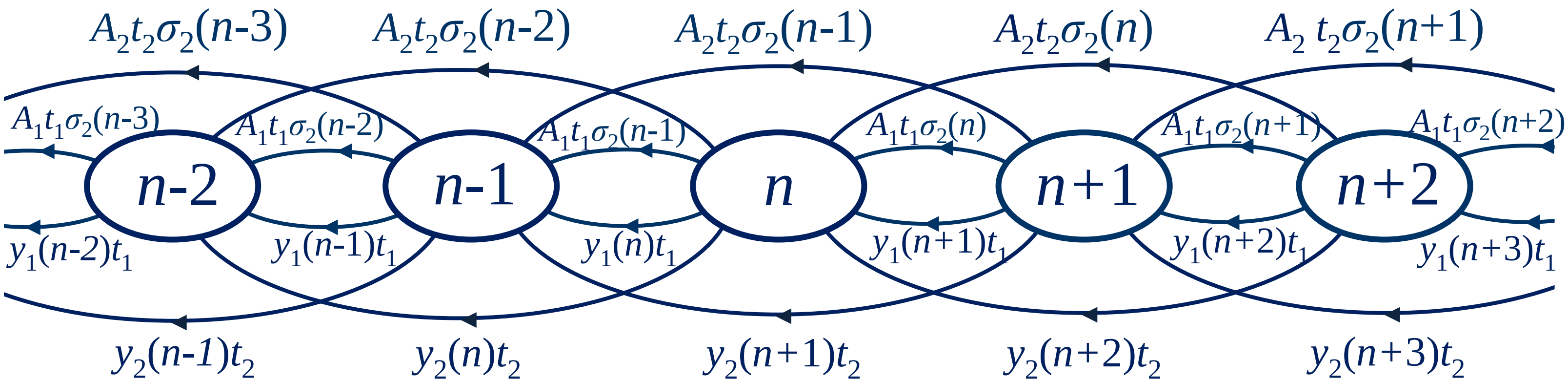
M – number of offered call streams in the system,

t_i – number of demanded AUs by class i call,

$[P_n]_V$ – occupancy distribution, probability of n AUs being busy,

$y_i(n)$ – average number of class i calls serviced in state n.

Multi-service Markov process



State-independent systems (FAG): $\sigma_i(n) = 1$ for $n \in \{0,...,V\}$.

State-dependent systems (EIG): $\sigma_i(n) \in [0,1]$ for $n \in \{0,...,V\}$.

FAG multi-service Markov process*

Occupancy distribution:

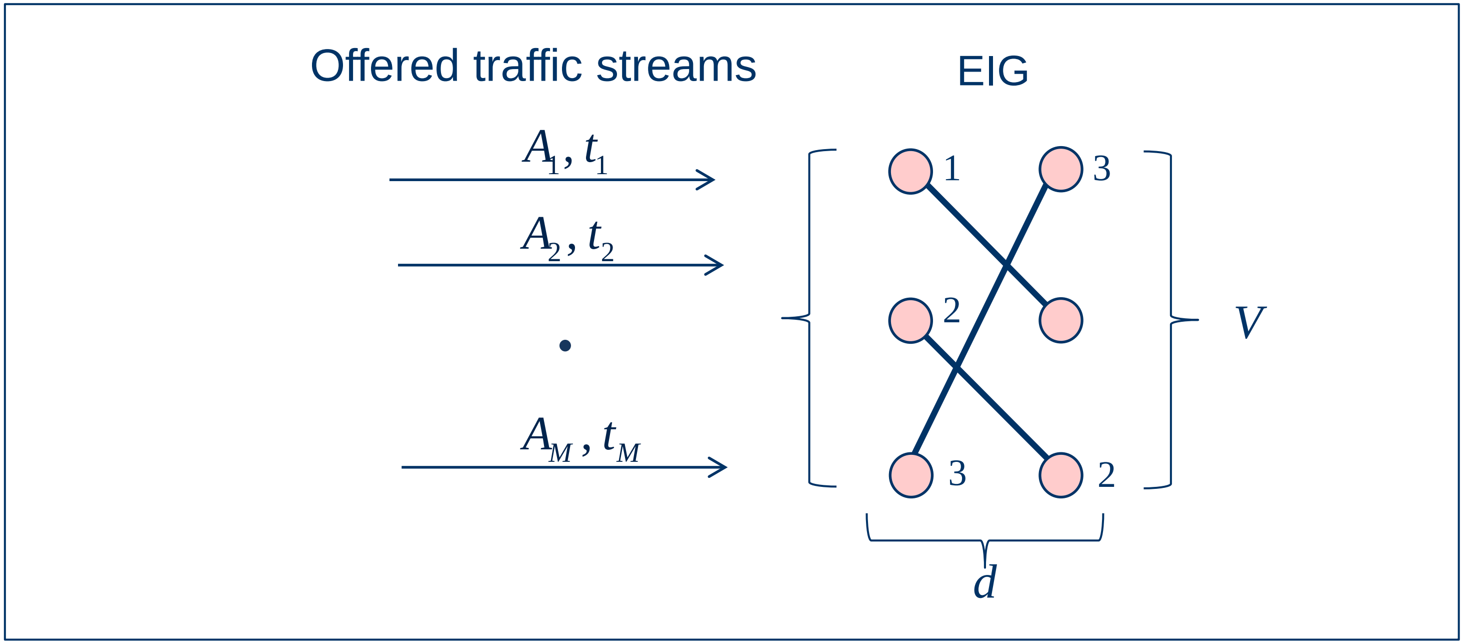
$$n\,[P_n]_V = \sum_{i=1}^{M} A_i\, t_i\, [P_{n-t_i}]_V$$

Blocking probability:

$$E_i = \sum_{n=V-t_i+1}^{V} [P_n]_V$$

* Kaufman J.S.: Blocking in a shared resource environment, **IEEE Transactions on Communications**, vol. COM-29, No. 10, 1981, pp. 1474-1481.
* Roberts J.W.: A service system with heterogeneous user requirements, [in] Performance of Data Communications Systems and their Applications, (Ed. G. Pujolle), **Elsevier**, Amsterdam, 1981, pp. 423-431.

Multi-service model of EIG



M - number of traffic classes offered to the system;

t_i - number of AUs required for class i call;

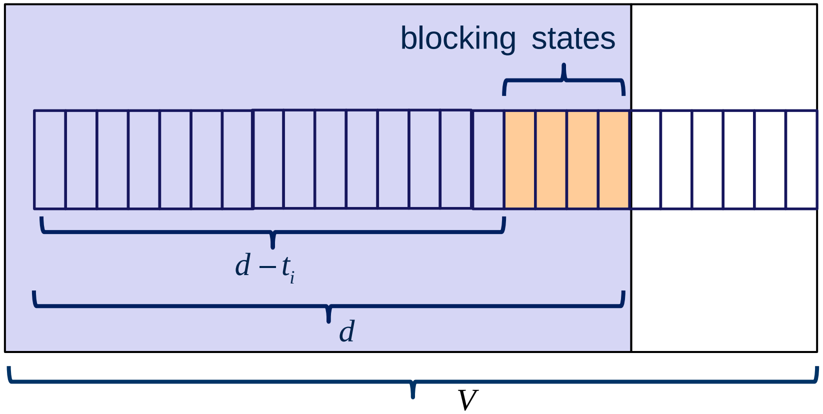
EIG multi-service Markov model*

Occupancy distribution: $n\left[p_n\right]_V = \sum_{i=1}^{M} A_i t_i \sigma_i(n-t_i)\left[p_{n-t_i}\right]_V$

Blocking probability: $E_i = \sum_{n=0}^{V} [1-\sigma_i(n)]\,[P_n]_V$

Transition coefficients: $\sigma_i(n) = 1 - \sum_{x=d-t_i+1}^{\min(n,d)} P_{V,d}(n,x)$

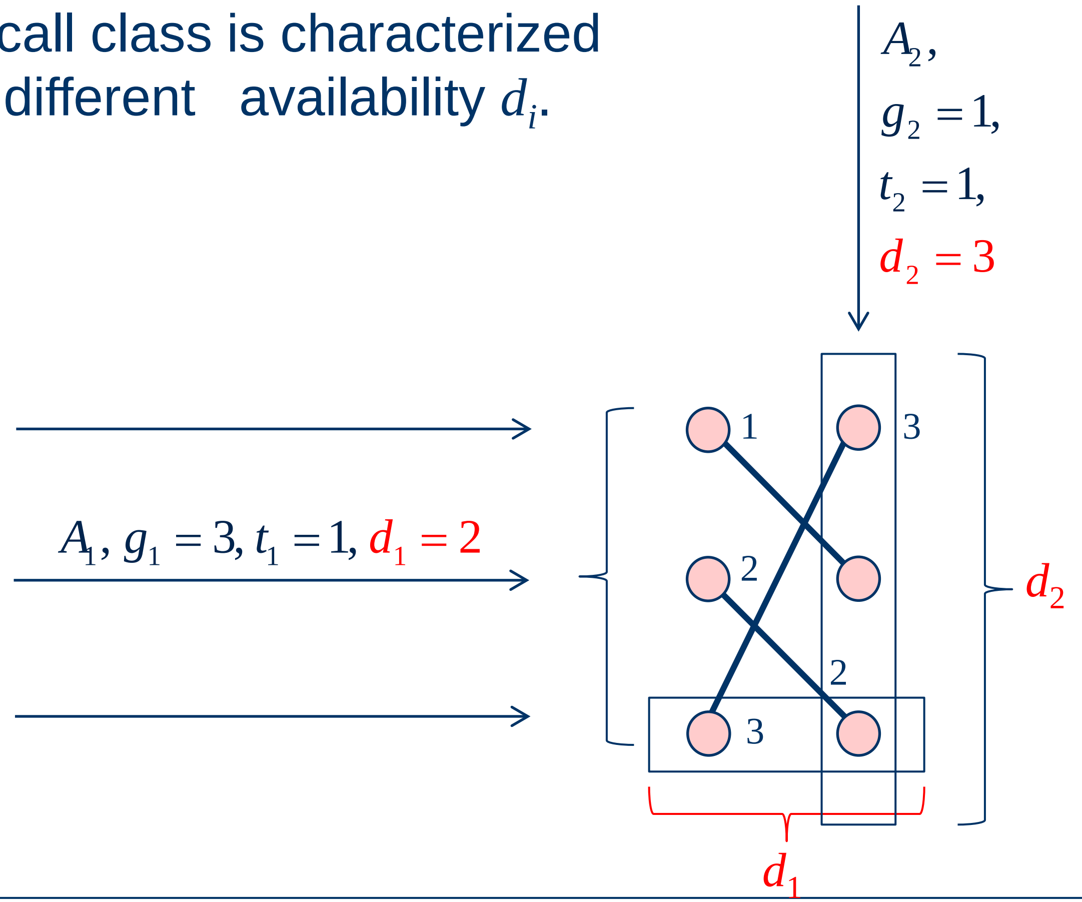
$$P_{V,d}(n,x) = \binom{d}{x}\binom{V-d}{n-x} / \binom{V}{n}$$



*Stasiak M.: An approximate model of a switching network carrying mixture of different multi-channel traffic streams. **IEEE Transactions on Communications**, vol. 41, No. 6, 1993, pp. 836-840.

Multi-service model of GEIG

Each call class is characterized by a different availability d_i.



GEIG multi-service Markov model*

Occupancy distribution: $n\left[p_n\right]_V = \sum_{i=1}^{M} A_i t_i \sigma_i(n-t_i)\left[p_{n-t_i}\right]_V$

Blocking probability: $E_i = \sum_{n=0}^{V} [1-\sigma_i(n)]\,[P_n]_V$

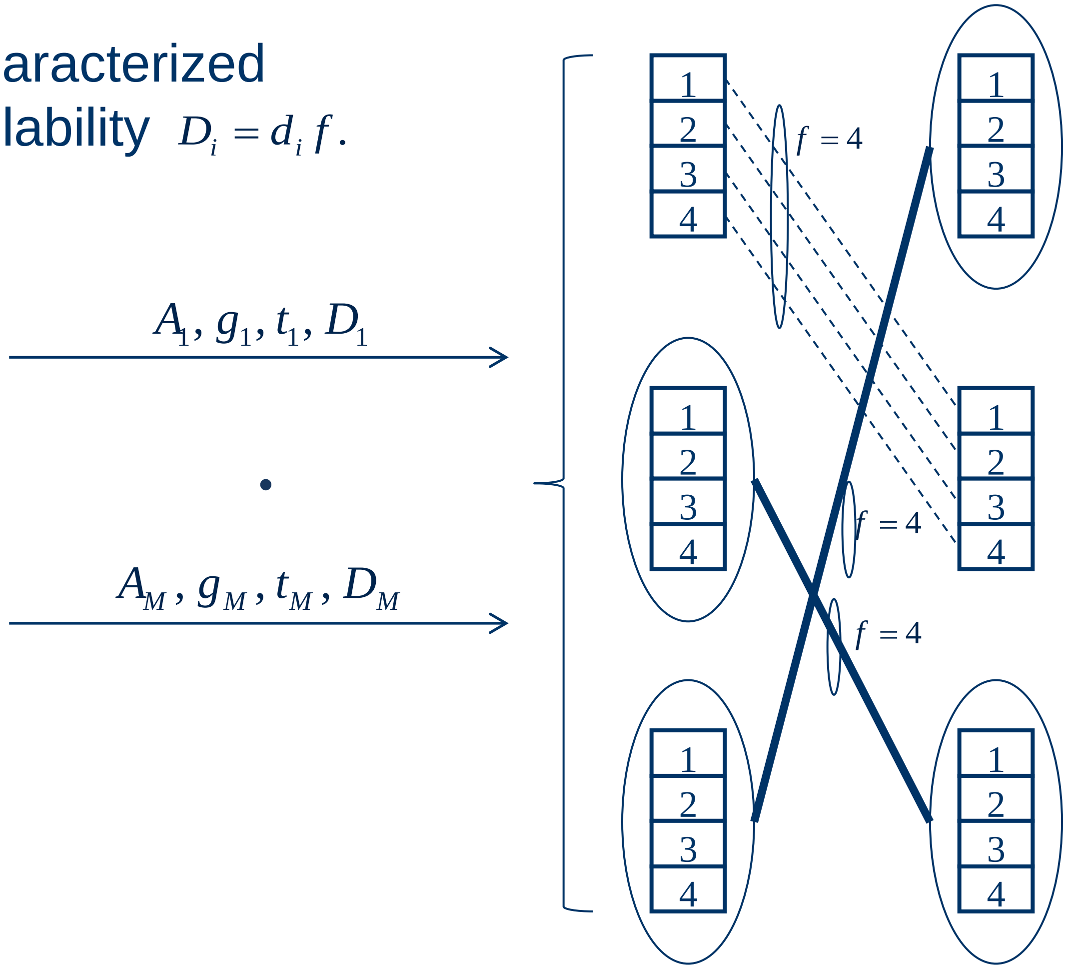
Transition coefficients: $\sigma_i(n) = 1 - \sum_{x=d_i-t_i+1}^{\min(n,d_i)} P_{V,d_i}(n,x)$

$$P_{V,d_i}(n,x) = \binom{d_i}{x}\binom{V-d_i}{n-x} / \binom{V}{n}$$

*Stasiak M., Hanczewski S., Approximation for Multi-service Systems with Reservation by Systems with Limited-Availability. **Lecture Notes in Computer Science**, Springer, vol. 5261, 2008, pp. 257-267.

Model of GEIG with multiplication of links GEIG(ml)

Each call class is characterized by a different availability $D_i = d_i f$.



GEIG(ml) multi-service Markov model*

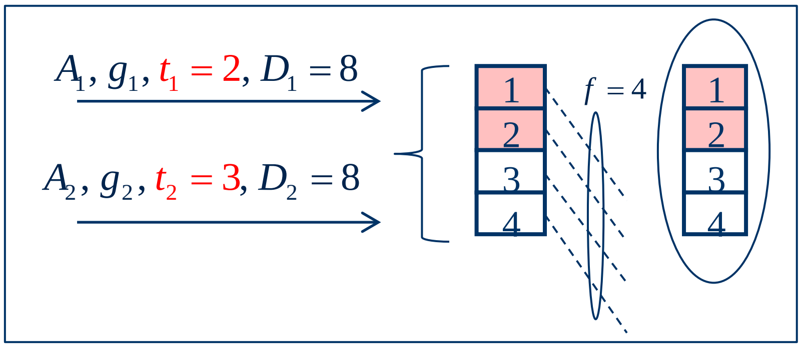
Occupancy distribution: $n\left[p_n\right]_V = \sum_{i=1}^{M} A_i t_i \sigma_i(n-t_i)\left[p_{n-t_i}\right]_V$

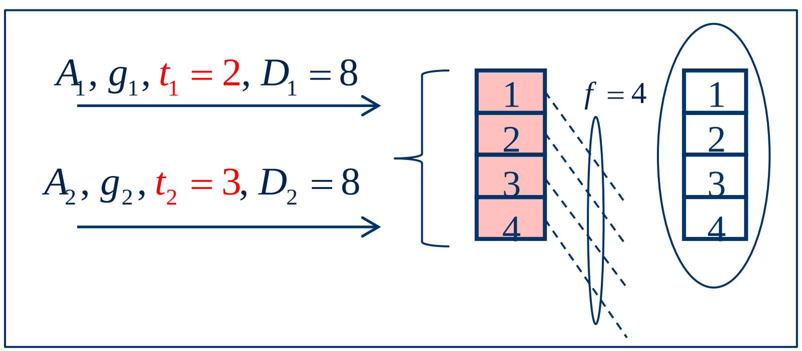
Blocking probability: $E_i = \sum_{n=0}^{V} [1-\sigma_i(n)]\,[P_n]_V$

*Głąbowski M., Hanczewski S., Stasiak S., Weissenberg J.: Modeling Erlang's Ideal Grading with Multi-rate BPP Traffic, **Mathematical Problems in Engineering**, 2012, pages 17, article ID 547909, doi:10.1155/2012/547909

GEIG(ml) multi-service Markov model*

Transition coefficients:





$$\sigma_i(n) = 1 - \sum_{x=D_i - t_i + 1}^{\min(n, D_i)} P_{V,D_i}(n,x)$$

$$P_{V,D_i}(n,x) = \frac{\binom{D_i}{x}\binom{V-D_i}{n-x}}{\binom{V}{n}} \frac{F(D_i - x, d_i, t_{i-1})}{F(D_i - x, d_i, f)}$$

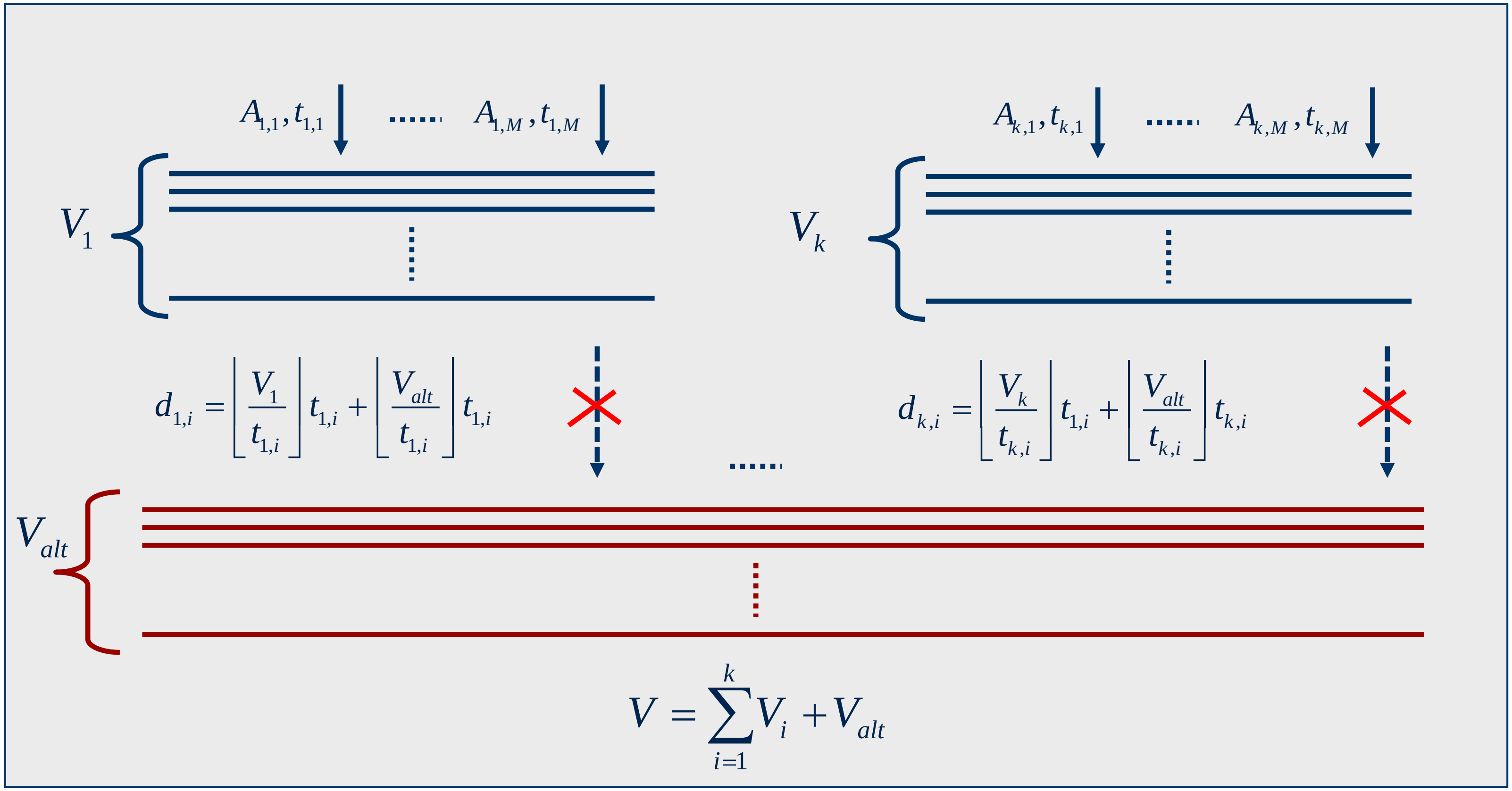
where:

$$F(z,k,f) = \sum_{i=0}^{\left\lfloor \frac{z}{f+1} \right\rfloor} (-1)^i \binom{k}{i} \binom{z+k-1-i(f+1)}{k-1}$$

Applications of multi-service EIG models

- Overflow systems
- VoD
- Switching networks
- Radio interface in mobile network

Overflow Scheme as GEIG*

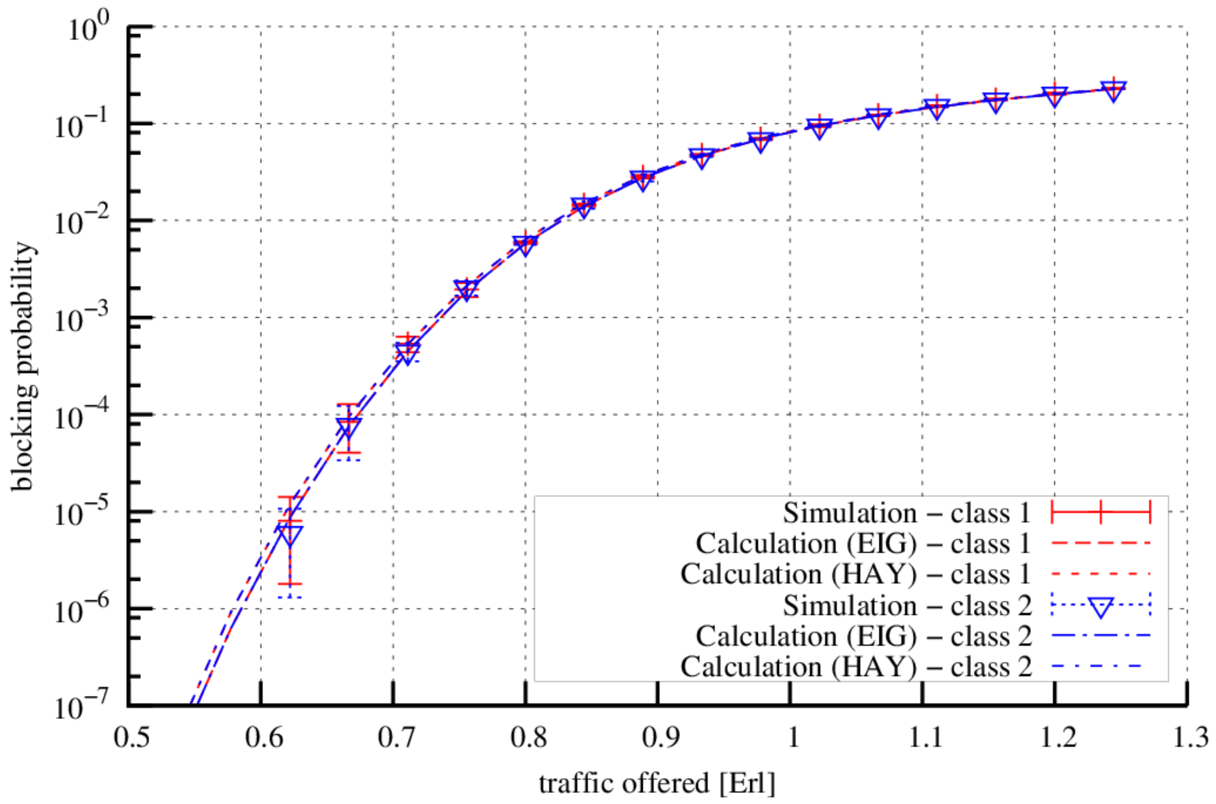


*Głabowski M., Hanczewski S., Stasiak M.: Modelling of Cellular Networks with Traffic Overflow, **Mathematical Problems in Engineering**, 2015, article ID 286490, 15 pages, doi:10.1155/2015/286490.

Overflow systems

Structure of the system:

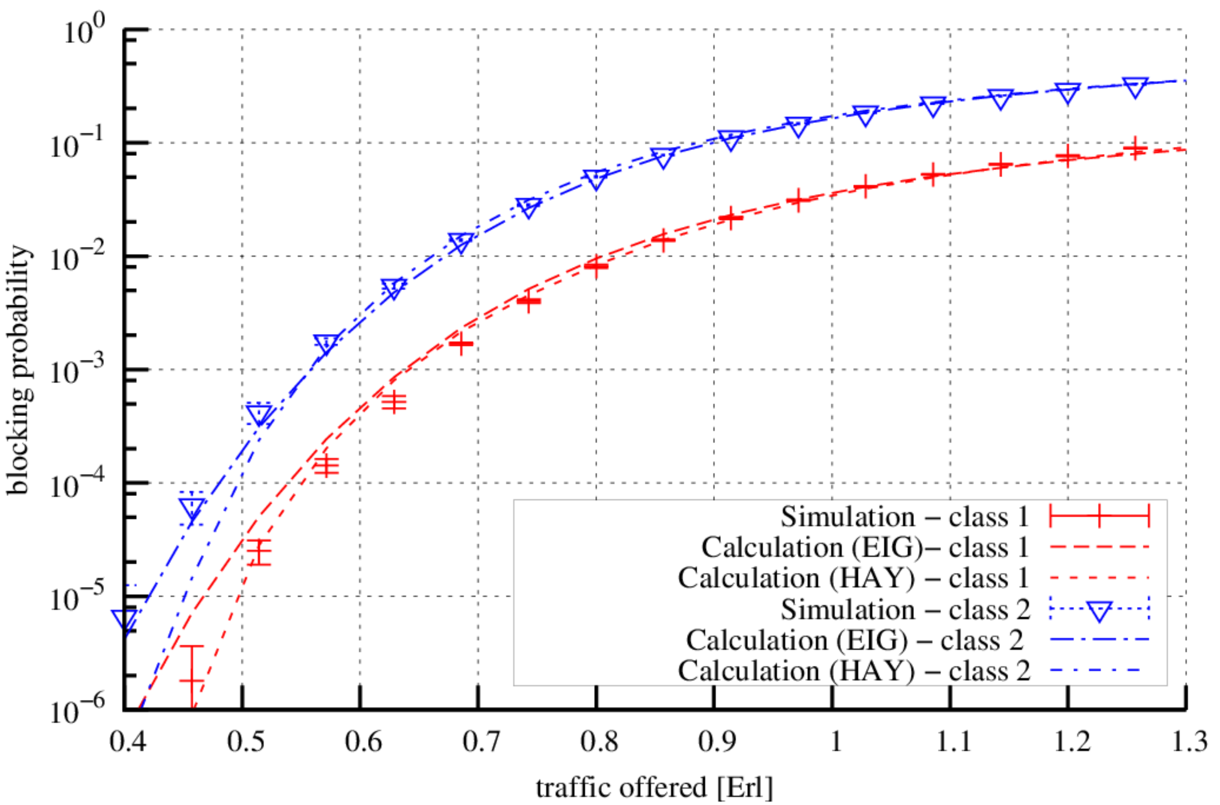
V_1=20, t_1=1, V_2=20, t_2=1, V_{alt}=50, A_1t_1:A_2t_2=1:1, $a = \frac{\sum_{i=1}^{k} A_i t_i}{V_{alt} + \sum_{i=1}^{k} V_i}$



Overflow systems

Structure of the system:

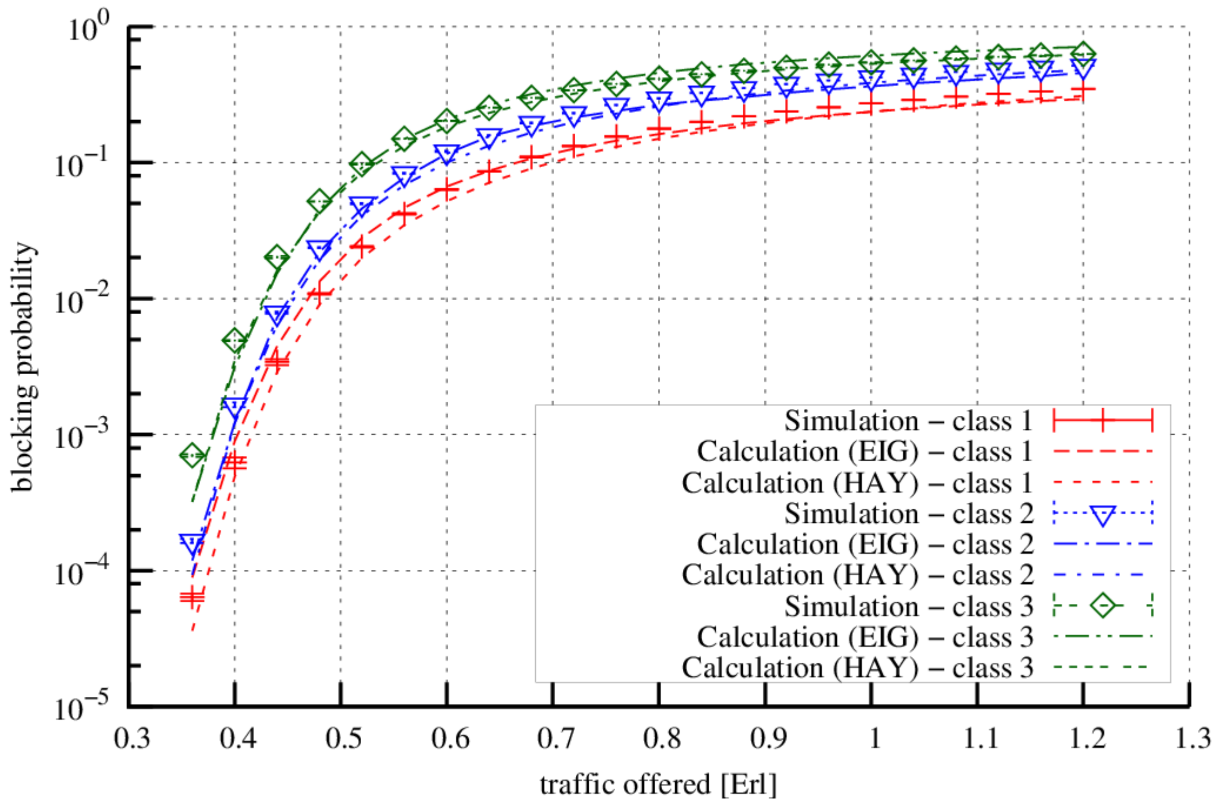
V_1=20, t_1=1, V_2=60, t_2=5, V_{alt}=60, A_1t_1:A_2t_2=1:1, $a = \frac{\sum_{i=1}^{k} A_i t_i}{V_{alt} + \sum_{i=1}^{k} V_i}$



Overflow systems

Structure of the system:

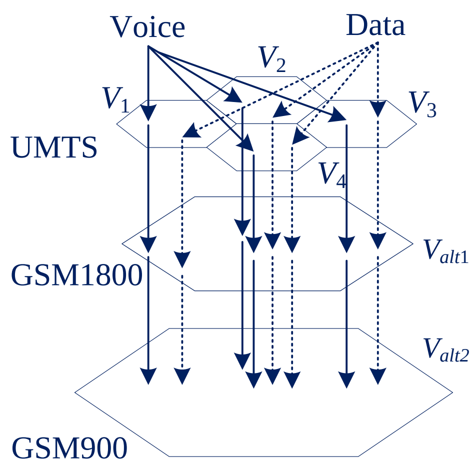
$V_1=20,\ t_1=1,\ V_2=40,\ t_2=2,\ V_3=80,\ t_3=4,\ V_{alt}=60,\ A_1t_1{:}A_2t_2{:}A_3t_3=1{:}1{:}1,\ a=\dfrac{\sum_{i=1}^{k} A_i t_i}{V_{alt}+\sum_{i=1}^{k} V_i}$

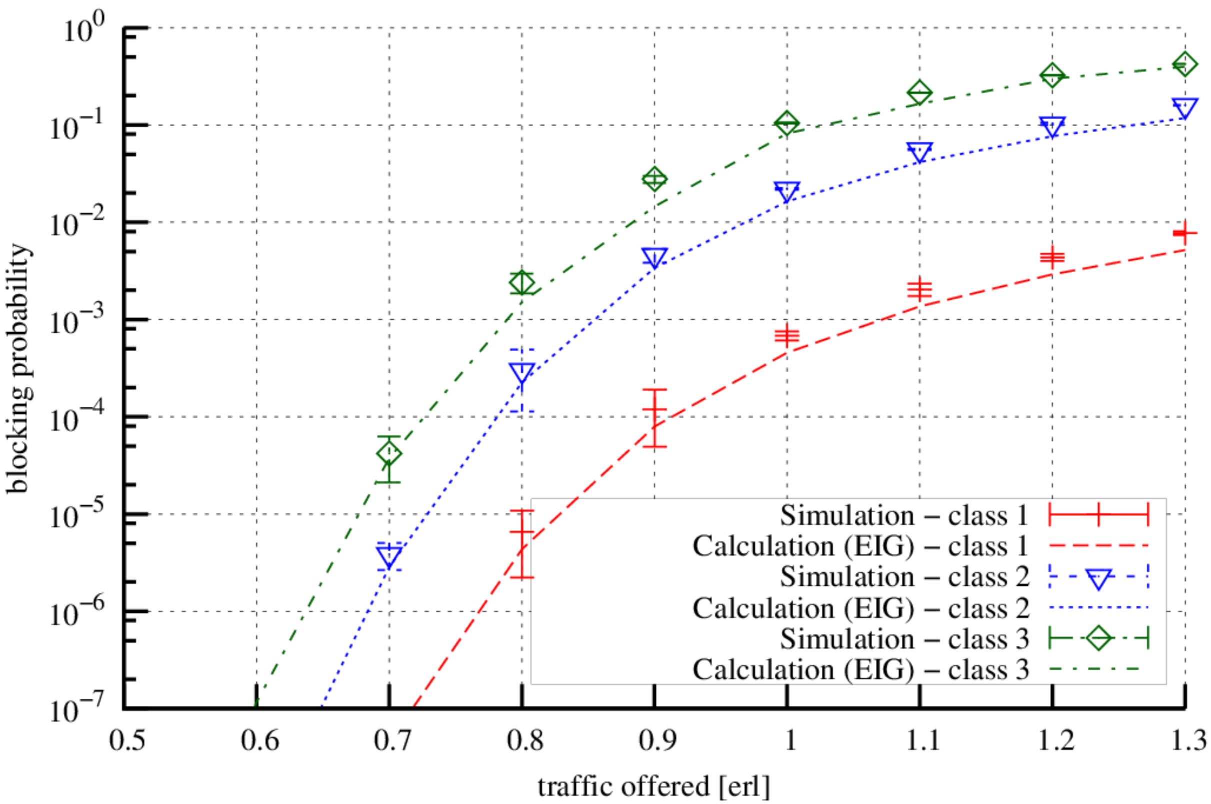


Overflow systems

Structure of the system UMTS to GSM:

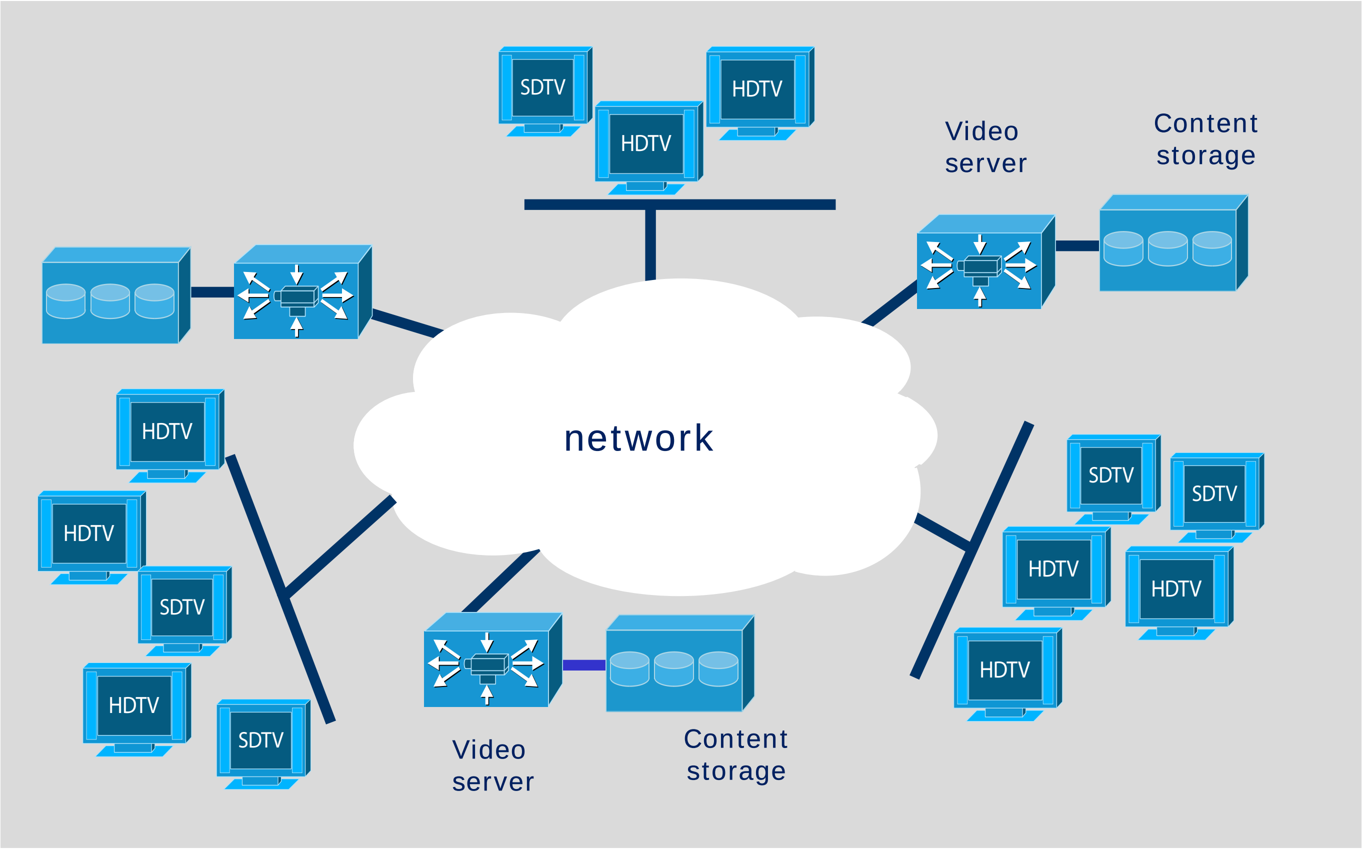
$V_1=V_2=V_3=V_4=140$, $V_{alt1}=32$, $V_{alt2}=32$,
$t_1=1$, $t_2=4$, $t_3=10$, $A_1t_1:A_2t_2:A_3t_3=1:1:1$,



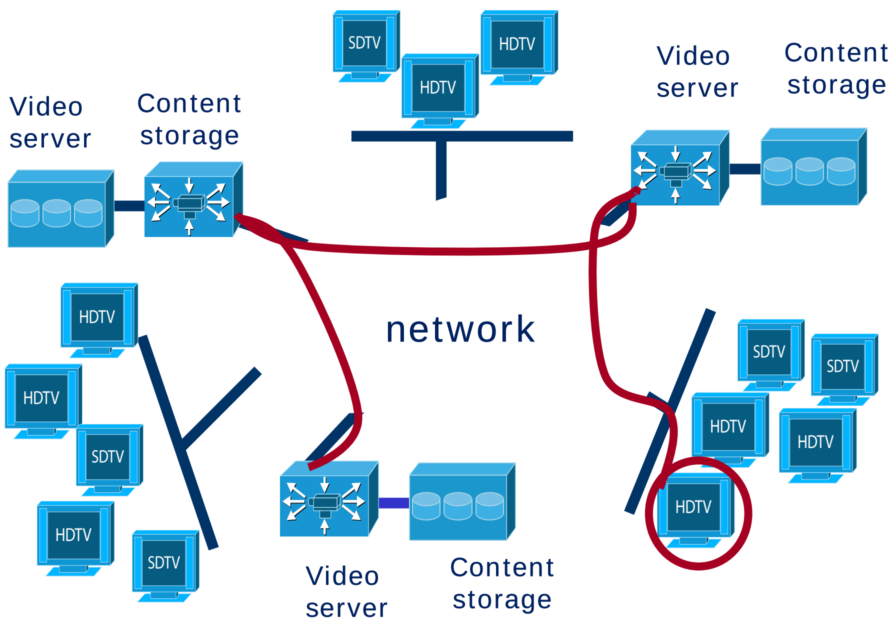


VoD systems

VOD system is built with: Nodes (video server + content storage),
Network (public or private),
User devices (TV set, computer, tablet)



Service process in Video on Demand System

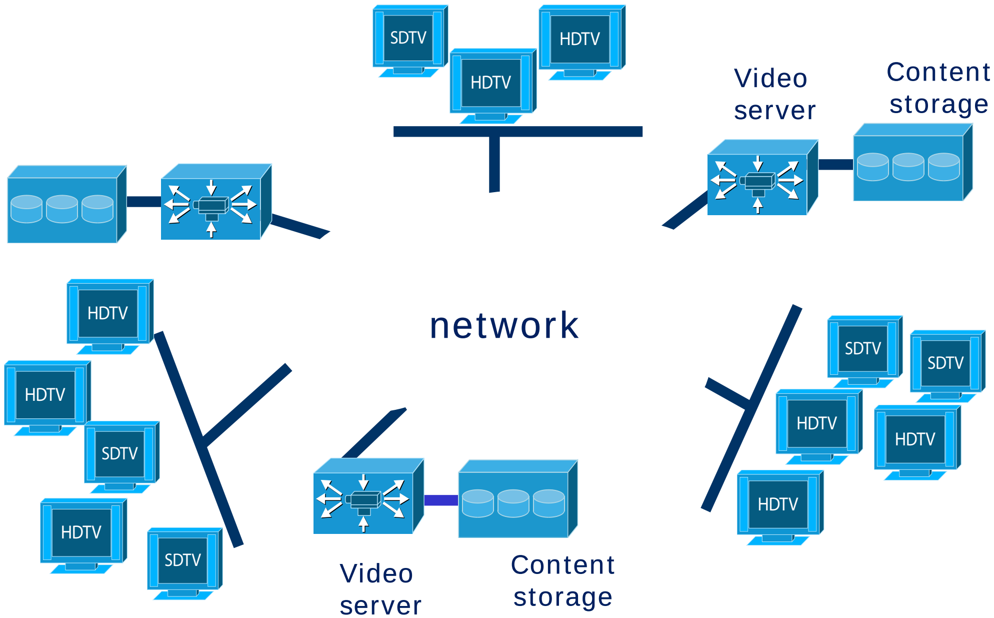


By default, users of the system are serviced by the closest video server to minimize the traffic load;

The user can be serviced by another, more distant, video server when a demanded multimedia file is not available in the closest server;

Demand access to the file will be then rejected (the VoD system is in blocking state) only when no other video server, from among all servers available in the system, can service this particular demand.

VoD system parameters



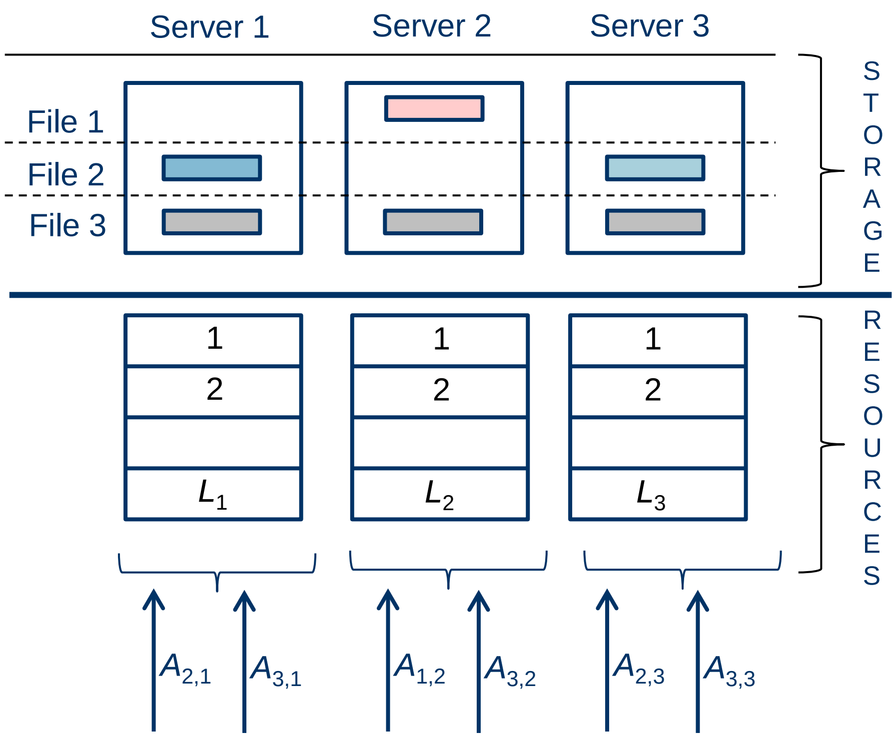
N - number of nodes,

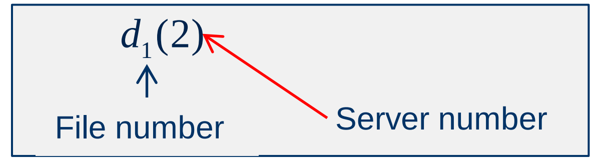
L_i - capacity of the server,

F_i - number of files on i server,

p_i - probability of choice of a given file (Zipf distribution)

VoD systems





$$d_1(2) = \frac{A_{1,2}}{A_{1,2} + A_{3,2}} L_2, \quad d_1 = d_1(2)$$

$$d_2(1) = \frac{A_{2,1}}{A_{2,1} + A_{3,1}} L_1, \quad d_2(3) = \frac{A_{2,3}}{A_{2,3} + A_{3,3}} L_3,$$

$$d_2 = d_2(1) + d_2(3)$$

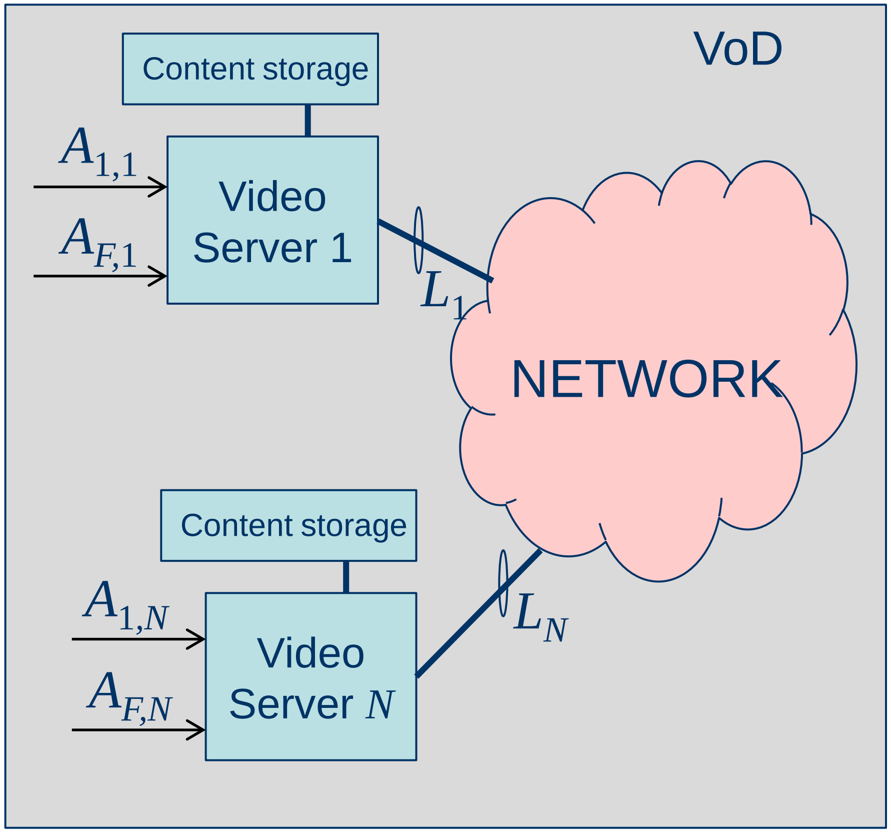
$$d_3(1) = \frac{A_{3,1}}{A_{2,1} + A_{3,1}}, \quad d_3(2) = \frac{A_{3,2}}{A_{1,2} + A_{3,2}},$$

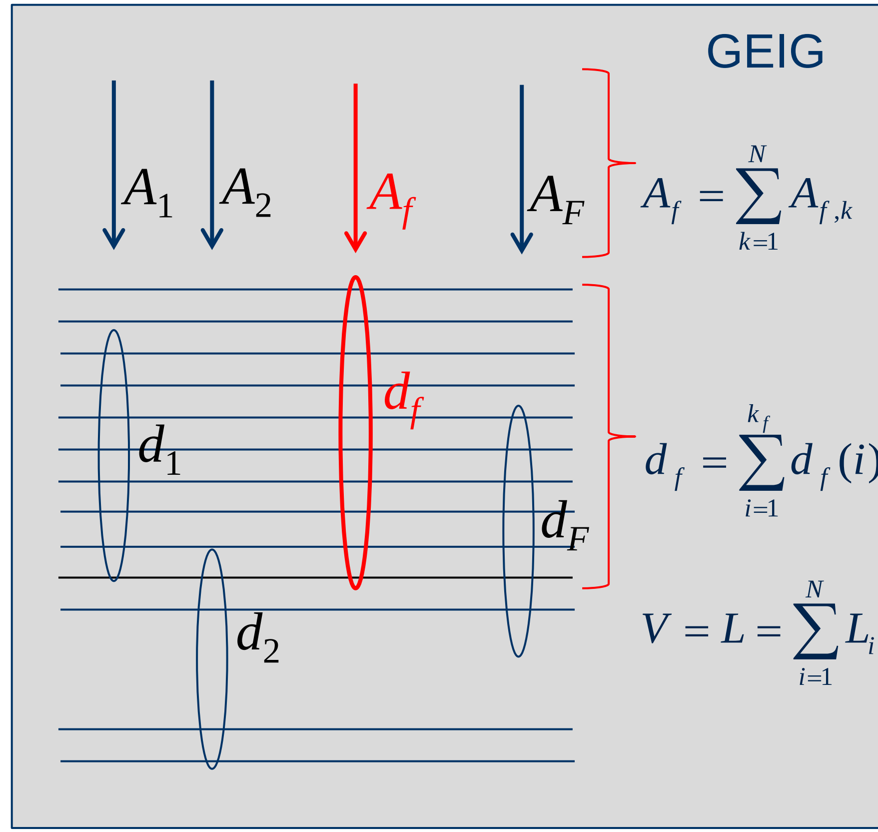
$$d_3(3) = \frac{A_{3,3}}{A_{2,3} + A_{3,3}},$$

$$d_3 = d_3(1) + d_3(2) + d_3(3)$$

$$L = L_1 + L_2 + L_3$$

Video on Demand system as GEIG*

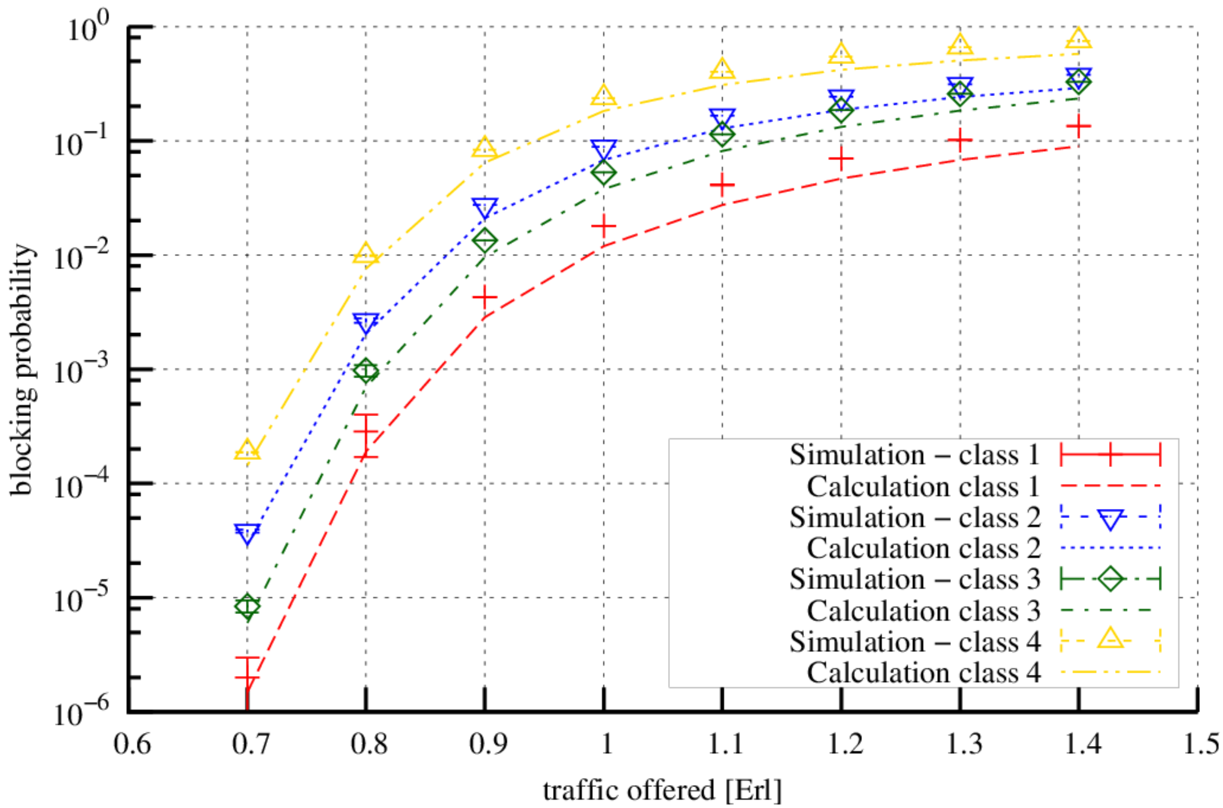




*Hanczewski S., Stasiak M.: Modeling of Video on Demand Systems, Computer Networks, A., **Communications in Computer and Information Science** (CCIS), Springer**,** vol. 431, 2013, pp. 233-242.

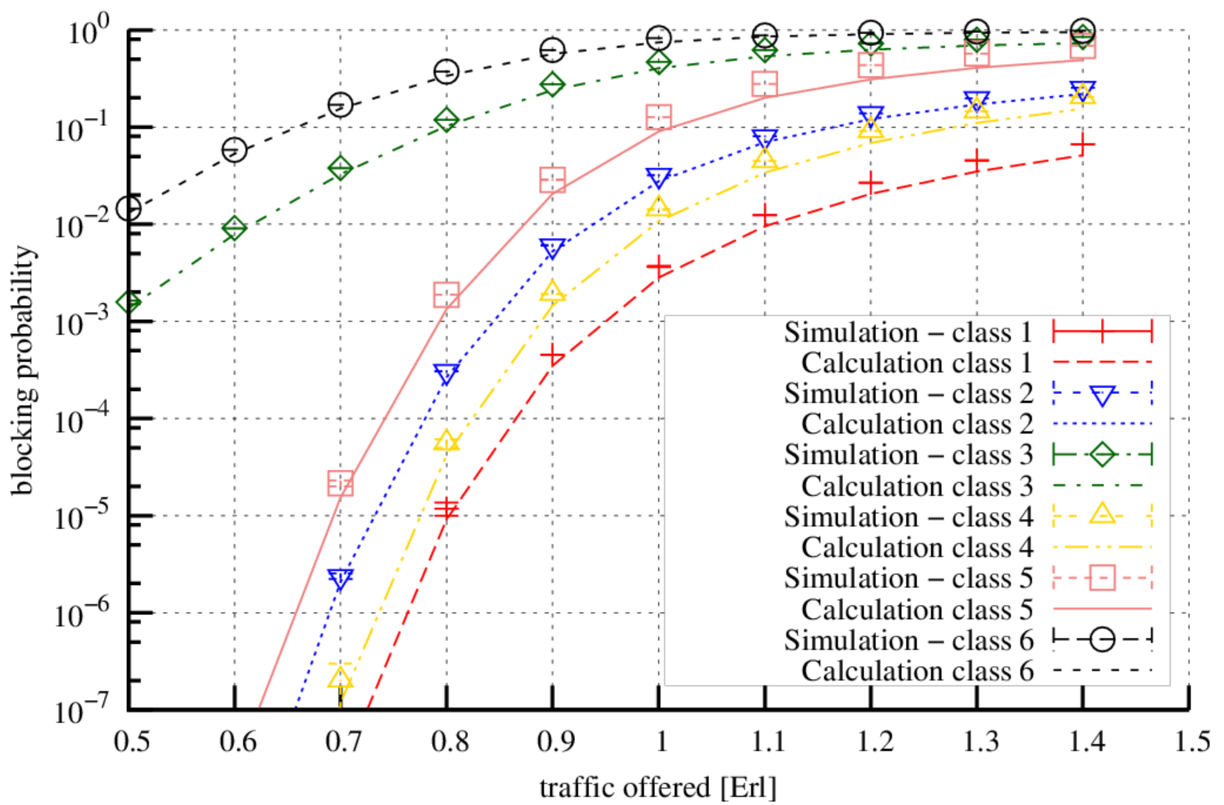
VoD systems

Structure of the system: N=5, L_i=50, F=200, SDTV (2 classes), HDTV (2 classes), the most popular – 5 copies, normal – 3 copies.

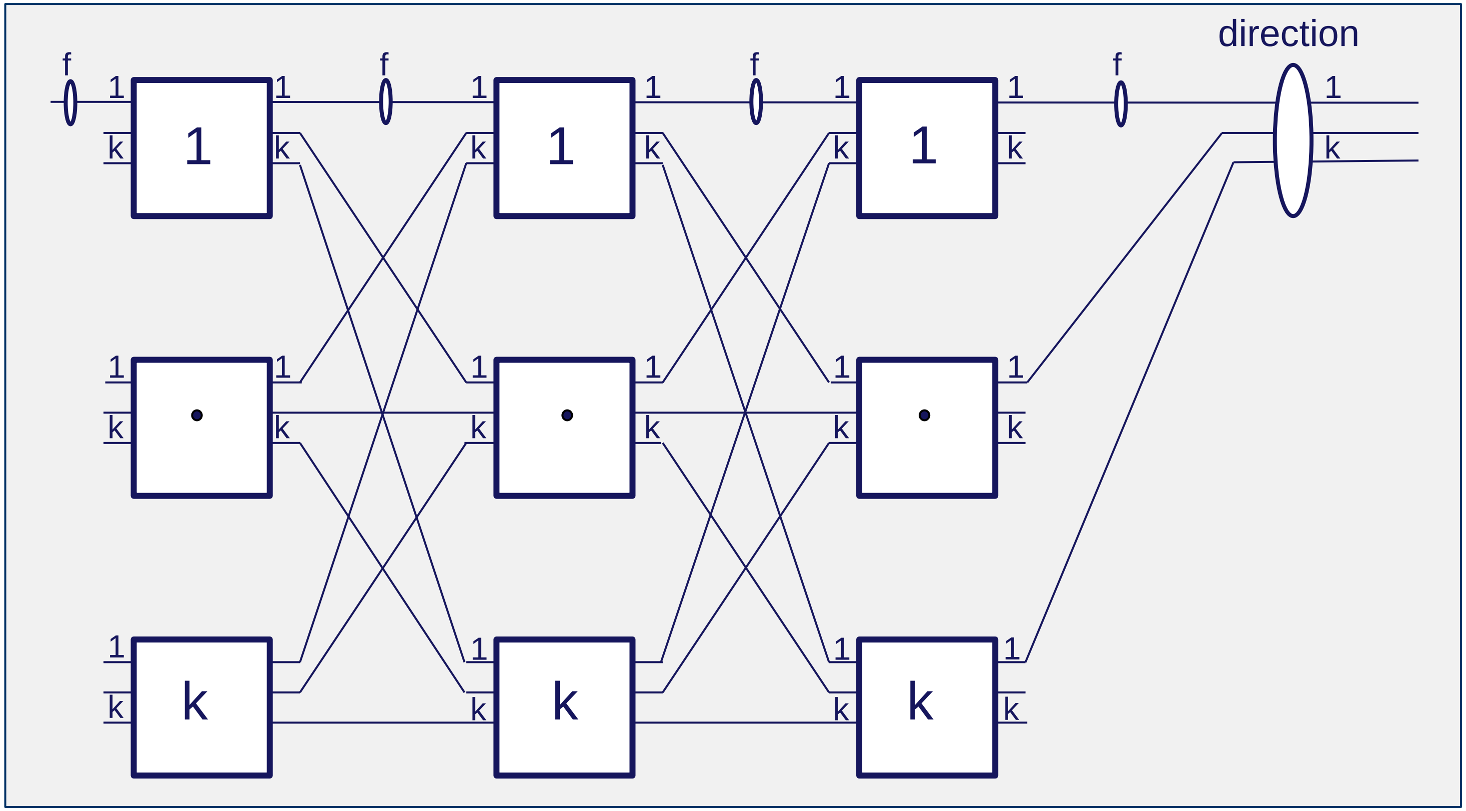


VoD systems

Structure of the system: N=6, L_i=50, F=300, SDTV (3 classes), HDTV (3 classes), the most popular – 6 copies, normal – 3 copies, rather unpopular– 1 copy

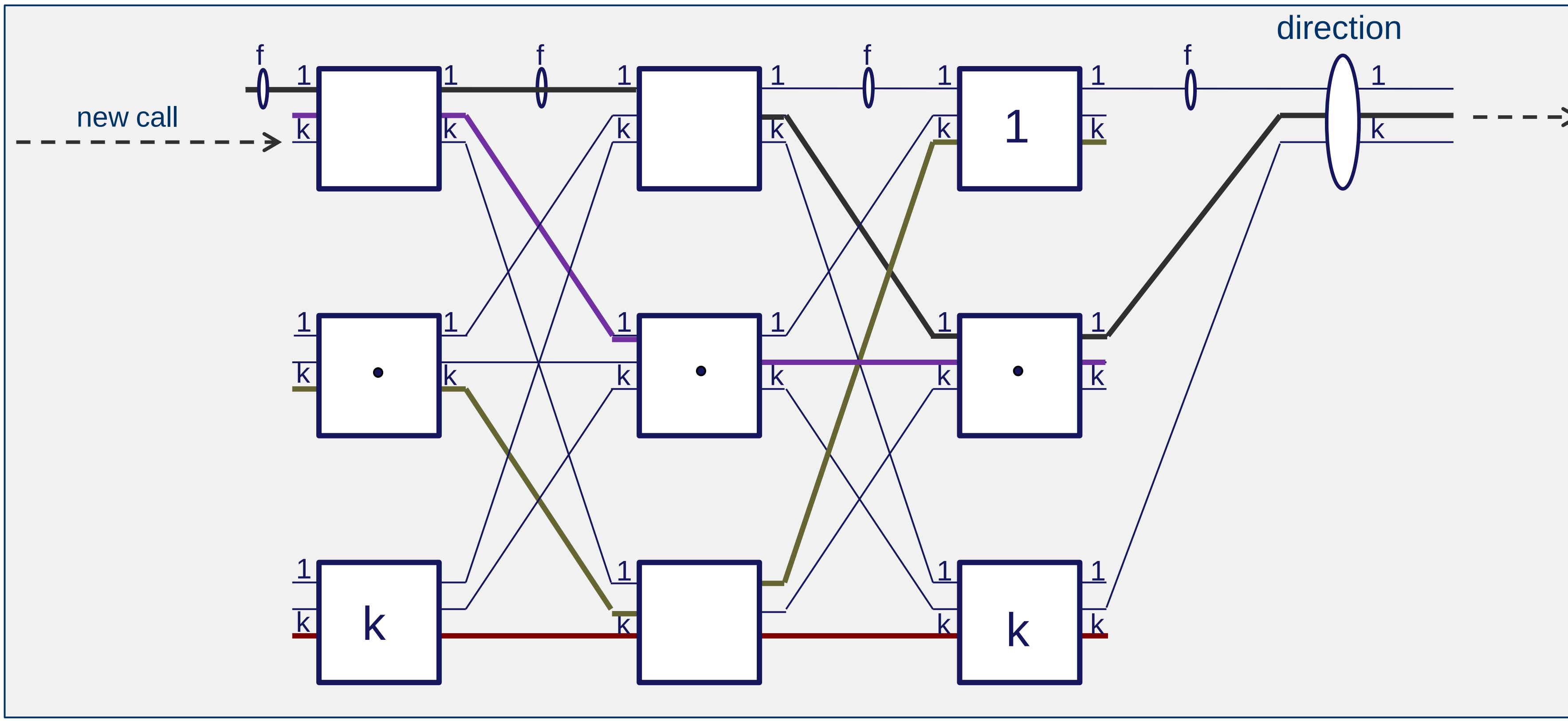


Multiservice switching network



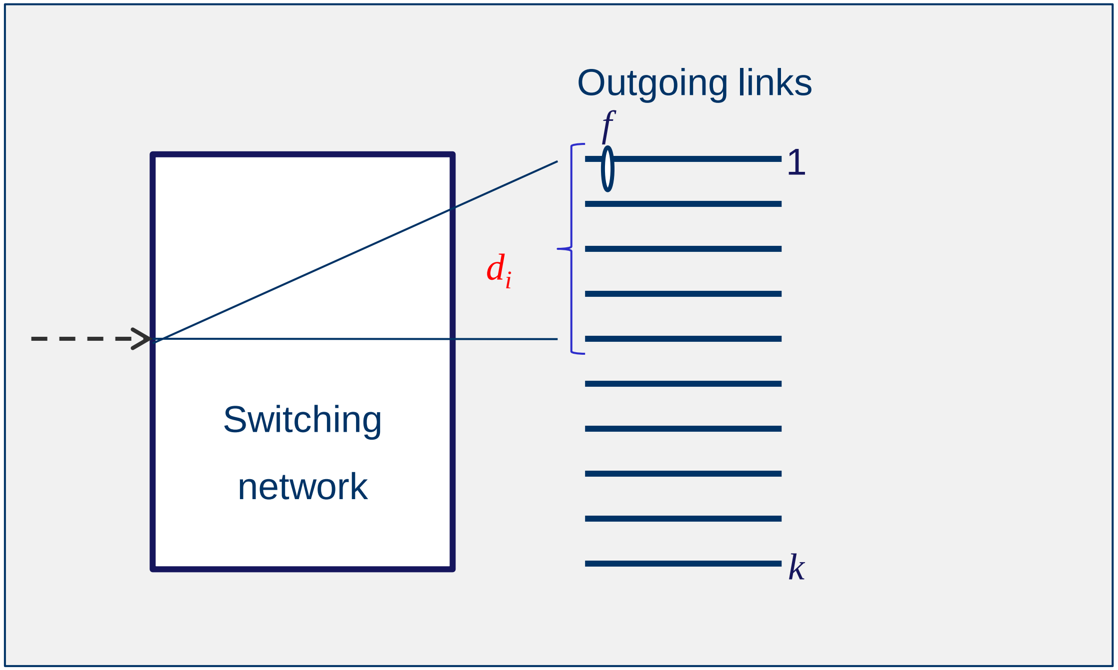
Three-stage Clos switching network

Blocking probability in switching networks



$$E_T = E_{in} + E_{ex}$$

Switching network as GEIG(ml)

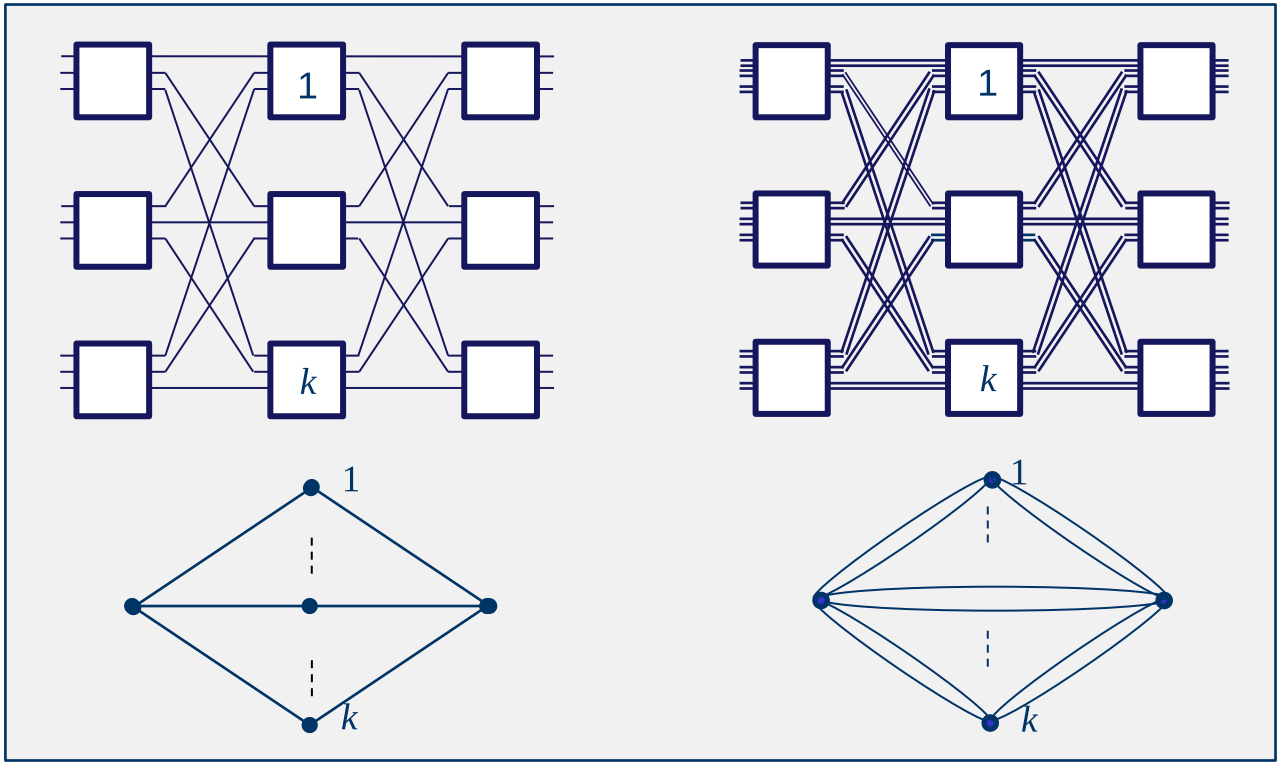


$$\text{Switching Network}\,(k, f, A_1, A_2, \ldots, A_M, t_1, t_2, \ldots, t_M)$$

$$\text{GEIG(ml)}\,(k, f, A_1, A_2, \ldots, A_M, t_1, t_2, \ldots, t_M, d_1, d_2, \ldots, d_M)$$

*Hanczewski S., Sobieraj M., Stasiak M.D.: The direct method of effective availability for switching networks with multi-service traffic, **IEICE Transactions on Communications**, vol.E99-B , No.6, 2016, pp.1291-1301.

Switching network as GEIG(ml)

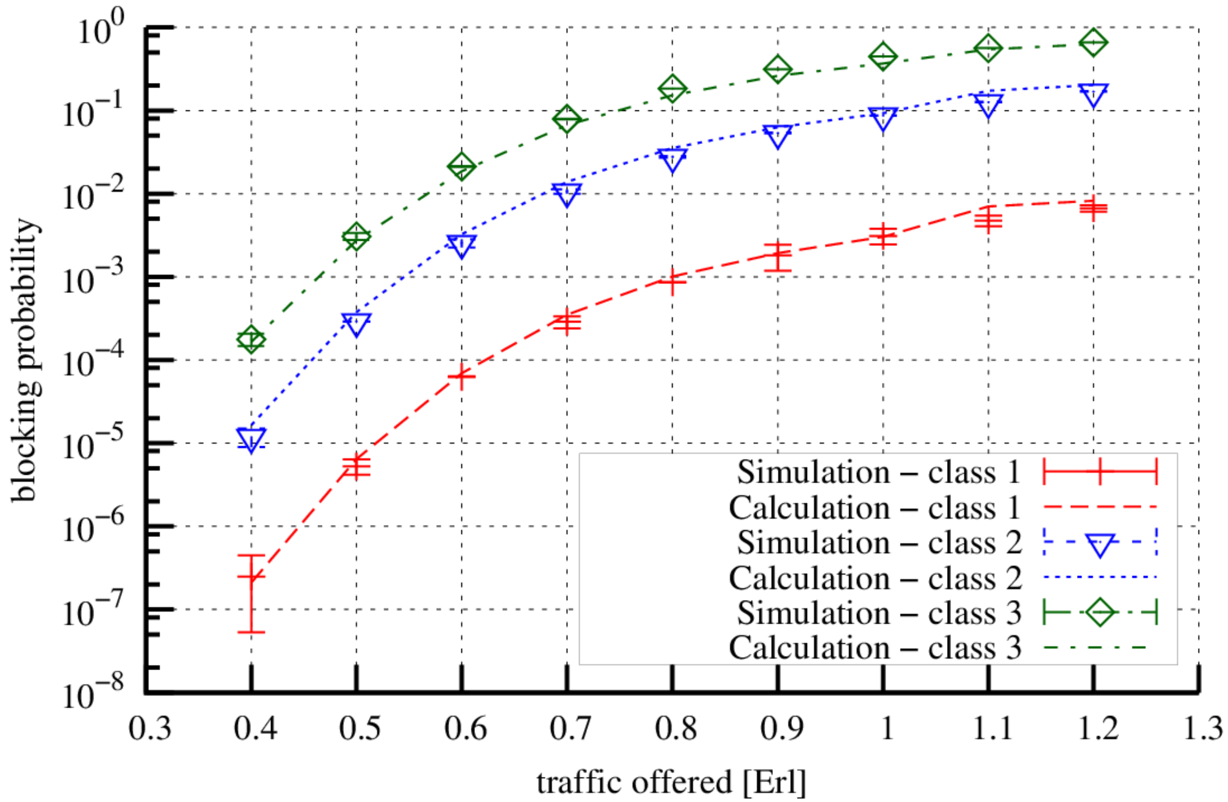


$$d_i = F(\text{graph}, A_1, A_2, \dots, A_M, t_1, t_2, \dots, t_M)$$

*Hanczewski S., Sobieraj M., Stasiak M.D.: The direct method of effective availability for switching networks with multi-service traffic, **IEICE Transactions on Communications**, vol.E99-B , No.6, 2016, pp.1291-1301.

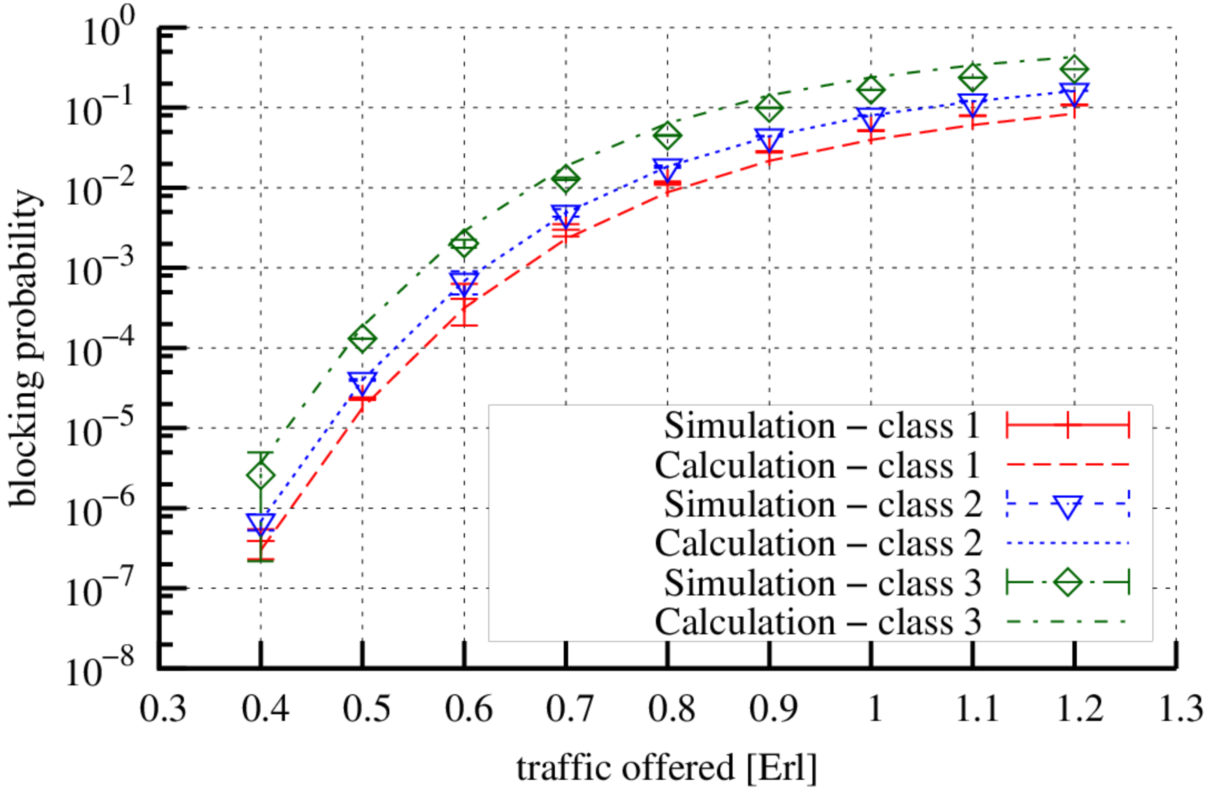
Switching networks (1)

Structure of the system: kxk=4x4, f=30, t_1=1, t_2=4, t_4=6

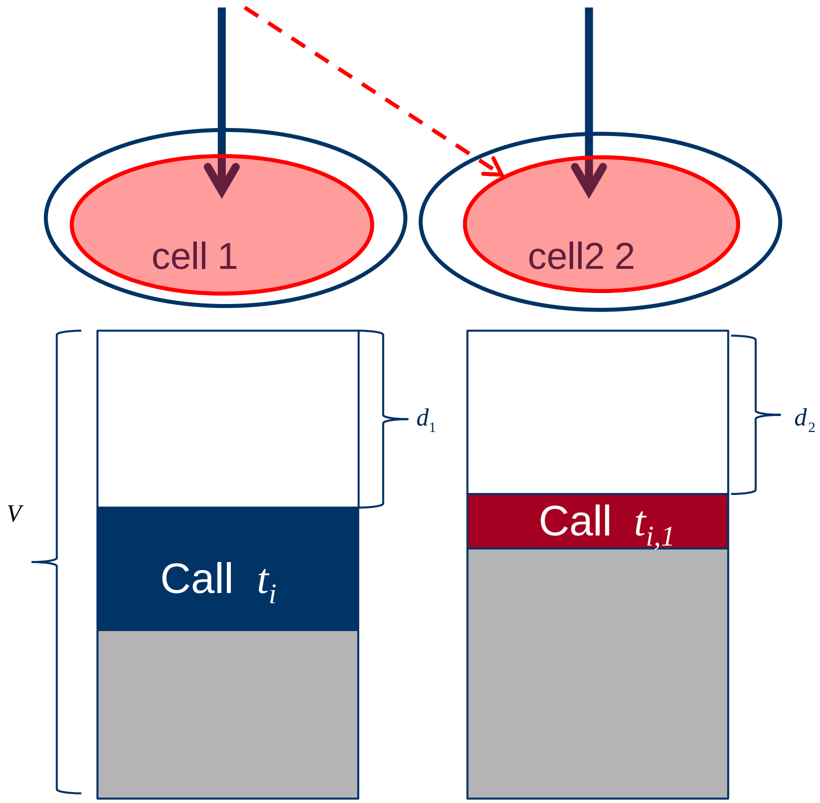


Switching networks (2)

Structure of the system: $kx2k=4x8$ (first stage), $2kx2k=8x8$ (second stage), $2kxk=8x4$ (third stage) $f_{ex}=30$, $f_{in}=20$ $t_1=1$, $t_2=2$, $t_4=6$



Radio interface



$$d_{cell} = V - \sum_{k=1}^{S} \sum_{i=1}^{M} t_{i,k} N_{i,k}$$

$t_{i,k}$ - number of AUs required by class i call in a given cell and generated in neighbouring cell k,

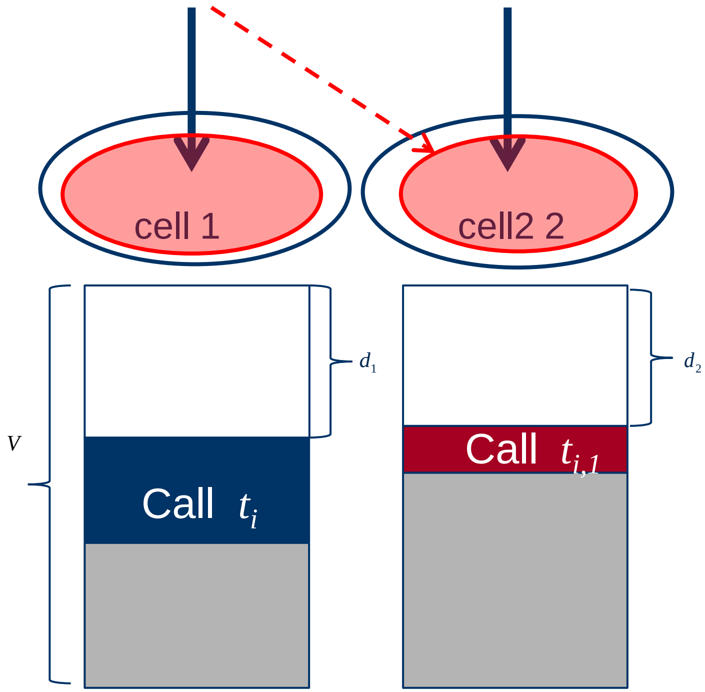
$N_{i,k}$ - average number of class i call serviced in neighbouring cell k,

S - number of neighbouring cells,

V - theoretical capacity of the cell.

$$t_{i,k} = F(t_i, \text{interferences})$$

Radio interface as EIG*



$$\mathrm{Cell}(S, V, A_1, A_2, \ldots, A_M, t_1, t_2, \ldots, t_M)$$

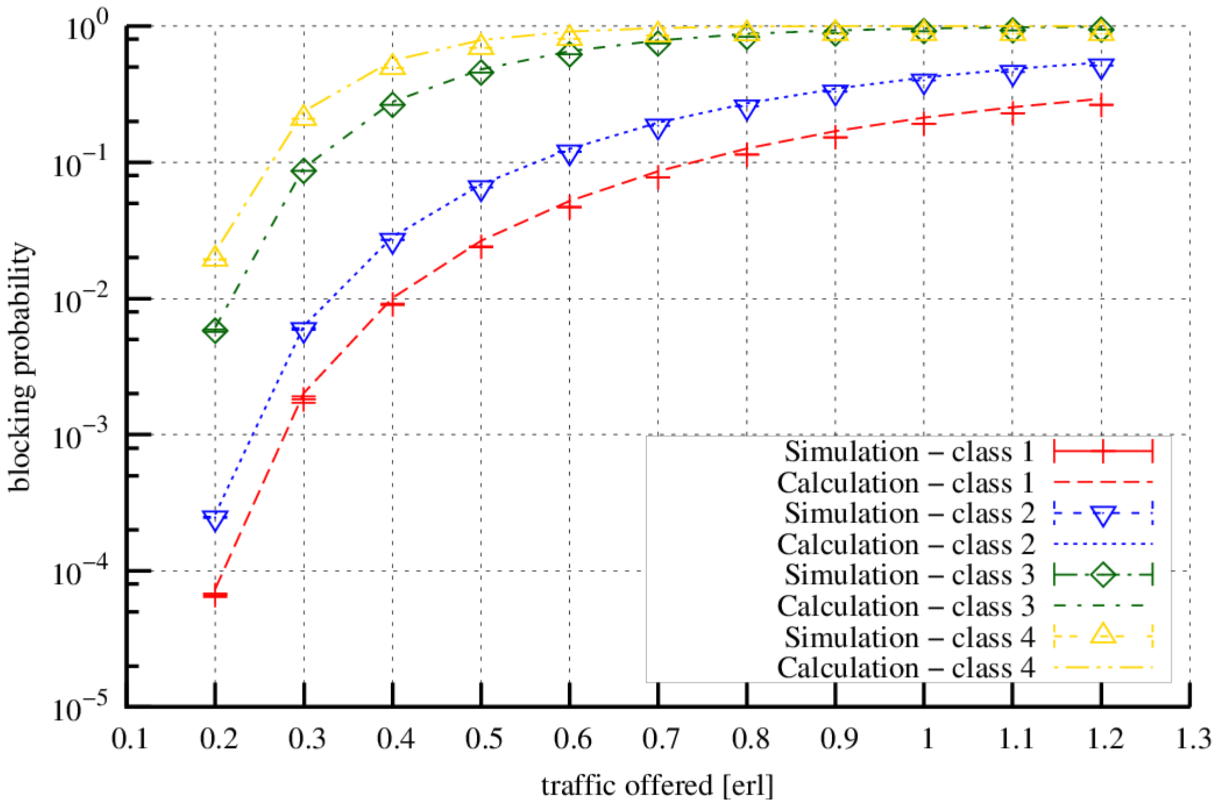
$$\Downarrow$$

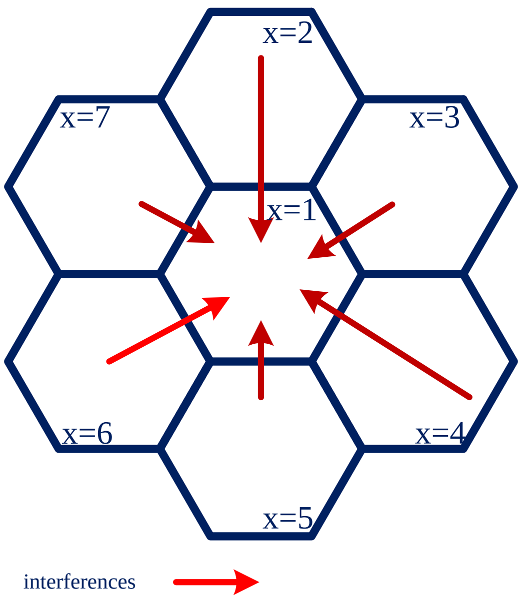
$$\mathrm{EIG}(S, V, A_1, A_2, \ldots, A_M, t_1, t_2, \ldots, t_M, d_{cell})$$

*Głąbowski M., S. Hanczewski, M. Stasiak: The application of the Erlang's Ideal Grading for modelling of UMTS cells, Proc. *8th International Symposium on Communication Systems, Networks & Digital Signal Processing* (CSNDSP), 2012, DOI: 10.1109/CSNDSP.2012.6292773.

Radio interface (1)

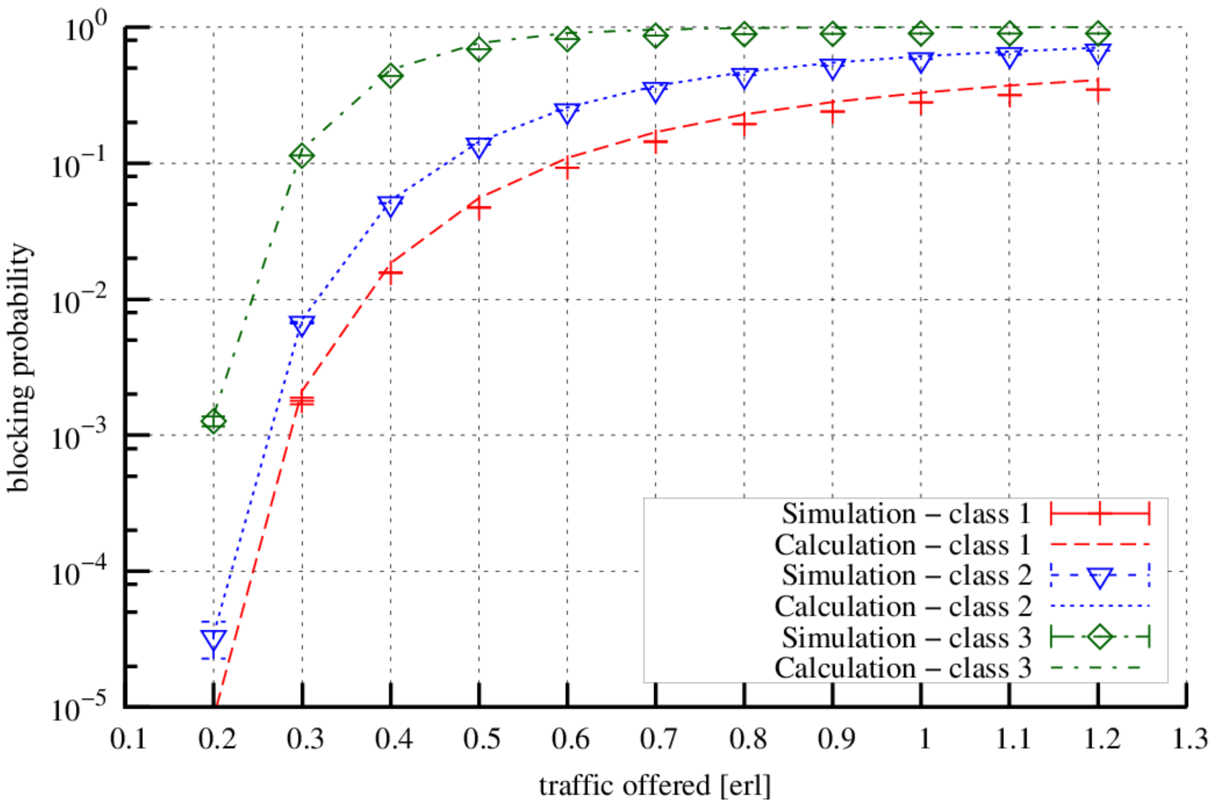
Structure of the system: V=200, t_1=1, t_2=5, t_3=10, t_4=20.

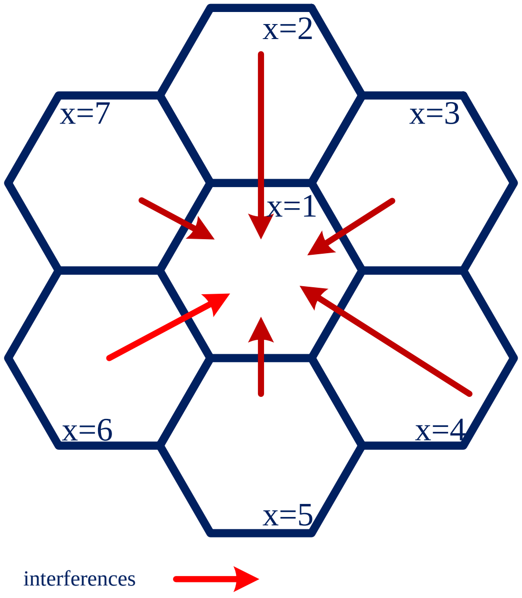




Radio interface (2)

Structure of the system: V=200, t_1=1, t_2=5, t_4=20.





Conclusion

- Multiservice state-dependent systems can be approximated by multi-service EIG models.
- The problem is to find the value of availability.
- The results of simulations have confirmed fair accuracy of the presented approach for modelling state-dependent systems on the basis of EIG.