Challenge of Time-Reversal MUSIC Imaging with a Limited Array

Heedong Choi, Yasutaka Ogawa, Toshihiko Nishimura, and Takeo Ohgane
Graduate School of Information Science and Technology, Hokkaido University
Sapporo 060-0814, Japan, {hdchoi, ogawa, nishim, ohgane}@m-icl.ist.hokudai.ac.jp

Abstract

This paper describes TR-MUSIC combined with a gating technique. TR-MUSIC needs a greater number of antenna elements than scatterers. This is not preferable for real environments due to the limitation of antenna arrays. In this paper, we propose to relax this restriction using a gating technique before applying TR-MUSIC.

Keywords : Time-Reversal Technique TR-MUSIC Imaging Gating Ultawideband

1. Introduction

The time-reversal (TR) based imaging technique [1] has been actively studied with unique capabilities for detection and imaging of obscured scatterers (targets). One of the methodologies, the TR-MUSIC (Time Reversal MUltiple SIgnal Classification) technique, has also been developed and considered in both single-frequency [2] and ultrawideband signals [3] due to superresolution. Fundamentally, TR-MUSIC is proposed under the condition that the number of transceivers N is larger than that of targets M ($N > M$) [2]. However, it may be difficult for this condition to be met because we cannot have a large number of antenna elements. This provides the main motivation of this research. The concept of the time-domain gating has been proposed for antenna measurement [4], and this is applicable also to the TR-MUSIC imaging method [5]. By using the time-domain gating technique for the TR-MUSIC imaging, we can eliminate the noise component outside of the gate; thus we obtain the reliable image due to the increase in SNR [5]. The wideband technique which has been developed recently provides a potential to beat the conventional range limitation with an unprecedented resolution capability. Therefore, if we extract the time-domain response around the desired corresponding time without damage to the signal(s), we can apply the TR-MUSIC imaging to the gated region where we have fewer targets. In this paper, we introduce a novel TR-MUSIC imaging approach for a case where the condition $N > M$ does not hold by using the gating technique.

2. Time-Reversal MUSIC Algorithm with the Gating Technique

2.1 Imaging Algorithm

When the number of antenna elements in the time reversal array (TRA) is N, the TR-MUSIC imaging method utilizes an $N \times N$ multistatic data matrix (MDM) denoted by $\mathbf{K}(\omega) = \{k_{i,j}(\omega) \text{ for } i, j = 1, 2, \ldots, N\}$, where ω is an angular frequency and bold letters denote vectors and matrices. Using the MDM, the time reversal operator (TRO) is defined as the self-adjoint matrix: $\mathbf{T}(\omega) = \mathbf{K}^{\dagger}(\omega)\mathbf{K}(\omega)$, where the superscript † denotes the Hermitian conjugate. Then, we apply the singular value decomposition (SVD) to the MDM to get: $\mathbf{K}(\omega) = \mathbf{U}(\omega)\mathbf{\Lambda}(\omega)\mathbf{V}^{\dagger}(\omega)$, where $\mathbf{U}(\omega)$ and $\mathbf{V}(\omega)$ are unitary matrices, $\mathbf{\Lambda}(\omega)$ is a diagonal and real-valued matrix with singular values $\lambda_1(\omega)$, $\lambda_2(\omega), \ldots, \lambda_N(\omega)$. Using SVD of the MDM, we can rewrite the TRO as $\mathbf{T}(\omega) = \mathbf{V}(\omega)\mathbf{S}(\omega)\mathbf{V}^{\dagger}(\omega)$, where $\mathbf{S}(\omega) = \mathbf{\Lambda}^{\dagger}(\omega)\mathbf{\Lambda}(\omega)$ is the real-valued diagonal matrix with eigenvalues $\lambda_1^2(\omega)$, $\lambda_2^2(\omega)$, …, $\lambda_N^2(\omega)$. The signal subspace Ss of the TRO is spanned by the eigenvectors with non-zero eigenvalues, and the noise subspace Ns is spanned by the eigenvectors having almost zero eigenvalues. That is, $Ss = \{\mathbf{v}_1(\omega), \mathbf{v}_2(\omega), \ldots, \mathbf{v}_M(\omega)\}$ with $\lambda_1(\omega) > \lambda_2(\omega) > \ldots > \lambda_M(\omega) > 0$ and $Ns =$

$\{\mathbf{v}_{M+1}(\omega), \mathbf{v}_{M+2}(\omega),..., \mathbf{v}_N(\omega)\}$ with $\lambda_{M+1}(\omega) \approx \lambda_{M+2}(\omega) \approx ... \approx \lambda_N(\omega) \approx 0$. Since the noise subspace *Ns* is always orthogonal to the signal subspace *Ss*, the Green's function vectors corresponding to the targets must be orthogonal to the noise subspace *Ns*. This provides the TR-MUSIC imaging pseudo-spectrum expressed in the form [2][3]

$$D(\mathbf{x}_p, \omega) = \left(\sum_{s=M+1}^{N} |< \mathbf{g}(\mathbf{x}_p, \omega), \mathbf{v}_s^*(\omega) >| \right)^{-1}, \tag{1}$$

where $\mathbf{x}_p$ represents the search point vector, the angular brackets $< \cdot >$ represent the inner product, the superscript asterisk * denotes the complex conjugate, and $\mathbf{g}(\mathbf{x}_p,\omega) = [\mathrm{G}(\mathbf{x}_p,\boldsymbol{\alpha}_1,\omega), \mathrm{G}(\mathbf{x}_p,\boldsymbol{\alpha}_2,\omega), \ldots, \mathrm{G}(\mathbf{x}_p,\boldsymbol{\alpha}_N,\omega)]^{\mathrm{T}}$ is the background steering vector. $\mathbf{g}(\mathbf{x}_p,\omega)$ is also called the background Green's function vector because $\mathrm{G}(\mathbf{x}_p,\boldsymbol{\alpha}_i,\omega)$ is the Green's function, where $\boldsymbol{\alpha}_i$ ($i = 1, 2,\ldots, N$) represents the position of the ith antenna element and the superscript $^{\mathrm{T}}$ denotes the transpose.

2.2 **Formulation of the Proposed Method**

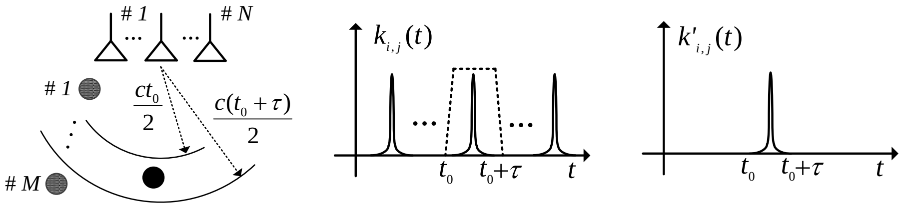
In this subsection, we propose the method that enables the TR-MUSIC imaging to detect more targets than the antenna elements. We introduce the frequency-domain response $R(\mathbf{r},\mathbf{r}',\omega)$ in the medium, which is the response at $\mathbf{r}'$ resulting from a unit amplitude source located at $\mathbf{r}$. Note that $R(\mathbf{r},\mathbf{r}',\omega)$ includes noise. When $\mathbf{x}_m = (x_m, y_m)$ is the position of the mth target, the frequency-domain response between the ith antenna element and the mth target is represented as $R(\boldsymbol{\alpha}_i,\mathbf{x}_m,\omega) = A(\omega) \cdot \exp(-j\omega(t_d/2)) + n(\omega)$ where $A(\omega)$ denotes a signal parameter, t_d represents the round trip time delay between the two points, and $n(\omega)$ is a noise component. By using this formulation, the ith row and jth column element of MDM is given by

$$k_{i,j}(\omega) = \sum_{m=1}^{M} \rho_m(\omega) R^*(\boldsymbol{\alpha}_i, \mathbf{x}_m, \omega) R(\mathbf{x}_m, \boldsymbol{\alpha}_j, \omega) \tag{2}$$

where $\rho_m(\omega)$ is the scattering coefficient of the mth target for $m = 1, 2, \ldots, M$. We can express the equation (2) in simple matrix notation by using steering vectors $\mathbf{r}(\mathbf{x}_m,\omega) = [R(\boldsymbol{\alpha}_1,\mathbf{x}_m,\omega), R(\boldsymbol{\alpha}_2,\mathbf{x}_m,\omega), \ldots, R(\boldsymbol{\alpha}_N,\mathbf{x}_m,\omega)]^{\mathrm{T}}$. In terms of these column vectors, the MDM is represented by

$$\mathbf{K}(\omega) = \sum_{m=1}^{M} \rho_m(\omega) \mathbf{r}^*(\mathbf{x}_m, \omega) \mathbf{r}^T(\mathbf{x}_m, \omega). \tag{3}$$

Next, we state a scheme to reduce the number of targets to apply the TR-MUSIC imaging. For simplicity, we only consider reflections from the targets embedded in the medium, and we ignore reflections between the targets. Therefore, the received signal at each antenna includes the combination of the reflected signals from all targets. As depicted in Fig. 1 (a), the remote sensing system can be conceptually made up of N antenna elements and M scatterers. In this case, the time-domain response of each target exists at the corresponding time as shown in Fig. 1 (b); thus we can extract the time-domain response(s) from t_0 to $t_0+\tau$ by the gating technique as shown in Fig. 1 (c). It should be noted that the number of the targets is reduced by the gating in such a way that we can apply the TR-MUSIC imaging.



(a) Schematic illustration of the remote sensing system

(b) Received signal responses

(c) Gated signal responses

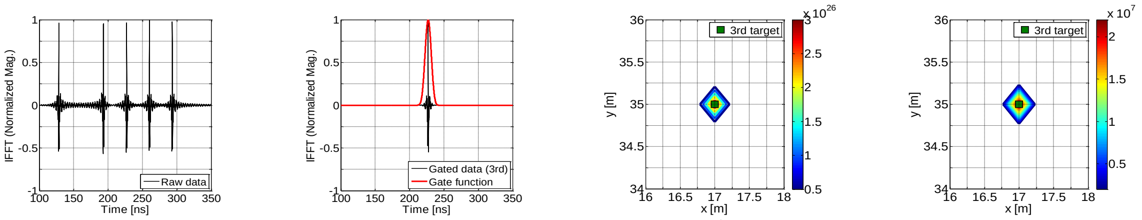
Figure 1: Concept of time-domain gating

Now, we state the steps of the proposed method. We first obtain the frequency-domain data $\mathbf{K}(\omega)$ using the TRA. By means of the inverse Fourier transform, we transform the frequency-domain response $k_{i,j}(\omega)$ into the time-domain response $k_{i,j}(t)$. After the transformation, we extract signal response(s) in the region from t_0 to $t_0+\tau$ by applying the gating technique. The region is determined in such a way that the number of target responses is fewer than that of antenna elements

N. Then, we obtain the gated response $k'_{i,j}(t)$ for $i, j = 1, 2, \ldots, N$ as shown in Fig. 1 (c). However, the gating technique is not effective when we cannot classify the responses or the sidelobes of responses are extremely overlapped with one another; thus wideband data are needed. We assume sufficiently wideband data throughout this paper. Next, we transform the gated response $k'_{i,j}(t)$ into the frequency-domain response $k'_{i,j}(\omega)$ using the Fourier transform, and we remake the gated MDM $\mathbf{K}'(\omega)$. The new MDM $\mathbf{K}'(\omega)$ consists of fewer target responses than the antenna elements. This enables the TR-MUSIC imaging to work without the limitation of the number of antenna elements. We detect the target(s) in the gating region applying the TR-MUSIC imaging technique to the modified MDM $\mathbf{K}'(\omega)$. Changing the gating region, we repeat this process. Then, we can detect all the targets in the whole area. It should be noted that the gating suppresses also noise outside the gating region, and increases the signal-to-noise ratio (SNR). This improves the performance of the TR-MUSIC imaging [5].

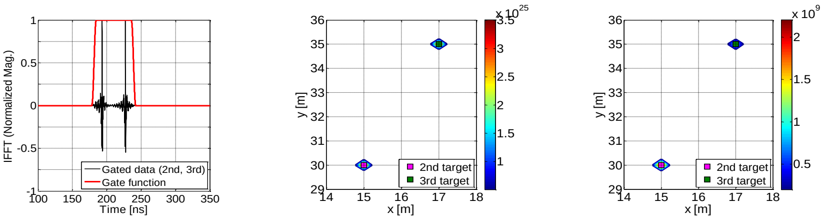
3. Numerical Simulation Results

Computer simulations have been conducted to evaluate the performance of the proposed scheme. The frequency band is from 2 GHz to 3 GHz (center frequency f_c = 2.5 GHz, and the sampling frequency separation δf = 1 MHz), and we used a linear array. The array had four antenna elements ($N = 4$) with $\lambda_c/2$ spacing where λ_c is the wavelength corresponding to f_c. It was aligned in parallel with the x-axis, and the central antenna was on the coordinate (15m, 1m). Five targets (M = 5) having identical strengths $\rho_m = 1$ (m = 1, 2, 3, 4 and 5) were located on (20m, 25m), (15m, 30m), (17m, 35m), (5m, 40m) and (30m, 45m). In order to verify the effectiveness of the proposed scheme, we considered the "Reference case" where the number of targets in the whole area is the same as that in the gating region. For example, if we select two responses in the gating region, there are only the two scatterers in the reference case. We used a single snapshot of data. The amplitude of the transmitted signal from each antenna was 1. For simplicity, we ignored the propagation loss and constant scalars in the Green's function. Thus, the amplitude of the received signal from each scatterer was 1. Also, we considered several noise conditions, ideal noiseless case and SNR from 20 to 60dB. Note that signal power is defined as the received signal power from each scatterer.



(a) Raw data in time domain (b) Gated data and window (c) Reference (d) Proposed method

Figure 2: Extraction of a single signal response (3rd target)

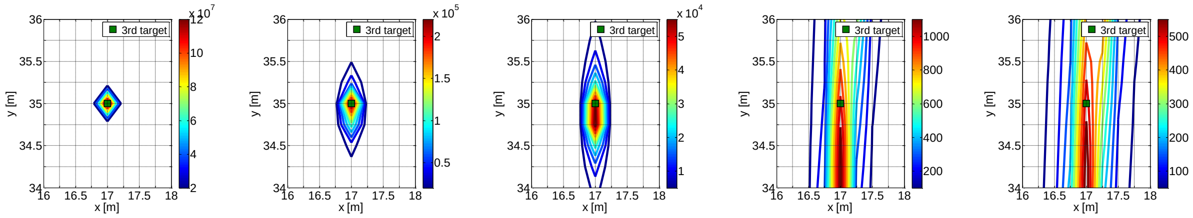


(a) Gated data and window (b) Reference (c) Proposed method

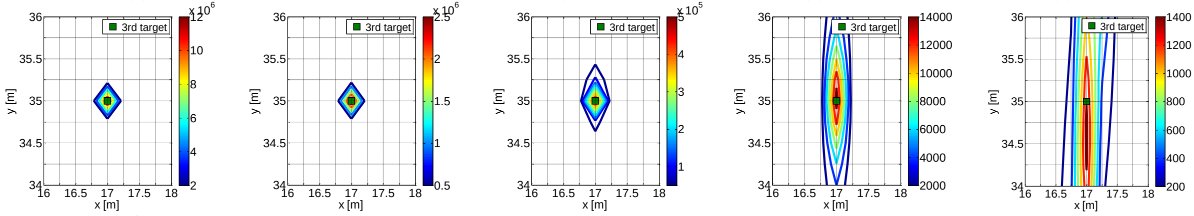
Figure 3: Extraction of two signal responses (2nd, 3rd target)

We first investigate an ideal noiseless environment. Figures 2 and 3 show the time-domain response of $k_{1,1}(t)$, gated responses, window functions and imaging functions obtained for the center frequency. As observed from Fig. 2 (a), the five peaks corresponding to the five scatterers can be detected by the 1GHz bandwidth data. Here, we consider imaging of the scatterer at (17m, 35m). The scatterer corresponds to the third response in the time domain shown in Fig. 2 (a). To obtain the imaging, we used the gate center T_0 = 227ns, gate span T_g = 30ns, and the Chebyshev window function as shown in Fig. 2 (b). As we see from Figs. 2 (c) and (d), the pseudo-spectra of TR-MUSIC are very similar to each other. We can say that the proposed technique works well for the target. Also, we carried out the simulation for extracting two targets at (15m, 30m) and (17m, 35m). They correspond to the second and third responses in the time domain. In this case, we used the gate center T_0 = 210ns, gate span T_g = 64ns, and the Raised Cosine window function with roll-off 0.2 as shown in Fig. 3 (a). Like the preceding case, we see from Figs. 3 (b) and (c) that the results of TR-MUSIC imaging are similar, and that the two targets are resolved precisely. From these results, we can say that the proposed method for the TR-MUSIC imaging algorithm can extract the targets even when $N > M$ does not hold.

Figure 4 shows the performance results of the proposed method in the presence of noise for the third target (17m, 35m). The upper row figures show the pseudo-spectra of TR-MUSIC for the reference, and the lower figures show the pseudo-spectra of TR-MUSIC for the proposed method in various noise environments. As observed, the proposed method provides more accurate images than the reference case. By applying the gating to the time-domain response, we can reduce the noise component. This causes the increase of SNR and gives the stable performance. From these results, it has been shown that the proposed TR-MUSIC with the time-domain gating technique can both relax the restriction of the number of antennas and reduce the noise power.



Reference (a) 60dB (b) 50dB (c) 40dB (d) 30dB (e) 20dB



Proposed (f) 60dB (g) 50dB (h) 40dB (i) 30dB (j) 20dB

Figure 4: TR-MUSIC pseudo-spectra with different noise (3rd target)

4. Conclusions

In this paper, we have described the TR-MUSIC imaging algorithm accompanied with the gating technique. Although the TR-MUSIC algorithm was developed for a condition where the number of transceivers N is larger than that of targets M, the proposed method can provide the image with the computational cost even when the condition does not hold. In this paper, we assumed sufficiently wideband data to separate targets in the time domain. Thus, further study should be done for this issue in future.

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