Effects of Raindrop Size Distribution on Millimeter Wave Attenuation, XPD and Their Relations

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1. Introduction

It is well known that rain-induced attenuation value depends on raindrop size distribution(DSD) . As typical DSD, Marshall and Palmer distribution(MP-DSD) has been well used for moderate rain case. Due to rapid measuring technology progress and increase of interest on DSD, more precise DSD model has been proposed using three parameters such as log-normal distribution model or Gamma distribution one(Gamma-DSD). These three parameter DSD models have different features from MP-DSD especially in small raindrop size region. As MP-DSD model adopts exponential function, DSD increases monotonically as diameter of raindrop decreases. On the otherhand, the above three parameter DSD has such feature that DSD has a peak at certain raindrop size value. Due to such difference of DSD in small raindrop size region, it is expected that, especially in millimeter wave bands, propagation characteristics such as attenuation and depolarization have difference between MP-DSD and Gamma-DSD cases.

In this paper, I calculate rain-induced attenuation and depolarization at from 8 to 100GHz assuming both MP-DSD and Gamma-DSD using Pruppacher-Pitter type raindrop shape. The relative differences of attenuation and XPD between the results are derived. Moreover, XPD prediction error when one uses the relation between attenuation and XPD which is recommended in ITU Recommendation which was derived under the assumption of MP-DSD.

2. MP-DSD and Gamma-DSD

In Fig.1, the two DSD, N(D) are expressed by equations as a function of raindrop diameter, D[cm]. In this paper, as three parameter DSD, I adopted Gamma distribution type one whose parameters are proposed by Prof.Kozu[1]. It is well known that the MP-DSD can also be expressed the same equation but the parameter m is equal to zero. By using DSDs, I can calculate rainfall rates, R [mm/h], and specific attenuation, k [dB/km], under the assumption of raidrop falling velocity, v(D), raindrop density, rho, and total cross section of raindrop, sigma.

In Fig.2, the DSDs are compared. As shown in the figure, Gamma-DSD has a peak at about 0.5mm of raindrop diameter. As in the paramters of DSD rainfall rate R is included and R can be calculated from DSD as shown in Fig.1, I can compare R values which is used to calculate DSD and is calculated from DSD. In this calculation, Gunn and Kinzer raindrop falling velocity relation was assumed. Fig.3 shows the comparison between the two Rs. The ordinate is expressed as relative difference [%] of two Rs. It is known that in the MP-DSD case calculated rainfall rate is larger than assumed one for DSD derivation. The relative difference is about 10-15% in the range of rainfall rate from 0 to 50mm/h. In case of Gamma-DSD, the relative difference ranges from -2 to 0% in the same rainfall rate range. From these results, it is expected that the estimated rain-induced attenuation and depolarization using MP-DSD may be overestimated.

3. Attenuation and XPD calculation

In Figs.4 and 5, calculated rain-induced attenuations at from 10 to 100GHz are shown as a function of rainfall rate for MP-DSD and Gamma-DSD, respectively. As I expected previously, it is observed that the attenuation values in MP-DSD case are larger than those in Gamma-DSD case. The relative difference of attenuation values are defined as follows and the dependence of that relative difference on frequency and ranfall rate are shown in Fig.6 as a contour map.

Relative difference [%] = (Attenuation(Gamma) – Attenuation (MP))/ Attenuation(MP) *100

From Fig.6, it is noticed that relative attenuation difference becomes very large up to -35% in frequency range less than about 20GHz.

I also calculated XPD values using both DSDs. In Fig.7, the relative difference of XPD values are shown as a coutour map similar to Fig.6. It is noticed that the relative XPD difference becomes large up to 20% at frequency about 20GHz. Also it is observed that relative difference becomes less than 4% in the ranges of frequency larger than 50GHz and rainfall rates less than 10mm/h. The results shown in both Figs.6 and 7 are interpreted from the fact that the MP-DSD has larger effective rainfall rates and degradation in attenuation and XPD are overestimated. This effect is dominant especially in attenuation calculation because of large relative difference shown in Fig.6.

4. XPD prediction from Attenuation value using ITU recommendation

In Fig.8, the relation between attenuation, A[dB], and XPD[dB] with several parameters such as rainfall rate, R[mm/h], standard deviation of raindrop canting angles, sigma, polarization canting angle, phi-tau, elevation angle, ipsiron, path length, l [km], and approximation constants, a, b, c and d. In Fig.9, these parameters are shown as a function of frequency, f[GHz]. The parameters in Fig.9 were derived using MP-DSD and a part of the parameters are reflected in ITU Recommendation 618[2]. It is expected that the estimated XPD values from measured attenuation value using the relation and parameters shownn in Figs.8 and 9 have errors from realistic XPD values.

In Figs.10 and 11, relative XPD prediction errors defined as the following equation are shown as coutour map in cases MP-DSD and Gamma-DSD, respectively.

Relative XPD error[%] =(XPD(estimated) – XPD(theory))/ XPD(theory) * 100

It is quite reasonable that the MP-DSD case in Fig.10 the relative error becomes small within -5% and +5% even in the ranges of frequency from 8 to 100GHz and rainfall rates from 0 to 50mm/h because of the relation between attenuation and XPD are derived from MP-DSD assumption. In Fig.11, the relative errors derived from theoretical attenuation and XPD calculation with Gamma-DSD are shown, it is interesting to note that although the each attenuation and XPD values have difference as shown in Figs.6 and 7, relative XPD prediction error remains between -5% and 5% in the practical rainfall rate range less than 30mm/h except the frequency less than about 10GHz. This observation may be caused by cancellation of errors in attenuation and XPD.

5. Conclusion

By assuming 2 kinds of DSD, MP-DSD and Gamma-DSD, theoretical calculation of rain-induced attenuation and depolarization has been done. Differences in attenuation and XPD between the DSDs are evaluated. Although those differences are not negligible, predicted XPD values using ITU recommendation have relative errors from -5% to 5% in the practical frequency and rainfall rate regions.

References

[1] T. Kozu, “Estimation of raindrop size distribution from spaceborne radar measurement”, Doctoral thesis from Kyoto University, pp.77-91, 1991.

[2] H. Fukuchi, “Relationship between rain attenuation and depolarization up to 100GHz”, Int. Symp. Ant. And Prop.(ISAP2008), Taipei, Oct. 2008.

$$N(D) = N_0 D^m \exp(-\Lambda D)$$

$$\textit{Gamma}: \quad m = 3, \quad N_0 = 1.19 \times 10^3 R^{-0.352}, \quad \Lambda = 67.8 R^{-0.176}$$

$$\textit{MP} \quad : \quad m = 0, \quad N_0 = 0.08 \quad , \quad \Lambda = 41.0 R^{-0.21}$$

Rainfall Rate $R = \int_0^\infty \frac{4}{3}\pi\rho\left(\frac{D}{2}\right)^3 v(D)\cdot N(D)\cdot dD$

Specific Attenuation :(dB/km) $k = 10^4 \log e \int_0^\infty \sigma_t(D)\cdot N(D)\cdot dD$

Fig.1 MP-DSD and Gamma-DSD

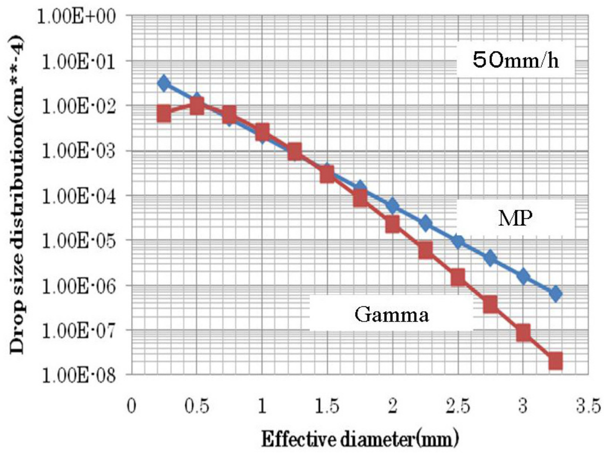

Fig.2 Comparison of DSDs

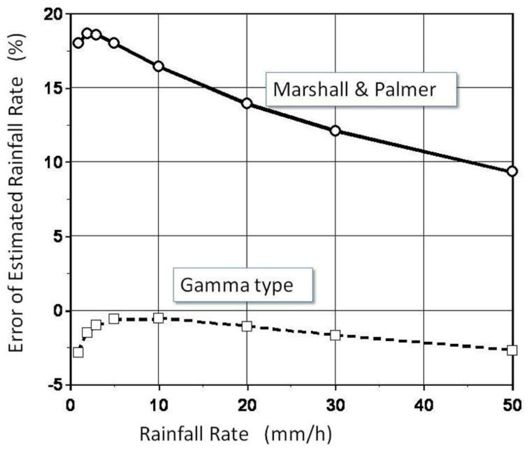

Fig.3 Error of derived rainfall rates

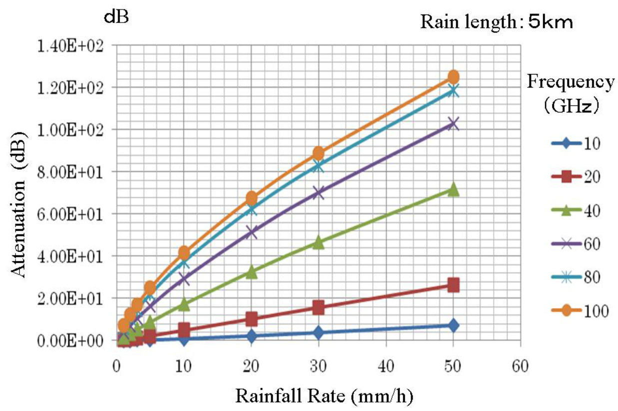

Fig.4 Attenuation calculation using MP-DSD

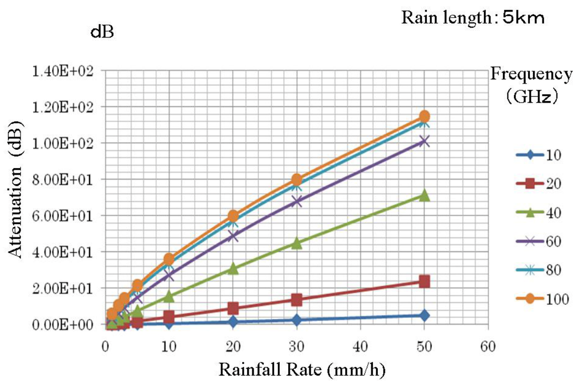

Fig.5 Attenuation calculation using Gamma-DSD

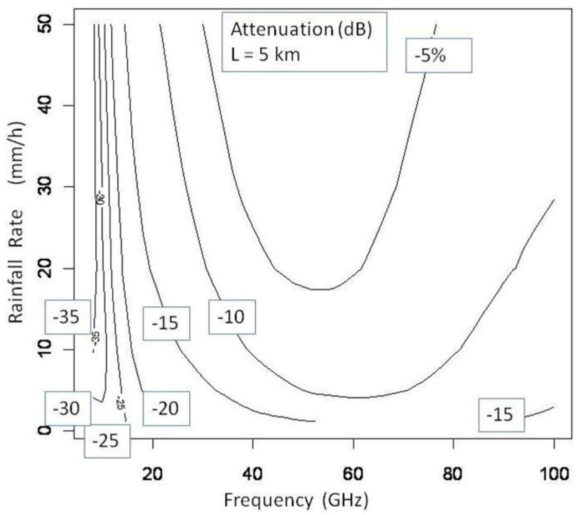

Fig.6 Contour of relative attenuation difference

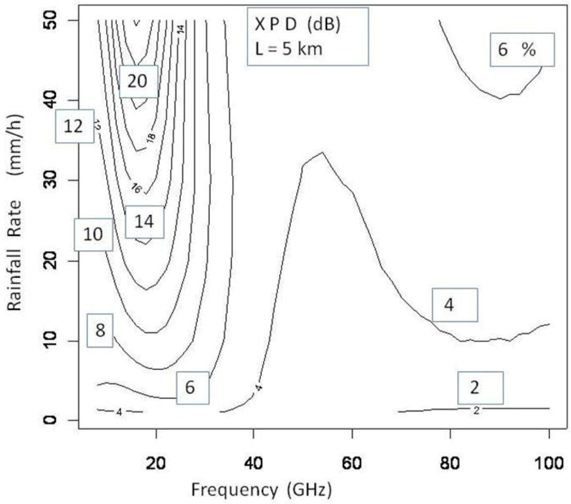

Fig.7 Contour of relative XPD difference

$$XPD = U - V \log A$$

$$A = a R^{b} l$$ Specific Attenuation

Parameter Relations

$$D = c R^{d}$$ Differential Propagation Constant

$$V = 20\, d / b$$

$$U = 0.0053\, \sigma^{2} - 20 \log(\sin 2|\phi - \tau|) - 40 \log(\cos \varepsilon) + u$$

$$u = -20 \log\left(\frac{c}{2 a^{d/b}}\right) + (V - 20) \log l$$

Fig.8 Relations between attenuation and XPD

Parameter V	Frequency (GHz) (fl < f < f2) f1	f2	Parameter u−(V−20) log l	Frequency (GHz) (fl < f < f2) f1	f2
$30.8\, f^{-0.21}$	6	9	60.0 log f −28.3	6	9
$12.8\, f^{0.19}$	9	20	26.0 log f +4.1	9	36
22.6	20	40	35.9 log f −11.3	36	100
$13.0\, f^{0.15}$	40	100			

Fig.9 Parameters in the relation between Attenuation and XPD

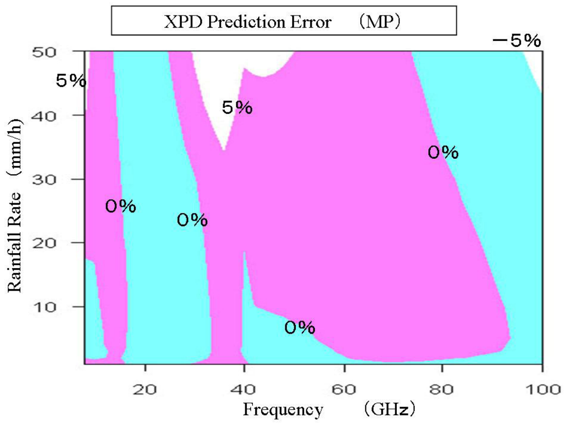

Fig.10 Relative XPD prediction Error (MP-DSD)

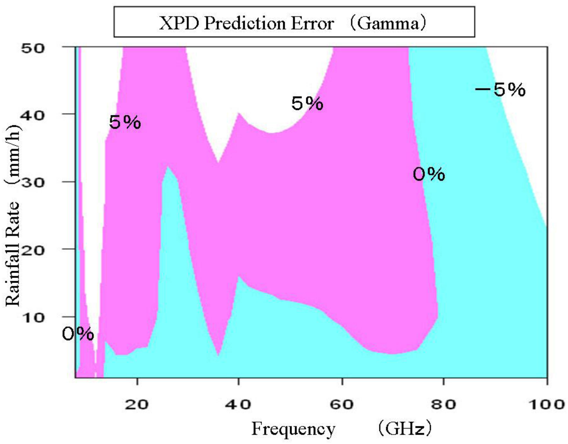

Fig.11 Relative XPD prediction Error (Gamma-DSD)