A Generalized Formula for Spatial Correlation in Three-dimensional Fading Environment

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Abstract

This paper presents a generalized formula of spatial correlation (SC) approximation with closed-form solution for three-dimensional (3D) incoming waves in multipath fading environment. The derivation of the formula is based on a novel model of angular power spectrum (APS) and the validity is verified by simulation results.

Keywords : spatial correlation, three-dimensional, angular power spectrum

1. Introduction

The research on modeling for multipath channels has been crucial in designing wireless communication system. To develop adequate channel models, the knowledge of spatial correlation (SC) performance of signals is highly anticipated. Prior works such as [1]-[4] assumed the power azimuth spectrum (PAS) as a cosine function, a uniform distribution, a Gaussian function, and a Laplacian function to approximated SC of two-dimensional (2D) angle of arrival (AoA). However it leads to a significant error in some multipath richness environments. In recent work [5], the three-dimensional (3D) AoA is considered under very specific spherically uniform assumption, which is not likely to accurately characterize propagations in general. In [6] the authors derive a generalized Doppler power spectrum for arbitrary 3D scattering environments, but the resultant form is in high complexity and can hardly be adapted to practical problems. In the present paper, a method that can approximate SC in general in 3D multipath fading environment is proposed. The model achieves a closed-form solution with neat terms and doesn't deprive the practical applicability. The numerical results confirm the validity of the proposed model in conclusion.

2. Spatial Correlation Approximation

2.1 General Analysis of SC

Figure 1 shows an incoming wave in 3D space along direction vector $\boldsymbol{\alpha}$, defined as

$$\boldsymbol{\alpha} = \mathbf{i}\cos\phi\cos\theta + \mathbf{j}\sin\phi\cos\theta + \mathbf{k}\sin\theta \quad (1)$$

where $\{\mathbf{i},\mathbf{j},\mathbf{k}\}$ are the corresponding unit vectors in Cartesian coordinates, and $\{\theta,\phi\}$ are the elevation and azimuth of AoA in spherical coordinates, respectively. The angular power spectrum (APS) at the reception point, $\Omega(\theta,\phi)$ is defined as

$$\Omega(\theta,\phi) \equiv G(\theta,\phi)\,\Omega_P(\theta,\phi) \quad (2)$$

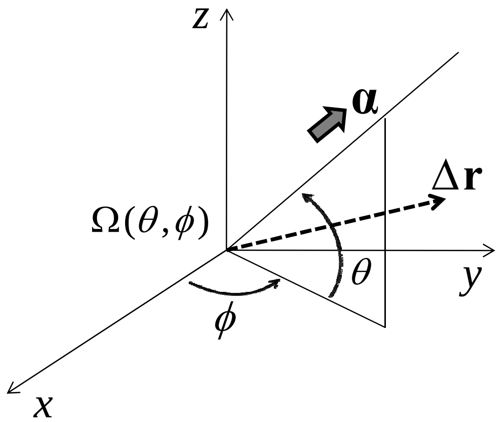

Fig. 1 An incident wave in 3D space.

where $G(\theta,\phi)$ denotes the antenna angular power gain and $\Omega_P(\theta,\phi)$ is the angular power distribution of multipath environment.

Given an arbitrary travel direction of the reception point $\Delta\mathbf{r}$, the spatial fading correlation of the incoming wave in (1) is calculated by [2]

$$\rho_a(\Delta\mathbf{r}) = \frac{1}{P_R}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\Omega(\theta,\phi)\exp(\ jk\Delta\mathbf{r}\cdot\boldsymbol{\alpha})\cos\ \theta\ d\phi d\theta \tag{3}$$

where k is the wave number defined as $k = 2\pi/\lambda$ with λ being the signal wavelength and P_R is the average received power under multipath condition represented as

$$P_R \equiv \int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\Omega(\theta,\phi)\cos\theta\, d\phi d\theta \tag{4}$$

Equation (3) provides a theoretical method of calculation of SC in spherical field. On the contrary side it drops practicability. SC is inclined to be approximated by three-dimensional Cartesian coordinates, given by

$$\Delta\mathbf{r} = \mathbf{i}\Delta x + \mathbf{j}\Delta y + \mathbf{k}\Delta z \tag{5}$$

where $\{\Delta x, \Delta y, \Delta z\}$ are the corresponding projections of $\Delta\mathbf{r}$ on Cartesian coordinates. According to (1) and (5), SC is approximated by

$$\begin{aligned}\rho_a(\Delta\mathbf{r}) &= \\ \frac{1}{P_R}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}&\Omega(\theta,\phi)\exp[\ jk(\Delta x\cos\phi\cos\theta+\Delta y\sin\phi\cos\theta+\Delta z\sin\theta)]\cos\theta d\phi d\theta \\ &\approx \rho_{a,x}(\Delta x)\rho_{a,y}(\Delta y)\rho_{a,z}(\Delta z)\end{aligned} \tag{6}$$

where $\{\rho_{a,x}(\Delta x)\rho_{a,y}(\Delta y)\rho_{a,z}(\Delta z)\}$ denote the corresponding spatial correlation functions of SC in Cartesian coordinates, and they are respectively given by

$$\rho_{a,x}(\Delta x) = \frac{1}{P_R}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\ \Omega(\theta,\phi)\exp(\ jk\Delta x\cos\phi\cos\theta)\cos\theta d\phi d\theta \tag{7}$$

$$\rho_{a,y}(\Delta y) = \frac{1}{P_R}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\ \Omega(\theta,\phi)\exp(\ jk\Delta y\sin\phi\cos\theta)\cos\theta d\phi d\theta \tag{8}$$

$$\rho_{a,z}(\Delta z) = \frac{1}{P_R}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}\ \Omega(\theta,\phi)\exp(\ jk\Delta z\sin\theta)\cos\theta d\phi d\theta \tag{9}$$

2.2 In The Case of $\Omega(\theta,\phi) = \cos^n\theta$

To analyze the effects of radio waves at the end of a link in different wireless scenarios which always result in a variety of distributions, we focus on the APS shape of

$$\Omega(\theta,\phi) = \cos^n\theta, \quad n \geq 0 \tag{10}$$

The key parameter n in (10) is used to develop 3D incident wave models in different multipath environments. For instance $n = 0$, we have $\Omega(\theta,\phi) = 1$, describing a reception pattern which is spherically uniform. More practically in another instance, considering a real antenna in 2D urban field such as half-wave dipole with $G(\theta,\phi) \approx \cos^2\theta$ and uniform PAS, n is reasonably around 2. For further consideration of cases of array antennas (AA), an equivalent pattern can be applied by behaving with an identical half-power beamwidth θ_{HPBW}.

Substituting (10) into (4), the result is given as

$$P_R = 2\pi\int_{-\pi/2}^{\pi/2}\cos^{n+1}\theta\, d\theta = 2\pi\sqrt{\pi}\,\Gamma(\frac{n}{2}+1)/\Gamma(\frac{n+3}{2}) \tag{11}$$

For $\rho_{a,x}(\Delta x)$, using the relation [7]

$$J_0(k\Delta x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(\,jk\Delta x\cos\phi)\,d\phi \qquad (12)$$

and (10), (11) to facilitate the integration, it is derived by

$$\begin{aligned}
\rho_{a,x}(\Delta x) &= \frac{2\pi}{P_R}\int_{-\pi/2}^{\pi/2}\cos^{n+1}\theta\, J_0(k\Delta x\cos\theta)d\theta \\
&= \frac{2\pi}{P_R}[\sqrt{\pi}\Gamma(\frac{n}{2}+1)\ {}_1F_2(\frac{n+2}{2};1,\frac{n+3}{2};-\frac{k^2\Delta x^2}{4})/\Gamma(\frac{n+3}{2})] \\
&= {}_1F_2(\frac{n+2}{2};1,\frac{n+3}{2};-\frac{k^2\Delta x^2}{4}) \qquad (13)
\end{aligned}$$

where $\Gamma(\cdot)$ is the Gamma function and ${}_pF_q(\cdot)$ is the generalized hypergeometric function.

For $\rho_{a,z}(\Delta z)$, with (10) and (11) it is derived by

$$\begin{aligned}
\rho_{a,z}(\Delta z) &= \frac{2\pi}{P_R}\int_{-\pi/2}^{\pi/2}\cos^{n+1}\theta\ \exp(\,jk\Delta z\sin\theta)d\theta \\
&= \frac{2\pi}{P_R}[\sqrt{\pi}\,\Gamma(\frac{n}{2}+1)\ {}_0F_1(\,;\frac{n+3}{2};-\frac{k^2\Delta z^2}{4})/\Gamma(\frac{n+3}{2})] \\
&= {}_0F_1(\,;\frac{n+3}{2};-\frac{k^2\Delta z^2}{4}) \qquad (14)
\end{aligned}$$

Because $\{\rho_{a,x}(\Delta x), \rho_{a,y}(\Delta y)\}$ hold symmetrical characteristics in azimuth plane, $\rho_{a,y}(\Delta y)$ is obtained straightforward from $\rho_{a,x}(\Delta x)$.

Classical analyses of SC in multipath environments turn out to be special cases of our proposed model. For example in the case $n=0$, which represents a spherically uniform pattern as we have explained, (13) and (14) reduce to [7]

$$\rho_{a,x}(\Delta x) = \mathrm{sinc}(k\Delta x) \qquad (15)$$

$$\rho_{a,z}(\Delta z) = \mathrm{sinc}(k\Delta z) \qquad (16)$$

respectively. For $n=2$, they reduce to [8]

$$\rho_{a,x}(\Delta x) = \frac{3}{2}\left\{\frac{\sin(k\Delta x)}{k\Delta x}\left[1-\frac{1}{(k\Delta x)^2}\right]+\frac{\cos(k\Delta x)}{(k\Delta x)^2}\right\} \qquad (17)$$

$$\rho_{a,z}(\Delta z) = \frac{3}{(k\Delta z)^2}\left(\frac{\sin k\Delta z}{k\Delta z}-\cos k\Delta z\right) \qquad (18)$$

respectively. For $n=\infty$, which signifies the case of 2D, they reduce to [7]

$$\rho_{a,x}(\Delta x) = \rho_{2D}(\Delta x) = J_0(k\Delta x) \qquad (19)$$

$$\rho_{a,z}(\Delta z) = 1 \qquad (20)$$

respectively.

3. Examples and Numerical Results

The patterns of $\Omega(\theta,\phi)=\cos^n\theta$ for $n=0,2,8$ are shown in Fig. 2. Additionally, a pattern of infinitesimal dipole array of two elements with vertical interval distance $d=\lambda$ is shown as an example. By normalizing their half-power beamwidth, the lobe of pattern of

$\Omega(\theta,\phi)=\cos^{23}\theta$ is found suitable to approach the main lobe of radiation pattern of the dipole array. As shown, increasing values of n intuitively describe antenna effects of strong directivity.

Finally for the same AA example, we approximate SC performances when travel directions of reception point are $\phi_r=30°$, $\theta_r=45°,60°,90°$ respectively. We compare the approximated results given by our formulas (13) and (14) with the theoretic values given by (3), as shown in Fig. 3. The good agreements for $\theta_r=45°,60°$ and an almost complete coincidence for $\theta_r=90°$ clarifies the validity of the proposed model.

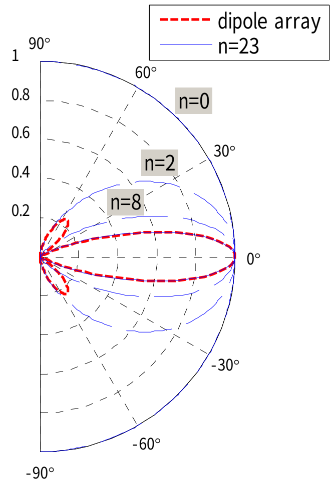

Fig. 2 The patterns of APS for values of $n=0,2,8$ and a value of $n=23$ which corresponds to θ_{HPBW} of infinitesimal dipole array.

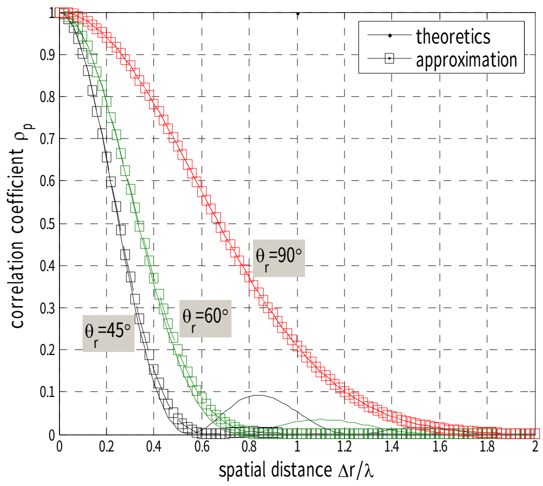

Fig. 3 SCs of theoretical values and approximated values, for n=23 with travel direction of $\phi_r=30°,\theta_r=45°,60°,90°$

4. Conclusion

A generalized model for approximation of SC for $\Omega(\theta,\phi)=\cos^n\theta$ is proposed in the paper. The proposed model is suitable for incoming waves with three-dimensional angular spread and includes the classical 2D model as a special case. Performance approximation applying the proposed model can be conducted by simple closed-form expression. The numerical results confirm the validity of the proposed model because of very good agreements to theoretic results.

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