Formulation of a Frequency Correlation for a Multiple-Antenna System

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Abstract

In this paper, we analyze the frequency correlation for a multiple-antenna system and compare it with the conventional frequency correlation defined for a single antenna. The proposed frequency correlation shows different behavior from that of the conventional one depending on an arrival angle and an antenna spacing.

Keywords : Frequency correlation Multiple-antenna Multipath Propagation

1. Introduction

In a wireless communication channel, we have multipath propagation due to a large number of reflected and diffracted waves. When the delay difference between multipath waves is not negligible, we encounter frequency selective fading.

Thus far, a frequency correlation defined for a single receive antenna has been used for evaluating the frequency selectivity [1, 2]. However, multiple antennas have been introduced to realize space diversity and/or spatial filtering. The conventional frequency correlation cannot be used for multiple-antenna systems. In this paper, we extend the frequency correlation to multiple-antenna systems.

2. Definition of Frequency Correlation in Multiple-Antenna System

Let us consider a multiple-antenna system with N elements as shown in Fig. 1. We express channels between transmit and receive antennas at frequency f as $h_1(f), h_2(f), \cdots, h_N(f)$ and weights for the receive antennas as $w_1, w_2, \cdots, w_N$. We define the vectors which consist of these as follows:

$$\boldsymbol{h}(f) = [h_1(f)\ h_2(f)\ \cdots\ h_N(f)]^T \tag{1}$$

$$\boldsymbol{w} = [w_1\ w_2\ \cdots\ w_N]^T, \tag{2}$$

where $(\cdot)^T$ denotes transposition. From (1) (2), the channel at the array output is given by

$$y(f) = \sum_{i=1}^{N} w_i h_i(f) = \boldsymbol{w}^T \boldsymbol{h}(f). \tag{3}$$

Using the conventional frequency correlation defined for a single antenna, we obtain

$$C(\Delta f) = \frac{< y^*(f) y(f+\Delta f) >}{< y^*(f) y(f) >}. \tag{4}$$

Substituting (3) into (4), we obtain

$$C(\Delta f) = \frac{< \boldsymbol{w}^H \boldsymbol{h}^*(f)\ \boldsymbol{w}^T \boldsymbol{h}(f+\Delta f) >}{< \boldsymbol{w}^H \boldsymbol{h}^*(f)\ \boldsymbol{w}^T \boldsymbol{h}(f) >}. \tag{5}$$

The above equation is the frequency correlation for a multiple-antenna system having weight vector $\boldsymbol{w}$. This is a natural extension of the conventional frequency correlation.

In this paper, we use maximum ratio combining (MRC) gains for the weights. Thus, we have

$$\boldsymbol{w} = \boldsymbol{h}^*(f). \tag{6}$$

Then, substituting (6) into (5), we obtain

$$C(\Delta f) = \frac{< \boldsymbol{h}^T(f)\boldsymbol{h}^*(f)\boldsymbol{h}^H(f)\boldsymbol{h}(f+\Delta f) >}{< \{\boldsymbol{h}^H(f)\boldsymbol{h}(f)\}^2 >}. \tag{7}$$

In the remainder of this paper, we consider $C(\Delta f)$ given by (7) as the frequency correlation for a multiple-antenna system.

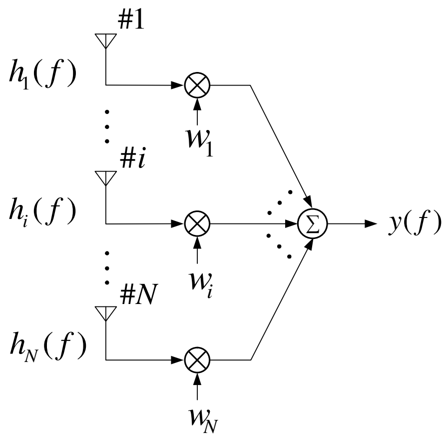

Figure 1: Multiple-antenna system configuration.

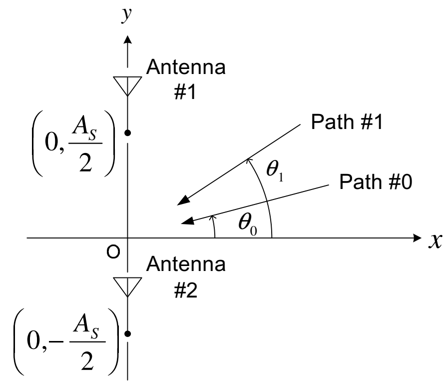

Figure 2: Antenna arrangement.

3. Considerations on the Frequency Correlation

In this section, we consider the behavior of the frequency correlation defined above in specific environments, and compare it with that for the conventional frequency correlation.

3.1. Two-Path Case

Here, we analyze the frequency correlation for a case where two multipath waves with equal power arrive at a two-antenna array with antenna spacing A_s as shown in Fig. 2. The preceding wave Path$\#0$ and the delayed wave Path$\#1$ arrive from θ_0 and θ_1, respectively. We express the delay difference between the multipath waves as τ. We assume that the phases of the waves are independent of each other and uniformly distributed from $-\pi$ to π.

In the following discussion, for the sake of simplicity, we consider the case of $\theta_0 = 0°$. Then, the frequency correlation defined by (7) is given by

$$C(\Delta f) = \frac{\begin{array}{l} 1 + \mathrm{e}^{-j2\pi\Delta f\tau}\cos(\frac{\pi A_s}{\lambda_o}\frac{\Delta f}{f}\sin\theta_1) \\ \quad + \frac{1}{2}\mathrm{e}^{-j2\pi\Delta f\tau}\cos(\frac{\pi A_s}{\lambda_o}\sin\theta_1)\cos(\frac{\pi A_s}{\lambda_o}(1+\frac{\Delta f}{f})\sin\theta_1) + \frac{1}{2}\cos^2(\frac{\pi A_s}{\lambda_o}\sin\theta_1) \end{array}}{2 + \cos^2(\frac{\pi A_s}{\lambda_o}\sin\theta_1)}, \tag{8}$$

where λ_0 is the wavelength corresponding to the frequency f. If the arrival angle of Path$\#1$ is also $0°$ ($\theta_1 = 0°$), (8) becomes the following equation and coincides with the conventional frequency correlation defined for a single antenna:

$$C(\Delta f) = \mathrm{e}^{-j\pi\Delta f\tau}\cos(\pi\Delta f\tau). \tag{9}$$

Next, let us consider the case of $|\Delta f/f| \ll 1$. Applying this to (8), we obtain

$$C(\Delta f) = \mathrm{e}^{-j\pi\Delta f\tau}\cos(\pi\Delta f\tau). \tag{10}$$

The above equation is equal to the conventional frequency correlation for a single antenna under the same condition. We can say that when $|\Delta f/f| \ll 1$, the frequency correlation for the array is approximately equal to that for a single antenna.

Figures 3 and 4 show the amplitudes of the frequency correlations when the antenna spacing is a half wavelength and 10 wavelengths, respectively, where the wavelength is the value at the frequency f (2 GHz). The conventional frequency correlation for a single antenna is also shown in the figures. From these figures, it is seen that when the antenna spacing is small (a half wavelength), all the curves are coincident. That is, the frequency correlation does not depend on the arrival angle θ_1, and it is the same as the conventional frequency correlation for a single antenna. In contrast, when the antenna spacing is large, dependence on the arrival angle θ_1 is apparent and we see that these frequency correlations differ from the conventional frequency correlation. However, in the case of $\theta_1 = 0°$ or $|\Delta f/f| \ll 1$, it coincides with the conventional correlation despite the antenna spacing as discussed above.

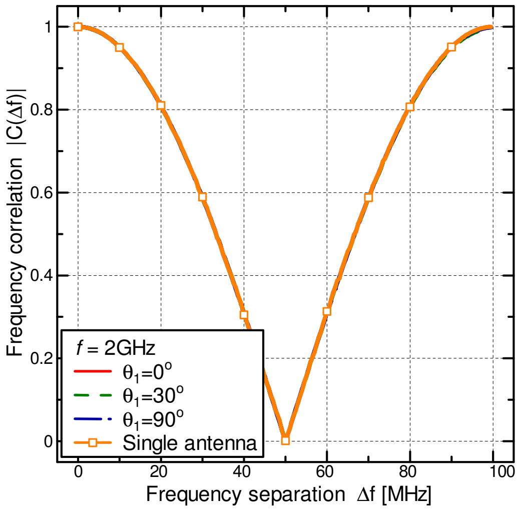

Figure 3: Frequency correlation in 2-path case where A_s is a half wavelength.

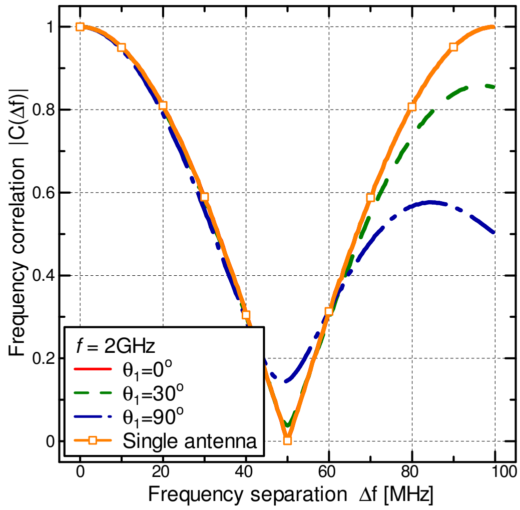

Figure 4: Frequency correlation in 2-path case where A_s is 10 wavelengths.

3.2. Exponential Delay Profile Case

In this subsection, we numerically investigate the frequency correlation in exponential delay profile cases. Table 1 shows the simulation parameters. We assume that 16 multipaths arrive at each antenna and the average power of each path decays successively by 1 dB. The antenna arrangement is the same as that shown in Fig. 2. In the case where the arrival angles $\theta_0, \theta_1, \cdots, \theta_{15}$ measured from the x-axis are specified, each multipath experiences independent Rayleigh fading. As for the Jakes' model, we assume 13 scatterers around the array.

Figures 5 and 6 show the amplitudes of the frequency correlations for the two-element array defined by (7) for the antenna spacing of a half wavelength and 10 wavelengths, respectively. Here, the delay spread σ_τ is 100 ns. The conventional frequency correlation for a single antenna is also shown in the figures.

From Fig. 5, it is seen that when the antenna spacing is small (a half wavelength), dependence on the arrival angle is very small and all the frequency correlations almost coincide with the conventional one. In this simulation, we assumed 16 multipaths with a time spacing of 23.04 ns. Then, the frequency correlation has a periodicity of 43.4 MHz. However, when the antenna spacing is large, the dependence on the arrival angle is seen as shown in Fig. 6, and the frequency correlations for the array differ from the conventional one. Moreover, this dependence is significant when Δf is large. In the case of the Jakes' model where the arrival angles distribute uniformly, the frequency correlation has the value between those for the cases of arrival angles of 30° and 90°.

Table 1: Simulation parameters

Number of received antennas	$N = 2$
Antenna spacing	Half wavelength, 10 wavelengths (at frequency f)
Frequency	$f = 2.0$ GHz
Delay spread	$\sigma_\tau = 100$ ns
Propagation model	16 multipaths with average power decaying successively by 1 dB
Arrival angles of multipaths	$\theta_0 = \theta_1 = \cdots = \theta_{15} = 0°$
	$\theta_0 = \theta_1 = \cdots = \theta_{15} = 30°$
	$\theta_0 = \theta_1 = \cdots = \theta_{15} = 90°$
	Jakes' model
Number of trials	100,000

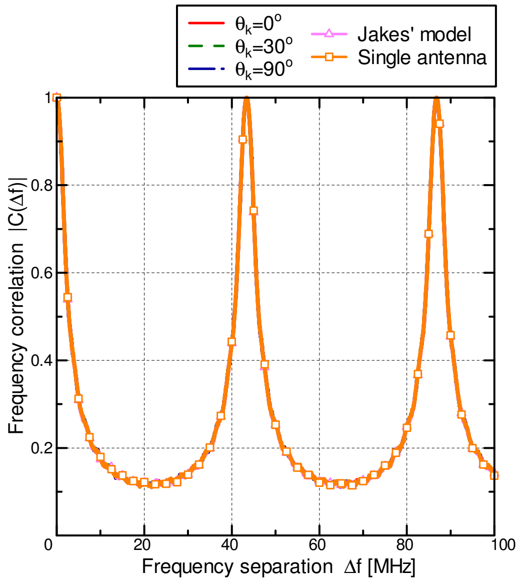

Figure 5: Frequency correlation in exponential delay profile case where A_s is a half wavelength.

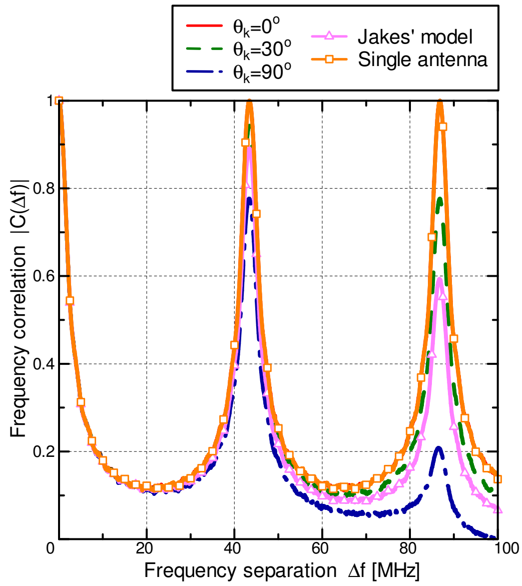

Figure 6: Frequency correlation in exponential delay profile case where A_s is 10 wavelengths.

4. Conclusions

In this paper, we have introduced the frequency correlation for a multiple-antenna system. Using the definition, we have shown the numerical examples for the two-wave case and the exponential delay profile case. It has been clarified that the frequency correlation for a small array has the same values as those of the conventional one for a single antenna and is independent of the arrival angles. However, as for a large array, it has been shown that the frequency correlation is different from the conventional one depending on the arrival angles.

References

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