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Matched input disturbance rejection for nonlinear systems in the Koopman framework

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Abstract—We study the problem of output regulation for a class of nonlinear systems subject to matched input disturbances, with the disturbance signal generated by an autonomous dynamical system. Using the Koopman operator, the problem is reformulated as bilinear output regulation. We show that a controller, inspired by the linear output regulation problem, is effective in disturbance rejection and achieving local asymptotic stability.

1. Introduction

The output regulation [6, 7] is a well-known control theory problem with applications in control of various dynamical systems, e.g. [9]. Output regulation for linear systems has been well studied in the literature [6, 7], necessary and sufficient conditions for regulating the output have been formulated. For nonlinear systems, solving the regulatory equations is challenging. In the literature, control designs, also using the internal-model principle, have been proposed for certain classes of systems [7, 10]. In this work, we study matched input disturbance rejection in Koopman framework. We show that using the Koopman operator [1, 5, 3], the nonlinear output regulation problem is reduced to a bilinear problem. Our results show that a controller, inspired by the linear output regulation problem, is effective in disturbance rejection and achieving local asymptotic stability. In what follows, we first present the problem and give a bilinear system representation using the Koopman operator. Thereafter, a controller is proposed and the main result is presented [11]. Finally, the work is concluded.

2. Problem formulation

Consider a nonlinear system of the form

$$\dot{x} = f(x) + g(x)(u + v), \quad e = h(x), \tag{1}$$

with $x \in X \subseteq \mathbb{R}^n$, $u, v \in \mathbb{R}$ and $f : X \mapsto X$, $g : X \mapsto X$, $h : X \mapsto \mathbb{R}^l$ nonlinear functions. The disturbance signal v is generated by the linear exosystem

$$\dot{w} = Sw, v = Ew \tag{2}$$

where $w \in W \subseteq \mathbb{R}^r$, and S is a skew-symmetric matrix. The goal is to find a controller u that asymptotically achieves regulation, that is, $\lim_{t\to\infty} e \to 0$. Based on the results in [5, 2], we deduce the following Lemma [11].

Lemma 1 *Assume that system* (1) *admits a set* $\mathcal{D}$ *of observable functions* ψ, *with* $N = dim(\mathcal{D})$, *that satisfies the following properties:*

- *if* $u, v = 0$ *and* $\psi \in \mathcal{D}$ *then* $\dot{\psi} \in span(\mathcal{D})$,
- *if* $\psi \in \mathcal{D}$ *then* $\frac{\partial \psi}{\partial x_i} g_i \in span(\mathcal{D})$,
- $h_i \in span(\mathcal{D})$; $\psi(x) = x_i \in span(\mathcal{D})$.

Then, system (1) *is equivalently described by*

$$\dot{z} = Az + B(u + v) + Nz(u + v), \ e = Cz, \tag{3}$$

where $z = \Psi(x)$.

Now, consider a linear dynamic error feedback controller (plus state feedback) of the form

$$\dot{\xi} = F\xi + Ge, \quad u = H\xi + Kz, \tag{4a}$$

with $\xi \in \Xi \subseteq \mathbb{R}^r$. Define

$$s = \begin{bmatrix} \xi - w \\ z \end{bmatrix}, \ \tilde{N} = \begin{bmatrix} 0 & 0 \\ 0 & N \end{bmatrix}, \ \tilde{K} = \begin{bmatrix} 0 & K \end{bmatrix}, \ \tilde{H} = \begin{bmatrix} H & 0 \end{bmatrix},$$

the closed loop dynamics then follows

$$\dot{s} = A_c s + \tilde{N} s \tilde{K} s + \tilde{N} s \tilde{H} s, \tag{5}$$

where $A_c = \begin{bmatrix} S & GC \\ -BE & A + BK \end{bmatrix}$.

Proposition 1 *Consider the system given by* (1) *and* (2), *mapped to system* (3) *using Lemma 1, with the controller in* (4) *satisfying the internal model principle,* $F = S$, $H = -E$. *Choose the matrix* K *such that* $A + BK$ *is Hurwitz and* G *such that* A_c *is Hurwitz. If* $\exists \epsilon > 0, 0 \preccurlyeq W = W^T \in \mathbb{R}^{(N+r)\times(N+r)}$, *such that*

$$\begin{bmatrix} WA_c^T + A_c W + \epsilon \tilde{N} W \tilde{N}^T & W \begin{bmatrix} H & K \end{bmatrix}^T \\ \begin{bmatrix} H & K \end{bmatrix} W & -\epsilon I \end{bmatrix} \prec 0, \tag{6}$$

is satisfied, then the output is regulated provided that

$$\begin{bmatrix} \xi_0 - w_0 \\ z_0 \end{bmatrix} = s_0 \in \mathcal{E} = \{ s \in \Xi \times Z \mid s^T W^{-1} s \leq 1 \}. \tag{7}$$

The proof builds upon a combination of techniques including the internal model principle, Petersen's Lemma, and quadratic Lyapunov functions [8, 11].

3. Conclusion

This works has shown the utilization of the Koopman operator to achieve output regulation of a nonlinear systems experiencing matched input disturbances.

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