2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Inferring dynamic networks driven by both unknown fast-varying noise and slow-varying signal

Zhang Zhaoyang†1, Chen Yang⋆1, Wang Shihong⋆ and Hu Gang‡

† Faculty of Science, Ningbo University
818 Fenghua Street, Ningbo, P. R. China
⋆ School of Sciences, Beijing University of Posts and Telecommunications
10 Xitucheng Avenue, Beijing, P. R. China
‡ Department of Physics, Beijing Normal University
19 Xinjiekouwai Street, Beijing, P. R. China
Email: ganghu@bnu.edu.cn

Abstract—Practical networks of biological systems are often driven by various unknown forces, some are varying fast (like white noise) and some other have the same time scale as the network dynamics (like slow- varying signal). It turns to be very difficult to depict the interaction structures of networks by the observable node-variable data only, as both noise and signal are unknown while both play important roles in producing the variable data. In this paper we present an effective method to overcome this difficulty. We show (i) how to recognize the influences from unknown noise and signal; (ii) how to separate these influences from different driving sources based on the recognition; and (iii) how to correctly infer the networks structures based on the understandings of (i) and (ii). Numerical results fully justify our theoretical analysis

1. Introduction

In the present world huge data have been accumulated, and day by day the data size increases exponentially. All these data have become a useful source available for public. It has turned to be a key issue in almost all fields of natural and social fields how to extract useful information for the database. In biological systems, data are often produced by dynamic networks [1, 2], among which neural networks [3, 4] and gene regulatory networks [5, 6] are most typical ones. The network structures which yield these data are however often unknown. Understanding these structures is crucial for understanding and controlling various biological functions. This paper focuses on the problem of inferring network structures by analyzing measurable data of network outputs, i.e. the so-called inverse problem [7, 8].

2. Model and task of network depiction

The dynamics of networks can be generally represented , around certain local phase space point, by a set of linearly coupled ordinary differential equations

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \hat{\boldsymbol{A}}\boldsymbol{x}(t) + \boldsymbol{\Gamma}(t) + \boldsymbol{S}(t) \qquad (1)\\ \boldsymbol{x}(t) &= (x_1(t), x_2(t), \cdots, x_N(t))^T \\ \boldsymbol{\Gamma}(t) &= (\Gamma_1(t), \Gamma_2(t), \cdots, \Gamma_N(t))^T \\ \boldsymbol{S}(t) &= (S_1(t), S_2(t), \cdots, S_N(t))^T \end{aligned}$$

where $\hat{\boldsymbol{A}}$ is $N \times N$ matrix with constant elements, which is the target for solving the inverse problem. The network dynamics is often driven by various unknown and complicatedly distributed forces. Some driving forces may vary very fast (often from the sources of microscopic world). we represent these impacts as white noise $\Gamma(t)$, approximated as

$$< \boldsymbol{\Gamma}(t) >= 0, < \boldsymbol{\Gamma}(t)\boldsymbol{\Gamma}^T(t') >= \boldsymbol{Q}\delta(t-t') \qquad (2a)$$

$$< \boldsymbol{\Gamma}(t')\boldsymbol{x}^T(t) >= 0 \text{ for all } t' > t \qquad (2b)$$

Eq.(2b) is crucial property for fast varying noise. Some other driving (often from the sources of macroscopic world) may vary much slower (like impacts by breaths and heart beats and so on). We represent them as signal $\boldsymbol{S}(t)$. The typical property of the slow impacts can be approximated as

$$0 \le |\boldsymbol{S}(t) - \boldsymbol{S}(t-\Delta t)| \ll 1, \text{ for } 0 \le \Delta t \ll 1 \qquad (3)$$

$$|\boldsymbol{S}(t)| = \sum_{i=1}^{N} |S_i(t)|/N$$

Now the task of the inverse problem reads: With all $x_i(t), i = 1, \cdots, N$ being known, we aim at depict all elements of matrix $\hat{\boldsymbol{A}}$ under the conditions that both $\boldsymbol{\Gamma}(t)$ and $\boldsymbol{S}(t)$ are unknown, apart the generally understandable properties of Eq.(2a) and Eq.(3). By known observables we mean a set of discrete $\boldsymbol{x}(t)$ data measured with certain frequency

$$\boldsymbol{x}(t_k); k = 1, 2, \cdots, L;\ t_{k+1} - t_k = \Delta t, \Delta t \ll 1 \qquad (4)$$

[1]The first two authors contributed equally to this paper

3. Decorrelation of fast varying noise

At the first glimpse, Eq.(1) is unsolvable since $\mathbf{\Gamma}(t_k)$ and $\boldsymbol{S}(t_k)$ may be random and the number of unknown quantities $\mathbf{\Gamma}(t_k)$, $\boldsymbol{S}(t_k)$, $k = 1, 2, \cdots, L$, and $\hat{A}_{ij}, i, j = 1, \cdots, N$ are more than the number of available equations with the known discrete variables $\boldsymbol{x}(t_k)$. Possibility to overcome the difficulty is to make various statistical computations, of which correlation computation is the most popular algorithm [9, 10, 11]. Multiplying the both sides of Eq.(1) by $\boldsymbol{x}^T(t_k)$, and computing the corresponding correlations we obtain a matrix equation

$$\hat{\boldsymbol{B}} = \hat{\boldsymbol{A}}\hat{\boldsymbol{C}} + \hat{\mathbf{\Gamma}} + \hat{\boldsymbol{S}} \tag{5}$$

with

$$\begin{aligned}
\hat{\boldsymbol{B}} &= <\dot{\boldsymbol{x}}\boldsymbol{x}^T>= \frac{1}{L-1}\sum_{k=1}^{L-1}\dot{\boldsymbol{x}}(t_k)\boldsymbol{x}^T(t_k) \\
\hat{\boldsymbol{C}} &= <\boldsymbol{x}\boldsymbol{x}^T>= \frac{1}{L}\sum_{k=1}^{L}\boldsymbol{x}(t_k)\boldsymbol{x}^T(t_k) \\
\hat{\mathbf{\Gamma}} &= <\mathbf{\Gamma}\boldsymbol{x}^T>= \frac{1}{L}\sum_{i=1}^{L}\mathbf{\Gamma}(t'_k)\boldsymbol{x}^T(t_k),\ t_{k+1} > t'_k > t_k \\
\hat{\boldsymbol{S}} &= <\boldsymbol{S}\boldsymbol{x}^T>= \frac{1}{L}\sum_{i=1}^{L}\boldsymbol{S}(t_k)\boldsymbol{x}^T(t_k)
\end{aligned}$$

$$\dot{\boldsymbol{x}}(t_k) = \frac{\boldsymbol{x}(t_{k+1}) - \boldsymbol{x}(t_k)}{\Delta t} \tag{6}$$

From formula Eq.(6), it is clear that $\mathbf{\Gamma}(t'_k)$ with $t_{k+1} > t'_k > t_k$ contribute to the velocity computation which is not correlated bo the variables $\boldsymbol{x}(t_k)$ for $t_k < t'_k$ due to the fast-vary property of Eq.(2b). This leads to decorrelation of fast-vary noise [11] as

$$\hat{\mathbf{\Gamma}} = 0 \tag{7}$$

and we achieve

$$\hat{\boldsymbol{B}} = \hat{\boldsymbol{A}}\hat{\boldsymbol{C}} + \hat{\boldsymbol{S}}$$

from which $\hat{\boldsymbol{A}}$ can be solved as

$$\hat{\boldsymbol{A}} = (\hat{\boldsymbol{B}} - \hat{\boldsymbol{S}})\hat{\boldsymbol{C}}^{-1} \tag{8}$$

If the network is driven by white-noise only, we have [11]

$$\hat{\boldsymbol{S}} = 0,\ \hat{\boldsymbol{A}} = \hat{\boldsymbol{B}}\hat{\boldsymbol{C}}^{-1} \tag{9}$$

In Fig.(1) we show the depiction of a dynamic network driven purely by fast-varying noise, and find that formula (9) works very well for the network inference.

However, if the slow-varing signals do not vanish, Eq.(9) dose no longer work. In Fig.2 we give two examples of slow-varying signal, one is colored noise with long correlation time (Fig.2(a)), and the other quasiperiodic signal (Fig.2(b)). In both cases the formula (9) fails to correctly depict the network structure (Fig.3). If we, however, know signal $\boldsymbol{S}(t)$ and can accordingly compute signal-correlation matrix $\hat{\boldsymbol{S}}$, the full algorithm (8) can perfectly depict the network structure, as shown in Fig.4.

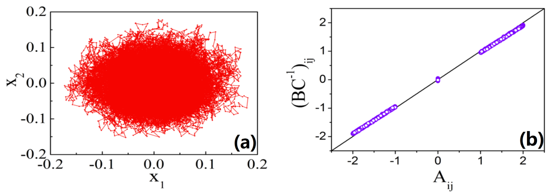

Figure 1: Trajectory of dynamical network under the investigation in a two-variable plane, driven by fast-varying noise only and the application of Eq.(9) in this case. Matrix $\hat{\boldsymbol{A}}$ for the network is given as follows: positive interactions $A(p)_{ij}$ is randomly chose within (1.0, 2.0) , negative interactions $A(n)_{ij}$ within (−2.0, −1.0) , diagonal entries $\hat{A}_{ii} = -8$. Nodes number $N = 200$ and mean degree $< K >= 20$ for the system. $\hat{Q}_{ij} = \sigma_i\delta_{ij}$ with $\sigma \in (0.005, 0.015)$ for white noises. (**a**) The trajectory of the system in the $x_1 - x_2$ subspace. (**b**) $\hat{\boldsymbol{B}}\hat{\boldsymbol{C}}^{-1}_{ij}$ v.s. actual $\hat{A}_{ij}$. It is clearly verified that Eq.(9) works very well for network inference in such case.

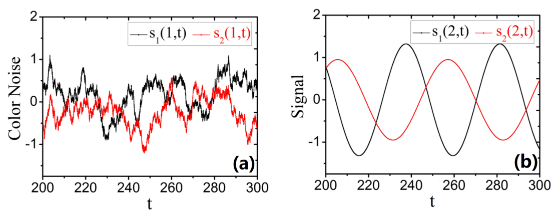

Figure 2: Examples of different kinds of slow-varying signal. (**a**) Time series of colored noises with long correlation time. (**b**) Time series for quasiperiodic signals.

4. Decorrelation of slow varying signals

Nevertheless, the computation of Eq.(8) is often not possible in practice, for the slow-vary signal is unknown and its correlation matrix $\hat{\boldsymbol{S}}$ is not computable. One can include Eq.(3), the only property of $\boldsymbol{S}(t)$, in the inference computation.

Initiated by Eq.(3) we can derive a set of increment equations from Eq.(3)

$$\Delta\dot{\boldsymbol{x}} = \hat{\boldsymbol{A}}\Delta\boldsymbol{x} + \Delta\boldsymbol{\Gamma} + \Delta\boldsymbol{S} \tag{10}$$

with

$$\Delta\dot{\boldsymbol{x}}(t_k) = \dot{\boldsymbol{x}}(t_{k+1}) - \dot{\boldsymbol{x}}(t_k), \quad \Delta\boldsymbol{x}(t_k) = \boldsymbol{x}(t_{k+1}) - \boldsymbol{x}(t_k)$$
$$\Delta\boldsymbol{\Gamma} = \boldsymbol{\Gamma}(t'_{k+1}) - \boldsymbol{\Gamma}(t'_k), \qquad \Delta\boldsymbol{S} = \boldsymbol{S}(t_{k+1}) - \boldsymbol{S}(t_k) \ll 1$$

Multiplying the both sides of Eq.(10) again by $\boldsymbol{x}^T(t_k)$ we have

$$\Delta\hat{\boldsymbol{B}} = \hat{\boldsymbol{A}}\Delta\hat{\boldsymbol{C}} + \Delta\hat{\boldsymbol{\Gamma}} + \Delta\hat{\boldsymbol{S}} \tag{11}$$

with $\Delta\hat{\boldsymbol{\Gamma}} = 0$ for the fast-noise decorrelation due to Eq.(2b) and $\Delta\hat{\boldsymbol{S}} \approx 0$ with the slow-varying property of $\boldsymbol{S}(t)$ Eq.(3), we can further reduce Eq.(10) to

$$\Delta\hat{\boldsymbol{B}} = \hat{\boldsymbol{A}}\Delta\hat{\boldsymbol{C}}$$

leading to

$$\hat{\boldsymbol{A}} = \Delta\hat{\boldsymbol{B}}\Delta\hat{\boldsymbol{C}}^{-1} \tag{12}$$

The final form of increment correlation matrices successfully decorrelate both unknown fast and slow varying unknown impacts and give a closed (approximated, of course) formula for successful network inference. In Fig.5 we show the inference results by applying Eq.(11) to the systems of Fig.2, which are very good. Unlike the almost totally wrong results of Fig.3, Fig.5 provide satisfactorily good network depiction. Moreover, unlike the restrict requirement of Fig.4 for the price of correct depiction, known slow-varying signal correlation $\hat{\boldsymbol{S}}$, the results of Fig.5 are achieved with these slow impacts (i.e. $\hat{\boldsymbol{\Gamma}}$ and $\hat{\boldsymbol{S}}$) totally unknown. We hope that the method in this paper may be used in practice to solve inverse problems of dynamic networks, in particular, to inferring neural networks.

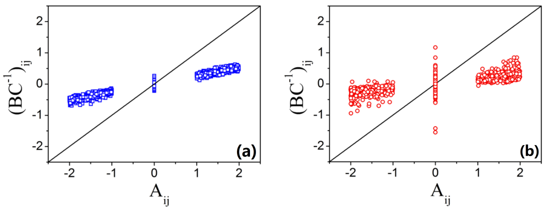

Figure 3: Direct applications of Eq.(9) to the system (1) driven by white noises and slow-varying signals at the same time. The network and statistical properties for the white noises is the same as in Fig.(1). **(a)** $(\hat{\boldsymbol{B}}\hat{\boldsymbol{C}}^{-1})_{ij}$ v.s. actual $\hat{A}_{ij}$ for the case of colored noises with long correlation time (signal Fig.2(a)). **(b)** $(\hat{\boldsymbol{B}}\hat{\boldsymbol{C}}^{-1})_{ij}$ v.s. actual $\hat{A}_{ij}$ for the case quasiperiodic signal (signal Fig.2(b)). In both case Eq.(9) fails to depict the network structure correctly.

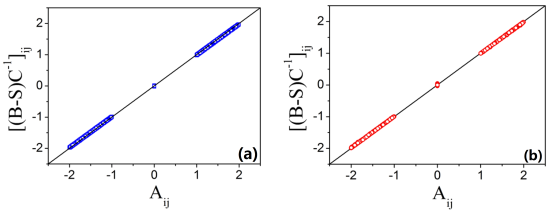

Figure 4: Depicting the network structure by Eq.(8) given signal $\boldsymbol{S}(t)$ and signal-correlation matrix $\hat{\boldsymbol{S}}$. The systems is the same as in Fig.(3). **(a)** $[(\hat{\boldsymbol{B}} - \hat{\boldsymbol{S}})\hat{\boldsymbol{C}}^{-1}]_{ij}$ v.s. actual $\hat{A}_{ij}$ for the case of colored noises with long correlation time. **(b)** $[(\hat{\boldsymbol{B}} - \hat{\boldsymbol{S}})\hat{\boldsymbol{C}}^{-1}]_{ij}$ v.s. actual $\hat{A}_{ij}$ for the case quasiperiodic signal. In both case Eq.(8) works perfectly well.

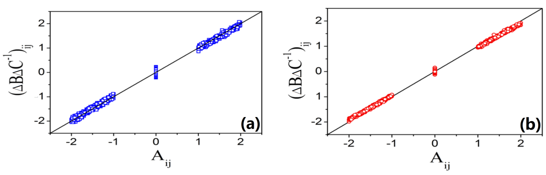

Figure 5: Numerical verification of Eq.(12) in the dynamical systems driven by both unknown fast-varying noise and slow-varying signals. The systems is adopted as in Fig.(3). **(a)** $[\boldsymbol{\Delta}\hat{\boldsymbol{B}}\boldsymbol{\Delta}\hat{\boldsymbol{C}}^{-1}]_{ij}$ v.s. actual $\hat{A}_{ij}$ for the case of colored noises with long correlation time. **(b)** $[\boldsymbol{\Delta}\hat{\boldsymbol{B}}\boldsymbol{\Delta}\hat{\boldsymbol{C}}^{-1}]_{ij}$ v.s. actual $\hat{A}_{ij}$ for the case of quasiperiodic signal.Though there is no any information about the signal $\boldsymbol{S}(t)$ (which is required for producing Fig.4), Eq.(12) depicts network structure very well in both cases.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No. 11135001, No. 11174034). The authors would like to thank NOLTA2015 organizing committee members for their fruitful suggestions and comments.

References

[1] A. L. Barabasi, Z. N. Oltvai, Network biology: understanding the cell's functional organization. Nature Reviews Genetics, 2004, 5(2): 101-113.

[2] H. Kitano, Systems biology: a brief overview[J]. Science, 2002, 295(5560): 1662-1664.

[3] E. Bullmore, O. Sporns. Complex brain networks: graph theoretical analysis of structural and functional systems. Nature Reviews Neuroscience, 2009, 10(3): 186-198.

[4] E. Schneidman, M. J.Berry, R. Segev and W. Bialek. Weak pairwise correlations imply strongly correlated network states in a neural population. Nature, 2006, 440(7087): 1007-1012.

[5] A. J. Butte, P. Tamayo, D. Slonim, et al. Discovering functional relationships between RNA expression and chemotherapeutic susceptibility using relevance networks. Proceedings of the National Academy of Sciences, 2000, 97(22): 12182-12186.

[6] D. Marbach, J. C. Costello, R. Kffner, et al. Wisdom of crowds for robust gene network inference[J]. Nature methods, 2012, 9(8): 796-804.

[7] K. Basso, A. A. Margolin, G. Stolovitzky, et al. Reverse engineering of regulatory networks in human B cells[J]. Nature genetics, 2005, 37(4): 382-390.

[8] M. Timme, J. Casadiego. Revealing networks from dynamics: an introduction[J]. Journal of Physics A: Mathematical and Theoretical, 2014, 47(34): 343001.

[9] R. Jansen, H. Yu, D. Greenbaum, et al. A Bayesian networks approach for predicting protein-protein interactions from genomic data[J]. Science, 2003, 302(5644): 449-453.

[10] J. Ren, W. -X. Wang, B. Li, Y. -C. Lai. Noise bridges dynamical correlation and topology in coupled oscillator networks[J]. Physical review letters, 2010, 104: 058701.

[11] Z. Zhang, Z. Zheng, H. Niu, Y. Mi, S. Wu, G. Hu. Solving the inverse problem of noise-driven dynamic networks[J]. Physical Review E, 2015, 91: 012814.