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The Dynamics of Two-Layer Continuous Attractor Neural Network Model With Moving Stimulus

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Abstract—The dynamics of single-layer continuous attractor neural networks (CANNs) model has gained extensive attention. We generalize the neural network model to a two-layer structure based on the original features, and take **feedback** and **feedforward** effects into consideration. We apply a static stimulus on one layer and a moving stimulus on the other layer. Under various strengths of the feedback and feedforward couplings, the two-layer network will show distinct behaviors. Under a relatively weak input in the first layer, when the feedback is inhibitory, the network dynamics displays kinks. The kinks will also behave differently with distinct inhibitory strengths. When both the feedback and feedforward couplings are excitatory, the activities in the two layers will attract each other. Therefore, a stronger moving stimulus is required to drag both layers to move simultaneously. We also consider the effect of inhibitory or excitatory in both layers in turn, which leads to some opposite behaviors.

1. Introduction

Our brain always performs computations following the rules determined by the structure of the network and the inputs. In simulation studies, people have built many kinds of neural network models to illustrate different functions and properties of the brain. In our study, we choose the CANN model to be the topic due to its property of translational invariance of neuronal interactions, with which the network is able to hold a continuous family of stationary states. The structure endows the model with the capacity to track a moving stimulus continuously, which is intuitive and practical for studying the functions of the brain. In this model, we can mimic visual, auditory or vestibular signals (stimuli) with Gaussian functions, and apply static or moving stimuli to the single layer network model to observe the responses.

There have already been extensive studies about the CANN model, and many properties of the model have been revealed [1–5]. In particular, the tracking dynamics of the CANN model is an important aspect and has been studied theoretically [1]. However, most of these studies were based on a single layer structure, namely, there is only one kind of neuron population producing responses. What we intend to realize is to generalize the CANN model to a two-layer structure and take feedback and feedforward couplings between the two layers into consideration. Under this structure, not only will there be interactions between the neurons in each layer, but also between the neurons in two different layers. The structure seems to be more complicated but the functions and the dynamics of the network will be improved and enhanced. With this model, we can study two different kinds of neuron populations and two different kinds of inputs, and this design achieves the combination of the visual and auditory signals.

2. Dynamics of The Network

2.1. Two-layer CANN Model

Zhang *et al.* [4] studied the dynamics of the two-layer CANN model. They considered one input to the first layer of the network. They found that positive feedbacks reduce the mobility of the network state while negative feedbacks enhance the mobility of the network state. In our study, we take both feedback and feedforward couplings into consideration and study the dynamics of the network.

Now we first consider a one-dimensional continuous stimulus x, which can be regarded as the direction of the movement, or the orientation, and so forth. We deal with networks whose neuronal interaction range is much less than the network range, so here we define the range to be $(-\infty, \infty)$.

We use the function $U(x, t)$ to denote the synaptic input to neurons at time t whose preferred stimulus is x, and use the function $r(x, t)$ to denote their firing rates. The firing rate increases with the synaptic input but cannot be an infinite quantity, so here we introduce a variable called global inhibition to make the firing rate saturate at some stage. The form of firing rate has been given [3] as:

$$r(x, t) = \frac{[U(x, t)]_+^2}{1 + k\rho \int_{-\infty}^{\infty} dx' [U(x', t)]_+^2}, \qquad (1)$$

in which $[U]_+ \equiv \max(U, 0)$, ρ is neural density, and k is the global inhibition, a small, positive constant.

The dynamics of the synaptic input $U(x, t)$ is determined by external input $I_{ext}(x, t)$, the network input from other neurons, and its own relaxation. The dynamical equations are:

$$\tau \frac{\partial U_1(x,t)}{\partial t} = I_{1ext}(x,t) + \rho \int_{-\infty}^{\infty} dx' J_{11}(x,x')r_1(x',t) +$$

$$\rho \int_{-\infty}^{\infty} dx' J_{fb}(x,x')r_2(x',t) - U_1(x,t),$$

$$\tau \frac{\partial U_2(x,t)}{\partial t} = I_{2ext}(x,t) + \rho \int_{-\infty}^{\infty} dx' J_{22}(x,x')r_2(x',t) +$$

$$\rho \int_{-\infty}^{\infty} dx' J_{ff}(x,x')r_1(x',t) - U_2(x,t). \quad (2)$$

Here τ is a time constant, typically of the order of 1 ms [1], which controls the rate at which the synaptic input relaxes to the total input of the neuron. $J_{11}(J_{22})(x,x')$ is the neural interaction between x and x' in the first (second) layers. J_{fb} (coupling from the second layer to the first layer) and J_{ff} (coupling from the first layer to the second layer) denote the feedback and feedforward couplings, respectively.

As for the interaction terms $J(x,x')$, we choose Gaussian function with a range a, which keeps the translational invariance,

$$J_i(x,x') = \frac{J_{i0}}{\sqrt{2\pi a}} \exp\left[-\frac{(x-x')^2}{2a^2}\right], \quad (3)$$

in which $J_{i0} = J_0$ for $i = 11$ and 22, and $J_{i0} = W_{ff}$ and W_{fb} for $i = ff$ and fb, respectively.

For the external input $I_{ext}(x,t)$, we define the input as a Gaussian function with width a.

2.2. Moving Stimulus

We study the case in which the input 1 is static while the input 2 is moving. Since the dynamics of the network can be complicated under some parameter settings, we study the dynamics in terms of the firing rate, average velocity and variance with respect to the stimuli.

2.2.1. The Firing Rate

When the input 1 is weak while the input 2 is a strong one, we record the behaviours of the network.

As shown in Fig. 1 to 2, we get some interesting results. The white (dashed) lines indicate the trajectory of the stimulus. We use a weak input 1 with magnitude of 0.3, and a strong input 2 of 0.8. Meanwhile, the velocity of input 2 is 0.01, a small value. Under this setting we may expect that the input 2 is likely to dominate the network dynamics due to the large magnitude difference. The number of neurons is 200. Global inhibition k is 0.7, and the range a of the Gaussian function is 0.5.

In Fig. 1, when the feedback and feedforward are both excitatory, the pattern shows that the firing rate in first layer will follow the second layer, although the stimulus in the first layer is static. For Fig. 2, when the feedback effect from second layer to first layer is inhibitory, we can see the bump in first layer is suppressed periodically. When

the stimulus 2 is approaching the same position as stimulus 1 (π), the bump in first layer will be suppressed, and in turn when the stimulus 2 is away from the position of the input 1, a small bump can hold for a while. Opposite to the Fig. 2, Fig. 3 has an inhibitory feedforward effect, but with the support of the strong input 2, the second layer is little influenced by this effect, and the first layer is still excited and follows the input 2.
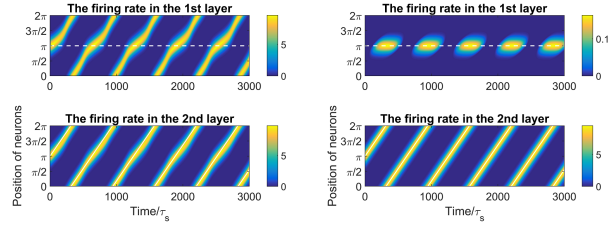

Figure 1 The firing rate patterns under slow moving stimuli.
$W_{fb} = 0.1, W_{ff} = 0.1$.

Figure 2 The firing rate patterns under slow moving stimuli.
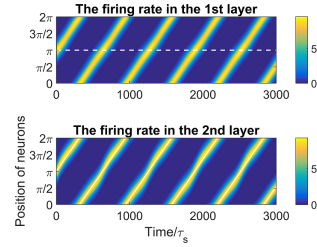$W_{fb} = -0.1, W_{ff} = 0.1$.

Figure 3 The firing rate patterns under slow moving stimuli. $W_{fb} = 0.1, W_{ff} = -0.1$.

There is another situation in which the velocity of input 2 can be faster. Here we pick one of these cases where $v_2 = 0.05$. We use the same parameter sets except the change of the velocity of input 2.
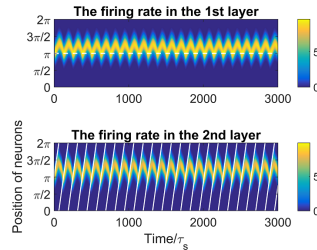

Figure 4 The firing rate patterns under fast moving stimuli. $W_{fb} = 0.1, W_{ff} = 0.1$.

Fig. 4 shows the firing rate patterns under a static stimulus in the first layer and a fast moving stimulus in the second layer. Both the feedback and feedforward couplings are excitatory. The dynamics of the network is very special.

Since the velocity of input 2 is fast, the bump in second layer cannot track the input so the dynamics looks like 'oscillating'. Because of the attraction between the two layers, the first layer is also 'oscillating'.

We also record the dynamics under a strong static input 1 and a strong moving input 2.

As shown in Fig. 5 to 7, the velocity of input 2 is 0.01, and the magnitude of the input 1 is increased to 0.7. The strength of input 2 is still 0.8. The number of neurons is 200, and the global inhibition k is 0.7. The initial condition of $U(x,t)$ is 0, and the range a of the Gaussian function is 0.5.
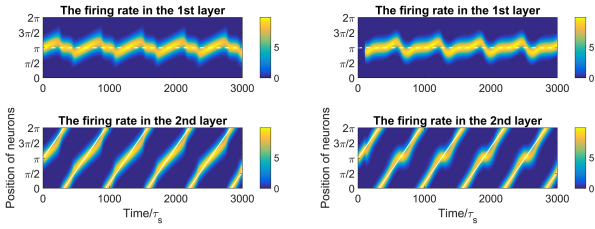

Figure 5 The firing rate patterns under strong input 1.
$W_{fb} = 0.1, W_{ff} = 0.1.$

Figure 6 The firing rate patterns under strong input 1.
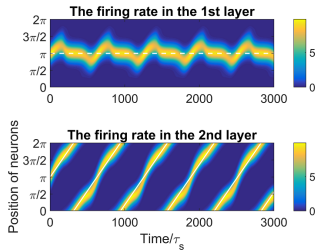$W_{fb} = -0.1, W_{ff} = 0.1.$

Figure 7 The firing rate patterns under strong input 1.
$W_{fb} = 0.1, W_{ff} = -0.1.$

It is natural to get the conclusion that when the strength of input 1 increases, the first layer will not be easily dragged by the second layer any more. Instead, the dynamics in the first layer will impact dramatically on the behaviours of the second layer. In Fig. 5, the first layer moves around the input position, which indicates that although the first layer is attracted by the second layer, it generally still follows its own stimulus due to the strong input 1. When the feedback becomes inhibitory, the second layer is affected by the first layer. The second layer in Fig. 6 displays small 'kinks' near the position of π (corresponding to input 1 position), which results from the attraction from the first layer to the second layer. In Fig. 7, the repulsiveness from first layer to the second layer arises. In the second layer, when the bump is around the same position as the input 1, its firing rate diminishes.

2.2.2. The Average Velocity

Now we take a look at the dynamics of the second layer under various magnitudes and velocities of the input 2. In order to get a summary figure, we calculate the average velocity of the responding moving bump in the second layer under each parameter set. We fix the magnitude of input 1 on 0.7, and other parameters are kept unchanged.
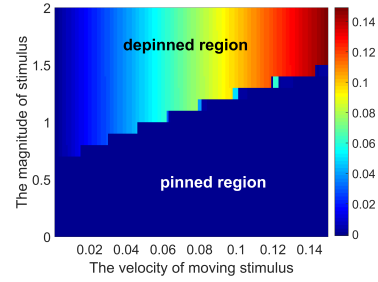

Figure 8 Average velocity of the moving bump in the second layer under various magnitudes and velocities of the input 2. $W_{fb} = 0.1, W_{ff} = 0.1.$
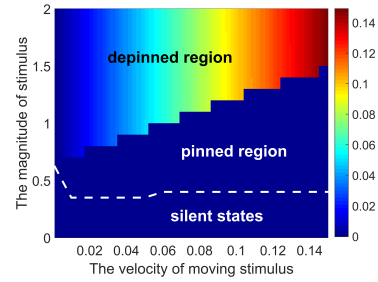

Figure 9 Average velocity of the moving bump in the second layer under various magnitudes and velocities of the input 2. $W_{fb} = 0.1, W_{ff} = -0.1.$
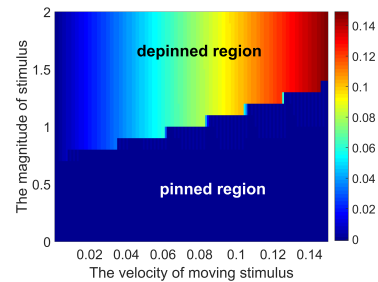

Figure 10 Average velocity of the moving bump in the second layer under various magnitudes and velocities of the input 2. $W_{fb} = -0.1, W_{ff} = 0.1.$

Figure 8, 9 and 10 show the average velocity of the moving bump in the second layer under various magnitudes and velocities of the input 2. We use 3000 τ_s as the time scale, and the magnitude of input 1 is 0.7. We find that there exist a boundary in each average velocity figure. Above the

boundary is the 'depinned' region in which the average velocity is not 0 while below the boundary is the 'pinned' region in which the average velocity is 0 or approximately to 0. The 'depinned' region indicates that the response is a moving bump and the displacement and average velocity are non-zero. However, the 'pinned' region suggests that the second layer will not move far away from its initial position, namely it tracks the input 1 so the displacement is very small even equals to 0, which leads to 0 average velocity. In particular, there exist a 'silent states' region in Fig. 9, in which the response in the second layer is suppressed completely due to the inhibitory feedforward coupling.

When the strength of input 2 is weak, no matter whether the couplings between the two layers are excitatory or inhibitory, the average velocity of the second layer is always about 0, which indicates that the bump of the second layer is 'oscillating' around the corresponding position of the input 1 (π). This is because the stimulus strength is not strong enough to overcome the attraction coming from the first layer, so that the second layer activity will be closer to the first layer activity instead of tracking the moving stimulus. On the other hand, when the moving velocity of the input 2 is too fast, the responding bump will not be able to track the stimulus, which corresponds to the 'pinned' region.

2.2.3. The Variance

A good method to evaluate whether the second layer is tracking the input 1 or input 2 is to calculate the variance of the difference between the bump position and the input position.

We denote the bump position as a function dependent of time: $x(t)$, and the stimulus input positions are $v_1 \cdot$ t and $v_2 \cdot$ t, respectively. So the variance of the two layers are:

$$\sigma_1^2 = \langle (x_2(t) - v_1 \cdot t)^2 \rangle_{time} - \langle x_2(t) - v_1 \cdot t \rangle_{time}^2$$
$$\sigma_2^2 = \langle (x_2(t) - v_2 \cdot t)^2 \rangle_{time} - \langle x_2(t) - v_2 \cdot t \rangle_{time}^2, \quad (4)$$

where the angular brackets denote the average value over time. If the value of the variance is larger, the tracking performance is worse.
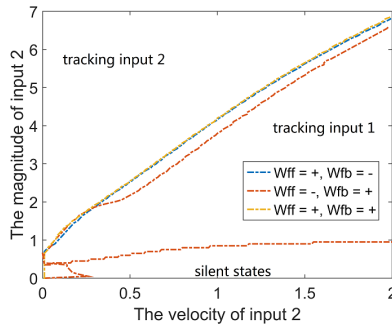

Figure 11 The phase diagram of tracking performance of second layer for three cases of couplings.

We plot the phase diagram as shown in Fig. 11. As the strength of input 2 increases, the second layer will track the moving stimulus. As for the region below the curve, it indicates that the bump in the second layer is tracking the input 1, which results from a weak or a fast input 2. When the strength of input 2 is not strong enough or the moving velocity of input 2 is too fast, the second layer cannot hold a static response under the inhibitory feedforward coupling. For inhibitory feedforward couplings, the phase boundary is spurious when both the velocity and magnitude of the input 2 are very small (the magnitude is about 0-0.4, the velocity is about 0-0.3), but this is an artifact due to the weak response.

3. Conclusion

According to what we have studied, when the magnitudes of the two inputs are comparable, there exist a competition between the inputs. Meanwhile, the couplings between the two layers are also important for the two layers to determine which input to track.

On the other hand, in the presence of a weak input 1 and a strong input 2, the network is shown to be more sensitive to the inhibitory feedback coupling than the inhibitory feedforward coupling.

Different kinds of couplings will lead to different dynamics of the network. In the average velocity studies, there is a sharp transition from the pinned to depinned region when the moving stimulus is sufficiently strong or slow. Furthermore, the inhibitory feedforward coupling gives rise to a special 'silent region', which is different from other two coupling forms. This also can be found in the study of the variance (Fig. 11).

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