2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Experimental observation of amplitude death in a delay-coupled circuit network with fast time-varying network topology

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Abstract—This paper experimentally demonstrates that amplitude death ocuurs in a delay-coupled circuit network with fast time-varying network topology. We implement the all-to-all network consisting of four double-scroll circuits with fast on-off connections. It is confirmed that our experimental results agree well with analytical results.

1. Introduction

In the field of nonlinear science, various phenomena in coupled oscillators have attracted researchers' interest. Amplitude death, stabilization of unstable fixed points in oscillators with a diffusive connection, is well known as one of such phenomena. This phenomenon never occurs in coupled identical oscillators [1]. Reddy *et al.* proved that a delay connection induces this phenomenon even in coupled identical oscillators [2]. After their work, many different kinds of delay connections inducing amplitude death in coupled oscillators have been proposed [3]. Amplitude death induced by the delay connections is regarded as an important phenomenon because practical systems always have transmission delays. In fact, some researches utilize this phenomenon for suppression of oscillations in practical systems, such as DC microgrid with constant-power loads [4], coupled permanent-magnet synchronous motors [5], and coupled thermoacoustic oscillators [6].

Many studies on amplitude death deal with coupled oscillators with time-invariant network topologies; in contrast, many of real systems have time-varying network topologies. However, it is difficult to analyze such time-varying systems. Sugitani *et al.* analytically and numerically investigated amplitude death in delay-coupled oscillators with a fast time-varying network topology [7]. Unfortunately, an experimental verification of their result has not been conducted.

This paper experimentally demonstrates that amplitude death occurs in a fast time-varying network implemented by four double-scroll circuits. It is shown that the death region in a parameter space on our experiments agrees with that on our analysis.

2. Time-Varying Network

2.1. Model and Analysis

Let us consider m-dimensional coupled oscillators,

$$\begin{cases} \dot{\boldsymbol{x}}^{(j)}(t) = \boldsymbol{F}\left(\boldsymbol{x}^{(j)}(t)\right) + \boldsymbol{b}u^{(j)}(t) \\ y^{(j)}(t) = \boldsymbol{c}\boldsymbol{x}^{(j)}(t) \end{cases} (j = 1, \ldots, N), \tag{1}$$

where $\boldsymbol{x}^{(j)}(t) \in \mathbb{R}^m$ is the state variable of the j-th oscillator and $y^{(j)}(t) \in \mathbb{R}$ is the output signal of the j-th oscillator. $\boldsymbol{b} \in \mathbb{R}^m$ and $\boldsymbol{c} \in \mathbb{R}^{1\times m}$ are the input matrix and the output matrix, respectively. $\boldsymbol{F}(\boldsymbol{x}) : \mathbb{R}^m \to \mathbb{R}^m$ is a nonlinear function which has at least one fixed point $\boldsymbol{x}^*$: $\boldsymbol{F}(\boldsymbol{x}^*) = 0$. The coupled signal is given by

$$u^{(j)}(t) = k \sum_{l=1}^{N} \varepsilon^{(jl)}(t) \left\{ y^{(l)}(t-\tau) - y^{(j)}(t) \right\}, \tag{2}$$

where k is the coupling strength and $y^{(l)}(t-\tau)$ is the delayed output with delay time τ of the l-th oscillator. $\varepsilon^{(jl)}(t)$ represents the network topology at time t. If the j-th oscillator and the l-th oscillator are coupled, then $\varepsilon^{(jl)}(t) = \varepsilon^{(lj)}(t) = 1$, otherwise $\varepsilon^{(jl)}(t) = \varepsilon^{(lj)}(t)=0$. The degree of the j-th oscillator at time t is given by $d^{(j)}(t) := \sum_{l=1}^{N} \varepsilon^{(jl)}(t)$.

The network topology always changes with time t according to the following rules.

(a) The network topology changes at intervals Δt.

(b) The network topology changes under the restriction on the constraint matrix $\boldsymbol{H}$: if the j-th oscillator and the l-th oscillator are allowed to be connected, then $\{\boldsymbol{H}\}_{jl} = \{\boldsymbol{H}\}_{lj} = 1$, otherwise $\{\boldsymbol{H}\}_{jl} = \{\boldsymbol{H}\}_{lj} = 0$.

(c) A pair of oscillators is connected with the probability p.

(d) The constraint matrix $\boldsymbol{H}$ satisfies

$$\sum_{l=1}^{N} \{\boldsymbol{H}\}_{jl} = D, \forall j \in \{1, \ldots, N\}, \tag{3}$$

where $D \in \mathbb{N}$ is a natural number.

The oscillators (1) with connection (2) have the homogeneous steady state,

$$\left[\boldsymbol{x}^{(1)^T} \quad \cdots \quad \boldsymbol{x}^{(N)^T}\right]^T = \left[\boldsymbol{x}^{*^T} \quad \cdots \quad \boldsymbol{x}^{*^T}\right]^T. \quad (4)$$

Coupled oscillators, (1) and (2), are linearized around the steady state (4),

$$\begin{aligned}\delta\dot{\boldsymbol{x}}^{(j)}(t) =& \boldsymbol{A}\delta\boldsymbol{x}^{(j)}(t) \\ &+ k\boldsymbol{bc}\sum_{l=1}^{N}\varepsilon^{(jl)}(t)\left\{\delta\boldsymbol{x}^{(l)}(t-\tau) - \delta\boldsymbol{x}^{(j)}(t)\right\},\end{aligned} \quad (5)$$

where $\delta\boldsymbol{x}^{(j)}(t) := \boldsymbol{x}^{(j)}(t) - \boldsymbol{x}^*$ and $\boldsymbol{A} := \partial\boldsymbol{F}(\boldsymbol{x}^*)/\partial\boldsymbol{x}$. This can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{X}}(t) =& (\boldsymbol{I}_N \otimes \boldsymbol{A})\boldsymbol{X}(t) + k\Big[(\boldsymbol{E}(t)\otimes\boldsymbol{bc})\boldsymbol{X}(t-\tau) \\ &- \mathrm{diag}\left\{d^{(1)}(t), \dots, d^{(N)}(t)\right\}\otimes\boldsymbol{bc}\boldsymbol{X}(t)\Big],\end{aligned} \quad (6)$$

where $\boldsymbol{X}(t) = \left[\delta\boldsymbol{x}^{(1)^T}\cdots\delta\boldsymbol{x}^{(N)^T}\right]^T$ and $\{\boldsymbol{E}(t)\}_{jl} = \varepsilon^{(jl)}(t)$. The coupling matrix $\boldsymbol{E}(t)$ and the degree of the j-th oscillator, $d^{(j)}(t)$, can be averaged over a period T_p ($\gg \Delta t$) for any t [8],

$$\begin{aligned}\frac{1}{T_p}\int_t^{t+T_p}\boldsymbol{E}(r)dr =& p\boldsymbol{H}, \\ \frac{1}{T_p}\int_t^{t+T_p}d^{(j)}(r)dr =& pD.\end{aligned} \quad (7)$$

For the fast time-varying topology, there exists the period Δt^* such that the stability of linearized system (6) is equivalent to that of the following linear time-invariant system,

$$\dot{\boldsymbol{X}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{A}_s)\boldsymbol{X}(t) + pk(\boldsymbol{H}\otimes\boldsymbol{bc})\boldsymbol{X}(t-\tau), \quad (8)$$

where $\boldsymbol{A}_s := \boldsymbol{A} - pkD\boldsymbol{bc}$. The characteristic equation of system (8) is described by

$$G(s) = \left|s\boldsymbol{I}_N - (\boldsymbol{I}_N\otimes\boldsymbol{A}_s) - pk(\boldsymbol{H}\otimes\boldsymbol{bc})e^{-s\tau}\right| = 0. \quad (9)$$

$\boldsymbol{H}$ is a real symmetric matrix; thus, it can be diagonalized by an orthognal matrix $\boldsymbol{T}$ as $\boldsymbol{T}^{-1}\boldsymbol{H}\boldsymbol{T} = \mathrm{diag}(\rho_1, \dots, \rho_N)$. Hence, Eq. (9) can be decomposed to

$$G(s) = \prod_{q=1}^{N} g(s, \rho_q), \quad (10)$$

where

$$g(s,\rho) = \left|s\boldsymbol{I}_N - \boldsymbol{A}_s - pk\rho\boldsymbol{bc}e^{-s\tau}\right|. \quad (11)$$

The steady state (4) is stable if and only if, for all $\rho_q (q = 1, \dots N)$, all the roots of $g(s, \rho_q) = 0$ have negative real parts.

2.2. Double-Scroll Circuits

We consider the double-scroll circuit [9] illustrated in Fig. 1,

$$\begin{cases} C_1\dfrac{\mathrm{d}v_1^{(j)}}{dt} = \dfrac{1}{R}\left(v_2^{(j)} - v_1^{(j)}\right) - h\left(v_1^{(j)}\right) \\ C_2\dfrac{\mathrm{d}v_2^{(j)}}{dt} = \dfrac{1}{R}\left(v_1^{(j)} - v_2^{(j)}\right) + i_L^{(j)} + i_u^{(j)} \\ L\dfrac{\mathrm{d}i_L^{(j)}}{dt} = -v_2^{(j)} \end{cases}, \quad (12)$$

where

$$h(x) := m_0 x + \frac{1}{2}(m_1 - m_0)\{|x + B_p| - |x - B_p|\}.$$

m_0, m_1, B_p are the parameters. The connection current from other oscillators to the j-th oscillator,

$$i_u^{(j)} = \frac{1}{r}\sum_{l=1}^{N}\varepsilon^{(jl)}(t)\left(v_2^{(l)}(t-T) - v_2^{(j)}(t)\right), \quad (13)$$

flows through the coupling resistance r, where T is the delay time.

This circuit can be described by oscillators (1), where

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{bmatrix}\eta\{x_2 - x_1 - g(x_1)\} \\ x_1 - x_2 + x_3 \\ -\gamma x_2\end{bmatrix}, \boldsymbol{b} = \begin{bmatrix}0\\1\\0\end{bmatrix}, \boldsymbol{c} = \begin{bmatrix}0\\1\\0\end{bmatrix}^T, \quad (14)$$

$$x_1 := \frac{v_1}{B_p},\ x_2 := \frac{v_2}{B_p},\ x_3 := \frac{i_L R}{B_p},\ \tau := \frac{T}{RC_2},\ \eta := \frac{C_2}{C_1},$$
$$\gamma := \frac{R^2 C_2}{L},\ a := m_1 R,\ b := m_0 R,\ k := \frac{R}{r},$$
$$g(x) := bx + \frac{1}{2}(b - a)\{|x-1| - |x+1|\}.$$

We note that the dimensionless time is given by t/RC_2. The oscillator with Eq. (14) has the three fixed points:

$$\boldsymbol{x}_\pm^* = \begin{bmatrix}\pm x_p & 0 & \mp x_p\end{bmatrix}^T, \boldsymbol{x}_0^* = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T, \quad (15)$$

where $x_p := (b-a)/(b+1)$. In order to simplify our discussion, we consider the stability of the fixed point $\boldsymbol{x}_+^*$. The dynamics of oscillators (1) with connection (2) at $\boldsymbol{x}_+^*$ is represented by Eq. (6) with

$$\boldsymbol{A} = \begin{bmatrix}-\bar{\eta} & \eta & 0 \\ 1 & -1 & 1 \\ 0 & \gamma & 0\end{bmatrix}, \quad (16)$$

where $\bar{\eta} := \eta(b+1)$.

Putting $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ into Eq. (11), we have

$$g(s,\rho) = (s+\bar{\eta})\left[s^2 + \{1 + kp(D - \rho e^{-s\tau})\}s + \gamma\right] - \eta s = 0. \quad (17)$$

If all the roots of $g(s, \rho_q) = 0$ for $q = 1, \cdots, N$ exist in the open left-half plane, the homogeneous steady state $\left[\boldsymbol{x}_+^{*T} \quad \cdots \quad \boldsymbol{x}_+^{*T}\right]^T$ is stable.

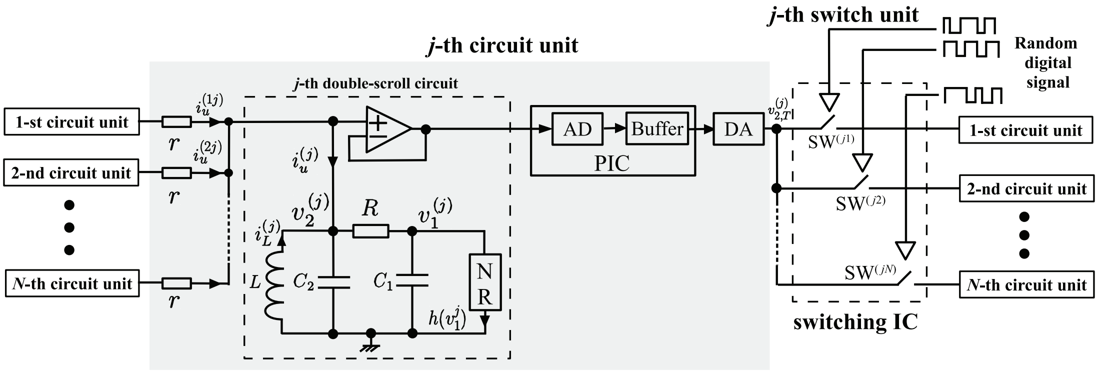


Figure 1: Circuit and switch units

3. Circuit Experiments

We implement a network consisting of the four double-scroll circuits. Let us focus on the j-th double-scroll circuits and its connections. Figure 1 shows the j-th circuit unit and the j-th switch unit. The circuit unit consists of a double-scroll circuit, a PIC (PIC18F2550), a DA converter, and coupling resistances r. The voltage $v_2^{(j)}(t)$ of the double-scroll circuit is applied to the PIC. The PIC samples the voltage through its AD converter and saves the sampled voltage into its buffer. The delayed voltage on 8-bit digital signal is converted into the analog voltage $v_{2,T}^{(j)} = v_2^{(j)}(t-T)$ via the DA converter. The switch unit, which governs the network topology, is implemented with switching ICs (ADG452). The switches $\mathrm{SW}^{(j1)}, \ldots, \mathrm{SW}^{(j4)}$ are turned on/off by digital signals. Each signal randomly switches between "on" and "off" every interval Δt. The signal turns on $\mathrm{SW}^{(jl)}$ (i.e., $\varepsilon^{(jl)} = 1$) with the probability p. The network consisting of these circuit and switch units is sketched in Fig. 2.

The parameters of double-scroll circuits are fixed as follows:

$$C_1 = 0.1\mu\mathrm{F},\ \ C_2 = 1.0\mu\mathrm{F},\ \ L = 180\mathrm{mH},\ \ B_p = 1.0\mathrm{V},$$
$$R = 1800\Omega,\ \ m_0 = -0.4\times10^{-3},\ \ m_1 = -0.8\times10^{-3},$$

where the double-scroll attractor is observed in each isolated oscillator. Further, we use the interval $\Delta t = 40\mu$s. This interval Δt is approximately one seventy-fifth of the natural period of the double-scroll circuit.

Let us consider the all-to-all network with

$$\boldsymbol{H} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, D = 3. \quad (18)$$

Figures 3(a) and 3(b) show the stability regions of the state (4) for $p = 0.5$ and $p = 1$ on our analytical results in the k-τ plane, respectively. The boundary curves of the regions are estimated by computing Eq. (17). When k crosses a bold (thin) line with an increase in it, the roots of Eq. (17) cross the imaginary axis from right (left) to left (right). The shaded areas are the stability regions, in which all roots of Eq. (17) exist in the open left-half plane. The upper stable region in Fig. 3(a) is expanded toward k axis compared with that in Fig. 3(b). This is because the coupling strength k in Eq. (11) is multiplied by the probability p.

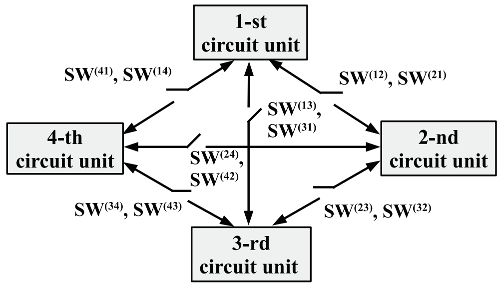


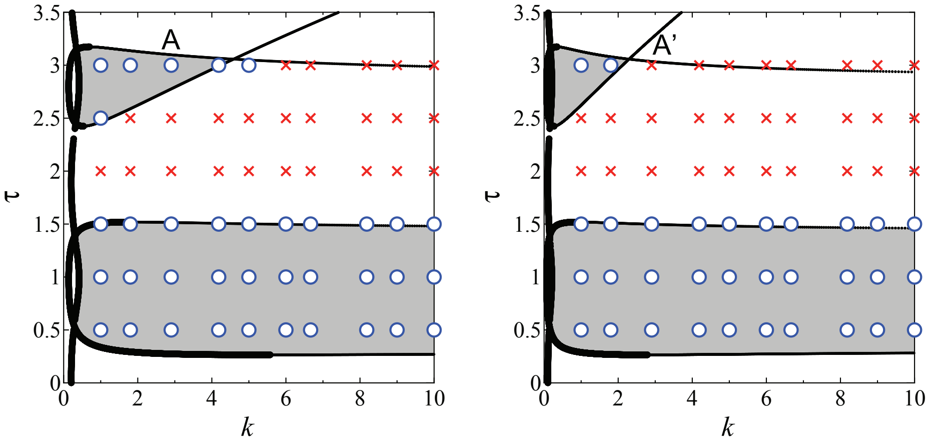
Figure 2: Time-varying network

The symbol $\bigcirc(\times)$ in Fig. 3 represents the parameter set (k, τ) where amplitude death is (is not) observed experimentally. The stable parameter sets on the circuit experiments agree with the stability region on our analytical estimation.

Figure 4 shows the time series data of all the double-scroll circuits for $k = 2.90, \tau = 3$ (Point A in Fig. 3(a) and Point A′ in Fig. 3(b)). Before $t = 150$ms, the network topology does not change (i.e. $p = 1$). At $t = 150$ms, the network topology starts to change with $p = 0.5$. This result indicates that $v_2^{(1)}, \ldots, v_2^{(4)}$, which oscillate in the time-invariant topology, are stabilized by the fast time-varying network topology.

4. Conclusion

This paper experimentally demonstrated that amplitude death is induced in the fast time-varying network consisting of the four double-scroll circuits. Our experimental results agreed with our analytical results.



(a) $p = 0.5$ (b) $p = 1$

Figure 3: Stability regions in $k - \tau$ plane: (a) time-varying network topology ($p = 0.5$), (b) time-invariant network topology ($p = 1$). The shaded regions and curves respectively represent the death regions and the boundaries based on Eq. (17). The symbol ◯(×) denotes the point where amplitude death is (is not) observed experimentally.

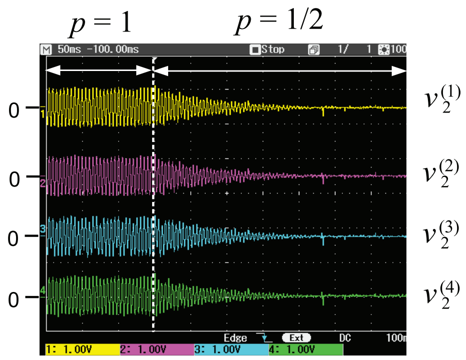


Figure 4: Time series data of voltage $v_2^{(1)}, \ldots, v_2^{(4)}$ ($N = 4, D = 3, k = 2.90, \tau = 3$). Horizontal axis: t (50ms/div); vertical axis: $v_2^{(1)}, \ldots, v_2^{(4)}$ (1V/div).

Acknowledgments

This research was partially supported by JSPS KAKENHI (26289131).

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