

Optimal Combiners in Pre-Amplified Optical Wireless Systems: Operation under Strong Atmospheric Turbulence

N. C. Sagias*, A. C. Boucouvalas*, K. Yiannopoulos*, M. Uysal†, and Z. Ghassemlooy‡

*Department of Informatics and Telecommunications, University of Peloponnese, Tripoli 22100, Greece.

†Faculty of Engineering, Ozyegin University, 34662, Istanbul, Turkey.

‡Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne NE1 8ST, U.K.

Abstract—In this paper we analytically investigate optimal combiners in pre-amplified diversity receivers that operate under strong atmospheric turbulence. We first demonstrate that the combiner operation is strongly affected by the existence of a signal-dependent noise component, which manifests at the optical detector output as the outcome of the beating process between the amplifier spontaneous noise and the signal itself. Due to the signal-dependent nature of noise, the optimal combiner acts as a hybrid between the well-known equal gain and maximal ratio combiner architectures. Having established the optimal design, we then proceed to assess the proposed combiner performance in the negative-exponential fading environment and demonstrate that it achieves an additional link gain of several dB in comparison with selection and equal gain combiners.

Index Terms—Bit-error-rate, outage probability, negative-exponential fading, outdoor optical wireless, optical amplifiers, diversity reception.

I. INTRODUCTION

Multi-Gb/s optical wireless communication (OWC) systems constitute a viable, low-cost and truly broadband interconnection alternative for the implementation of data networks with a radius of a few kms. The capacity that is provided by optical technologies, however, can only be fully utilized by properly taking into account the adverse aspects of infrared beam transmission through the atmosphere and compensating for them. Due to the volatile nature of the atmosphere's refractive index, transmission effects such as beam wander, spreading and time-varying losses [1] ultimately manifest as scintillations in the received optical power and the OWC link may suffer an outage when the scintillation becomes severe enough to lower the received power under the required sensitivity. The deleterious impact of atmosphere-induced scintillations on the OWC link performance has been extensively studied in the literature and a number of techniques have been proposed with a goal to immunize the OWC system against the stochastic response of the atmospheric channel. Applicable techniques include beam focusing [2], aperture averaging [3], spatial and temporal diversity schemes [4]–[8], coding [9]–[12], relaying [13]–[15] and amplification [15]–[18]. Combinations of these techniques have also been proposed, while the co-utilization of spatial diversity and amplification has previously reported high link gains by using multiple optical amplifiers and equal gain combiners (EGCs) in the electronic domain [18], [19].

Within this context, we recently demonstrated that a significant link gain can be obtained by deploying optimal combiners in a multi-branch receiver arrangement with amplification [20]. Optimal combiners in amplified systems operate as a hybrid between EGCs and maximal ratio combiners (MRCs), due to the existence of a signal-with-optical-noise beating term that dominates the receiver. The beating noise power is signal dependent and this leads to a dual operation of optimal combiners: whenever a branch enters a fade state, thus the signal (and beating noise) power is low, the combiner treats the branch similar to MRC and applies a gain that is signal dependent. If the branch signal is strong enough, then the corresponding gain stabilizes so as not to further aggravate the impact of the beating noise, and the combiner treats the branch similar to an EGC. Our previous study was limited to moderate turbulence and the results presented therein suggested that optimal combiners are better suited for deployment in more adverse conditions, which are typically expected in the saturated turbulence regime. In this work we extend our analysis with a goal to investigate the optimal combiner performance in negative-exponential fading [21], and demonstrate that the proposed combiners can indeed attain an additional benefit when compared with the equal gain and selection combiners (SCs).

The rest of this paper is organized as follows: in Section II we present a basic mathematical model for the description of the OWC channel, the amplifier and receiver. Sections III and IV calculate the outage probability and the average BER of the SC and the EGC in a negative-exponential fading environment, respectively, while Section V presents analytical relations for the optimal combiner structure, operation and performance in terms of the aforementioned metrics. Numerical results that quantify the comparison between the optimal combiner and the SC and EGCs are illustrated in Section VI, where it is also demonstrated that the optimal combiner provides a link gain improvement of several dB. Finally, Section VII summarizes the findings of this work and concludes the presentation.

II. CHANNEL, AMPLIFIER AND RECEIVER MODELS

The system under study is presented in Fig. 1. The OWC signal propagates through the atmosphere and experiences turbulence induced fading. At the receiver side, L identical

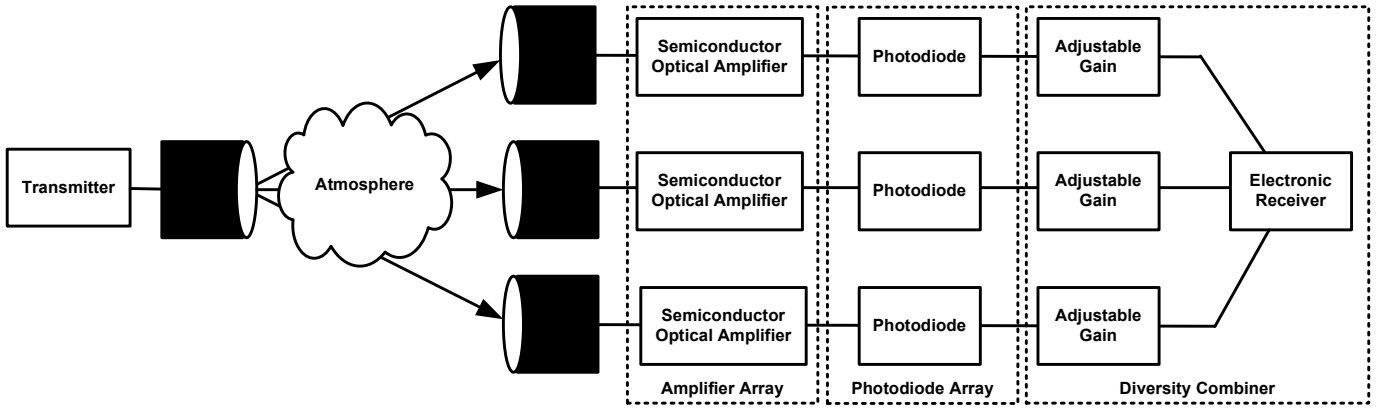


Fig. 1. Optically pre-amplified system with diversity.

receiving elements (optical antennas) are deployed and the output of each element is fed to an optical amplifier. The role of the amplifier is to improve the corresponding branch sensitivity and therefore enhance its resilience against fades. In the multi-branch setup of Fig. 1, the amplifier outputs are applied to photodetectors (PD), where the photocurrent outputs are then linearly combined prior to signal detection. The combiner scales the signal from each PD by a gain factor prior to combining them. Adjustable gains are provided at each branch of the combiner, so as to implement the most popular combiner types (selection, equal-gain or optimal).

For the rest of the analysis, we assume that the received signals are identically distributed and statistically independent (*i.i.d.*) random variables. Given that the channel stochastic response follows negative-exponential statistics, the optical signal x that is detected is distributed according to

$$f_X(x) = \frac{\exp\left(-\frac{x}{\bar{P}_{in}}\right)}{\bar{P}_{in}}, \quad (1)$$

where \bar{P}_{in} is the average input optical power at the receiver. For a fair comparison between receivers with multiple elements, we assume that the received power is split equally among branches, thus the optical power at each branch $z_i = \frac{x}{L}$ is distributed according to

$$f_{P_{in}}(z_i) = L \frac{\exp\left(-\frac{L z_i}{\bar{P}_{in}}\right)}{\bar{P}_{in}}. \quad (2)$$

The received optical signals traverse the corresponding amplifiers and receive a static gain equal to G . In addition, each amplifier generates an optical noise component due to the amplified spontaneous emission (ASE), which is described by the ASE spectral density

$$P_n = n_{sp} h \frac{c}{\lambda}, \quad (3)$$

where c is the speed of light in a vacuum, h is the Planck constant, n_{sp} is the population inversion factor and λ denotes the wavelength. The optical signal and the ASE beat on the photodiodes (square-law detectors) of the receiver and as a

result a number of electrical noise components will be present at the PD output [22], [23]. The associated noise variances are denoted as thermal, shot, signal-spontaneous beating and spontaneous-spontaneous beating, respectively, and are calculated from

$$\sigma_{th}^2 = \frac{4 k_B T F_n B_e}{R_L}, \quad (4a)$$

$$\sigma_{shot}^2(z_i) = 2 q R (G z_i + (G - 1) P_n B_o) B_e, \quad (4b)$$

$$\sigma_{sig-sp}^2(z_i) = 4 R^2 G z_i (G - 1) P_n B_e, \quad (4c)$$

and

$$\sigma_{sp-sp}^2 = R^2 ((G - 1) P_n)^2 (2 B_o - B_e) B_e, \quad (4d)$$

where B_e and B_o are the electrical and optical bandwidths, R is the photodiode responsivity, T is the receiver temperature, k_B denotes the Boltzmann constant, F_n is the electric noise figure and R_L is the resistor load. Given (4), the signal and noise powers for the '1' and '0' bits are directly given by

$$I_1(z_i) = R G z_i, \quad (5a)$$

$$\sigma_1^2(z_i) = \sigma_{th}^2 + \sigma_{shot}^2(z_i) + \sigma_{sig-sp}^2(z_i) + \sigma_{sp-sp}^2, \quad (5b)$$

and

$$I_0 = 0, \quad (6a)$$

$$\sigma_0^2 = \sigma_{th}^2 + \sigma_{shot}^2(0) + \sigma_{sp-sp}^2, \quad (6b)$$

respectively.

III. SELECTION COMBINER

The selection combiner samples all the received signals and selects the one with the highest irradiance level, therefore

$$z_{sc} = \max z_i. \quad (7)$$

An outage occurs when all branches simultaneously experience a fade, and since the received signals are independent it follows that

$$\begin{aligned} P_{out,sc} &= \prod_{i=1}^L \Pr\{BER(z_i) > BER_0\} \\ &= \Pr\{BER(z) > BER_0\}^L, \end{aligned} \quad (8)$$

where BER_0 is the desired BER level of the OWC system. Equivalently, one can calculate the outage probability from the receiver sensitivity P_s

$$P_{\text{out,sc}} = \Pr\{z \leq P_s\}^L = \left(\int_0^{P_s} f_{P_{\text{in}}}(z) dz \right)^L \quad (9)$$

$$= \left(1 - \exp\left(-\frac{LP_s}{\overline{P}_{\text{in}}}\right) \right)^L.$$

The corresponding sensitivity is obtained after solving

$$\frac{1}{2} \operatorname{erfc}\left(\frac{Q(P_s)}{\sqrt{2}}\right) = BER_0, \quad (10)$$

where the Q-factor of the receiver is given by

$$Q(z) = \frac{I_1(z)}{\sigma_0 + \sigma_1(z)}, \quad (11)$$

assuming that each branch is capable of estimating the channel state (CSI-capable) and setting its decision threshold on a bit-by-bit fashion to

$$I_{th}(z_i) = \frac{\sigma_0 I_1(z_i)}{\sigma_0 + \sigma_1(z_i)}. \quad (12)$$

With respect to the average BER, it is calculated from the probability density function of z_{sc} as

$$\overline{BER}_{sc} = \int_0^\infty BER(z_{sc}) f_{sc}(z_{sc}) dz_{sc} \quad (13)$$

$$= \frac{1}{2} \int_0^\infty \operatorname{erfc}\left(\frac{Q(z_{sc})}{\sqrt{2}}\right) f_{sc}(z_{sc}) dz_{sc}.$$

$f_{sc}(z_{sc})$ is obtained by differentiating the cumulative distribution function of the selection combiner from (9) and the result is

$$f_{sc}(z_{sc}) = L^2 \frac{\exp\left(-\frac{Lz_{sc}}{\overline{P}_{\text{in}}}\right)}{\overline{P}_{\text{in}}} \left(1 - \exp\left(-\frac{Lz_{sc}}{\overline{P}_{\text{in}}}\right)\right)^{L-1}. \quad (14)$$

After combining (13) and (14) we finally obtain

$$\overline{BER}_{sc} = \frac{L^2}{2} \int_0^\infty \operatorname{erfc}\left(\frac{Q(z_{sc})}{\sqrt{2}}\right) \frac{\exp\left(-\frac{Lz_{sc}}{\overline{P}_{\text{in}}}\right)}{\overline{P}_{\text{in}}} \times \left(1 - \exp\left(-\frac{Lz_{sc}}{\overline{P}_{\text{in}}}\right)\right)^{L-1} dz_{sc}. \quad (15)$$

IV. EQUAL GAIN COMBINER

The EGC adds the electrical signals from all branches after providing a common gain. Assuming that the combiner is CSI-capable, the instantaneous BER is calculated as

$$BER_{egc} = \frac{1}{2} \operatorname{erfc}\left(\frac{Q_{egc}}{\sqrt{2}}\right), \quad (16)$$

where Q_{egc} is the EGC Q-factor that equals

$$Q_{egc} = \frac{\sum_{i=1}^L I_1(z_i)}{\sqrt{\sum_{i=1}^L \sigma_1^2(z_i) + \sqrt{L} \sigma_0^2}}. \quad (17)$$

Eq. (17) can be written in a simpler form as

$$Q_{egc}(z_{egc}) = Q_A \frac{z_{egc}}{\sqrt{z_{egc} + Lz_0 + \sqrt{L}z_0}}, \quad (18)$$

where

$$Q_A = \frac{RG}{\sigma_A}, \quad (19a)$$

$$\sigma_A^2 = 2qRG B_e + 4R^2 G(G-1) P_n B_e, \quad (19b)$$

$$z_0 = \frac{\sigma_0^2}{\sigma_A^2}, \quad (19c)$$

and

$$z_{egc} = \sum_{i=1}^L z_i \quad (20)$$

is a random variable that is obtained from the sum of *i.i.d.* negative exponential variables. The pdf of z_{egc} is calculated in a straightforward manner from the Erlang distribution

$$f_{egc}(z_{egc}) = \left(\frac{L}{\overline{P}_{\text{in}}}\right)^L z_{egc}^{L-1} \frac{\exp\left(-\frac{Lz_{egc}}{\overline{P}_{\text{in}}}\right)}{\Gamma(L)}, \quad (21)$$

with $\Gamma(\cdot)$ denoting the Gamma function. Following (21), the EGC outage probability is given by

$$P_{\text{out,egc}} = 1 - \frac{\Gamma\left(L, \frac{L P_{s, \text{egc}}}{\overline{P}_{\text{in}}}\right)}{\Gamma(L)}, \quad (22)$$

where the EGC sensitivity $P_{s, \text{egc}}$ is calculated from

$$\frac{1}{2} \operatorname{erfc}\left(\frac{Q_{egc}(P_{s, \text{egc}})}{\sqrt{2}}\right) = BER_0. \quad (23)$$

Finally, the average BER is obtained from (18) and (21) as

$$\overline{BER}_{egc} = \frac{1}{2} \left(\frac{L}{\overline{P}_{\text{in}}}\right)^L \int_0^\infty \operatorname{erfc}\left(\frac{Q_{egc}(z_{egc})}{\sqrt{2}}\right) \times z_{egc}^{L-1} \frac{\exp\left(-\frac{Lz_{egc}}{\overline{P}_{\text{in}}}\right)}{\Gamma(L)} dz_{egc}. \quad (24)$$

V. OPTIMAL COMBINER

The optimal combiner provides unequal gains w_i to branches, thus the Q-factor is written as

$$Q_{\text{opt}} = \frac{\sum_{i=1}^L w_i I_1(z_i)}{\sqrt{\sum_{i=1}^L w_i^2 \sigma_1^2(z_i) + \sqrt{\sum_{i=1}^L w_i^2} \sigma_0^2}}. \quad (25)$$

By using the definitions of (19) we re-write (25) as

$$Q_{\text{opt}} = Q_A \frac{\sum_{i=1}^L w_i z_i}{\sqrt{\sum_{i=1}^L w_i^2 (z_i + z_0) + \sqrt{\sum_{i=1}^L w_i^2} z_0}}. \quad (26)$$

Eq. (26) indicates that the optimal combiner in pre-amplified OWC systems with diversity is fundamentally different from

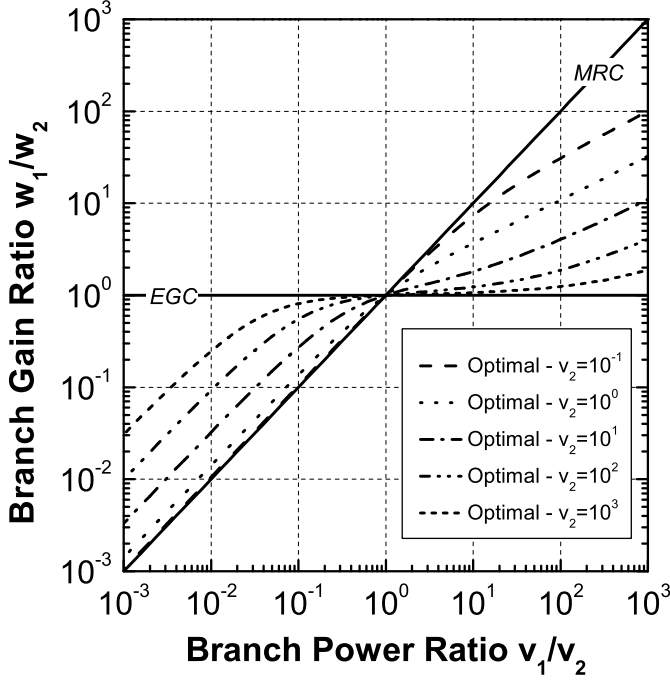


Fig. 2. Branch gain ratio $\frac{w_1}{w_2}$ as a function of the normalized branch powers $v_1 = \frac{z_1}{z_0}$ and $v_2 = \frac{z_2}{z_0}$ for a dual branch optimal combiner arrangement.

the optimal combiner in non-amplified systems (MRC), due to the existence of the signal-spontaneous beating noise that grows with the signal itself. By differentiating with respect to the branch gains w_i , we find that the optimal gain values are given after solving

$$w_{i,opt} \left(\frac{z_i + z_0}{\sqrt{\sum_{i=1}^L w_{i,opt}^2 (z_i + z_0)}} + \frac{z_0}{\sqrt{\sum_{i=1}^L w_{i,opt}^2 z_0}} \right) = \frac{z_i \left(\sqrt{\sum_{i=1}^L w_{i,opt}^2 (z_i + z_0)} + \sqrt{\sum_{i=1}^L w_{i,opt}^2 z_0} \right)}{\sum_{i=1}^L w_{i,opt} z_i}. \quad (27)$$

Eq. (27) can not be solved in a closed form, but some insight can be provided by numerically solving it. A solution for a combiner with $L = 2$ branches is presented in Fig. 2, where the branch gain ratio $\frac{w_1}{w_2}$ is plotted against the normalized branch powers

$$v_i = \frac{z_i}{z_0}. \quad (28)$$

The figure demonstrates that the optimal combiner operates in a fashion similar to an MRC when the input powers are low compared to z_0 and the branch gain increases almost linearly with the input power. At higher input powers, however, the branch gain saturates and becomes relatively insensitive to further increases in power. In this regime, the branch gain is almost constant and the optimal combiner operation bears a close resemblance to the EGC one.

TABLE I
SYSTEM PARAMETERS

Parameter	Symbol	Value
Amplifier gain	G	20 dB
Wavelength	λ	1550 nm
Population inversion factor	n_{sp}	4.0
Optical bandwidth	B_o	50 GHz
Photodiode responsivity	R	1.25 A/W
Receiver temperature	T	300° K
Resistor load	R_L	100 Ω
Electrical noise figure	F_n	3 dB
Electrical bandwidth	B_e	7 GHz

The optimal combiner Q-factor also proves challenging to calculate, but we have previously shown that it is bound by

$$Q_{opt}(z_{opt}) = Q_A \sqrt{L z_{opt} + L z_0} - \sqrt{L z_0} \quad (29a)$$

$$z_{opt} = \max z_i. \quad (29b)$$

Since the pdf of z_{opt} has been calculated in (14), a lower limit for the outage probability of the optimal combiner is given by

$$P_{out,opt} \geq \Pr \{z_{opt} \leq P_{s,opt}\} = \left(1 - \exp \left(-\frac{L P_{s,opt}}{\bar{P}_{in}} \right) \right)^L, \quad (30)$$

where $P_{s,opt}$ corresponds to the optimal combiner sensitivity

$$\frac{1}{2} \operatorname{erfc} \left(\frac{Q_{opt}(P_{s,opt})}{\sqrt{2}} \right) = BER_0. \quad (31)$$

Finally, the average BER is also bound by

$$\begin{aligned} \overline{BER}_{opt} &\geq \frac{1}{2} \int_0^\infty \operatorname{erfc} \left(\frac{Q_{opt}(z_{opt})}{\sqrt{2}} \right) f_{opt}(z_{opt}) dz_{opt} \\ &= \frac{L^2}{2} \int_0^\infty \operatorname{erfc} \left(\frac{Q(z_{opt})}{\sqrt{2}} \right) \frac{\exp \left(-\frac{L z_{opt}}{\bar{P}_{in}} \right)}{\bar{P}_{in}} \\ &\quad \times \left(1 - \exp \left(-\frac{L z_{opt}}{\bar{P}_{in}} \right) \right)^{L-1} dz_{opt}. \end{aligned} \quad (32)$$

VI. RESULTS AND DISCUSSION

The performance of the amplifier-assisted diversity setup is investigated for a 10 Gb/s optical wireless link with channel, amplifier and receiver parameter values that are summarized in Table I. Fig. 3 illustrates the outage probability of the proposed setup for $L = 1, 2$ and 5 diversity branches and a required BER of 10^{-3} . The outage probability of a system that relies solely on diversity is also plotted in the figure for comparison purposes. As it is shown in Fig. 3, both amplification and diversity offer a significant link gain, while the combination of the two methods amounts to a gain of over 25 – 30 dB depending on the required BER, the desired outage probability and the number of receiving elements. The results also suggest that the optimal combiner outperforms the SC by a significant factor and a link budget improvement of up to 5 dB is demonstrated. A less pronounced, but still

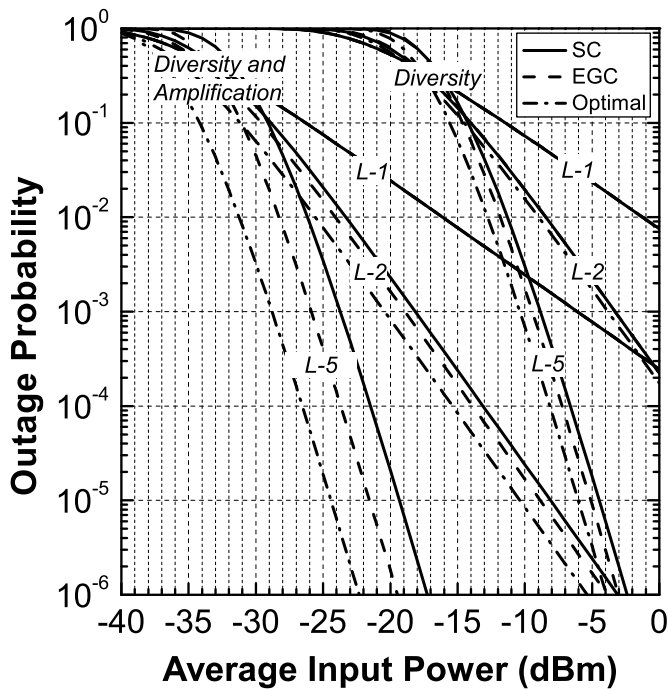


Fig. 3. Outage probability versus the average input power for $BER_0 = 10^{-3}$.

valuable, improvement of 3 dB is predicted when the optimal combiner is compared with the EGC for the parameter set under consideration. Similar conclusions can be deduced from the average BER plots in Fig. 4, where link margins of the same magnitude are observed for the pre-amplified system with optimal diversity combining.

VII. CONCLUSION

We have presented an analytical description of the outage probability and the average BER in pre-amplified OWC systems that optimally combine signals from multiple receivers. Based on the analytical model, we derived results on an electronic combiner that optimizes the BER performance of the system under negative-exponential fading. The presented results show that the optimal combiner performs closely to an EGC for increased signal powers, while its per-branch operation reverts to an MRC when a fade occurs. An additional benefit of 3-5 dB is expected from the deployment of the proposed combiner in practical high-capacity OWC links operating under strong turbulence.

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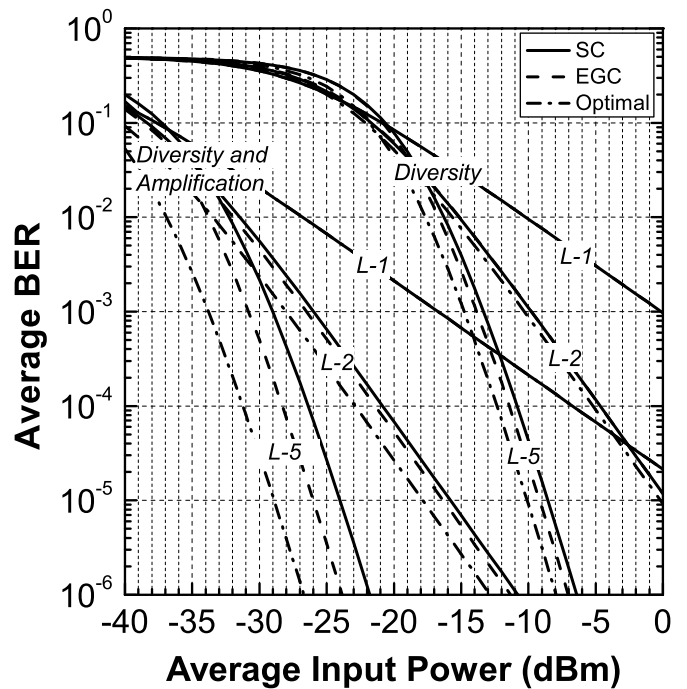


Fig. 4. Average BER versus the average input power.

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