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Valuation of Information Sharing in a Two-level Supply Chain facing Demands with Jump Diffusion Processes

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Abstract—This paper deals with the valuation of information sharing in a two-level supply chain facing demands with jump diffusion processes. We show basic model for demand time series in a supply chain described by variables including jump diffusion processes as well as ordinary Brownian motion. Then, the optimization of evaluations functions is attained. For simplicity, we reduce the problem to cases where only 4 parameters are to be shared. We show simulation results for valuing the information sharing with deteriorated data.

Information sharing, Valuing scheme, Jump diffusion processes, Deterioration.

1. Introduction

Recently, the design of sensing and transmission systems are widely discussed for environmental observation and security systems where information is exchanged over the broadband Internet[1][2]. At the same time, many industries embarked on reengineering efforts to improve of their supply chains [3]-[5]. One key initiative that is commonly mentioned is information sharing between partners in a supply chain. Conventional works demonstrated that information sharing alone could provide significant inventory reduction and cost saving to manufacturer, but these works depend on relatively simple models.

In the paper, firstly we summarize the basics of information sharing in a supply chain for a simple demand process [3]-[5]. Then, we extend the model of demand process into cases where time series exhibits enormous spikes in which besides ordinary Brownian motions there exist jumps in a short period of time then return to normal level just as quickly [6]-[10]. Then, we formalize operating objective to seek the strategy that maximize the expected cash flows. Consider a simple two-level supply chain that consists of one manufacture and one retailer where the information about underlying demand process faced by the retailer is transmitted to the manufacturer. We assume four parameters about the demand process shared by the manufacturer and retailer , where the are limited due to rate distortions. Then we evaluate the value of information sharing by comparing the optimal production scheme and cases with inefficient information (deterioration).

2. Information sharing and rate-distortion

We concisely summarize the value of information sharing shown in conventional works.

(1) Information sharing to reduce inventory cost

Consider a simple two-level supply chain that consists of a one manufacturer and one retailer. Let D_t be the underlying demand process faced by a retailer, and is written as $D_t = d + \rho D_{t-1} + \varepsilon_t$ where is a normal distribution with mean zero and variance σ^2. We consider a periodic review system in which each site review its inventory level and replenishes its inventory from the upstream site every period with length L and l, respectively. After demand D_t has been realized the retailer observes the inventory level and place an order of size Y_t. Next, the manufacturer handle this ordering process for required order Y_t and if the manufacturer does not have enough stock to fill order by paying additional cost for shortfall. After manufacturer receives retailer's order, the manufacture immediately places an order so as to bring the inventory position to an order level ready for till the end of period $t + L + 1$. Then, the optimal order-up-to level (theoretically necessitated order) with no information sharing is given by

$$T_t^* = M_t + K\sigma\sqrt{V} \tag{1}$$

where $K = \Phi^{-1}[P/(P + H)]$ and the function $\Phi(.)$ is a normal distribution , and H, P are holding cost and storage cost of a manufacturer. Parameters M_t and V come from the normal distribution F_t with mean M_t and variance $\sigma^2 V$ of the manufacturer's total shipment quantity over lead time.

With information sharing, the manufacturer now know the retailer's order quantity, and the error term ε_t, the optimal order-up-to level is given by.

$$T_t^* = M_t' + K\sigma\sqrt{V}' \tag{2}$$

In this formula, we have

$$M_t' = M_t - f_1(\rho, l, L)\varepsilon_t < M_t, V' = f_2(\rho, l, L) < V_t \tag{3}$$

where functions $f_1(.), f_2(.)$ are certain functions. It is seen from the result, the quantity is corrected by the error term for forecast of demand process D_t. Thus, the information sharing would reduce the variance of the total shipment quantity of the manufacturer.

(2) rate distortion function

If the data of information sharing is effective (not deteriorated), shared information will contribute inventory and cost reduction. However, retailers usually pay money to monitor the demand in the market, and the efficiency of information sharing depends on the cost of monitoring and analysis of demand. Then, we summarize the relation between the cost and efficiency based on well-known rate-distortion function of the information theory. Under the restriction on transmission rate for each decoder-encoder pair, the region of feasible realization of rate-distortion functions are defined [1][2].

We evaluate the efficiency of information sharing by using the relation so that we find complementary characteristics of cost-benefit feature in monitoring the market demand.

3. Optimization of evaluation function including jumps

A general valuation and control algorithm must be employed to deal with wide range of potential spike time series model while maintaining computationally tractable. It is assumed that there exits a manufacturer facing the production of final good by using parts procured from other firms in the period $t = 0 \sim T$. We define the symbols at time t to describe the production process as follows:

unit price of parts: $f(t)$.
demand for goods in the market: $x(t)$.
unit price of good in market: $P(t)$.
quantity of production by manufacturer: $c(t)$.
inventory level of at manufacturer: $R(t)$.

For the purpose of the paper, we assume that the jump processes are included only in the demand time series, and other time series such as price of final goods are assumed to be stable. The demand time series exhibits enormous spikes in which demands may jumps several orders of magnitude in a short period of time and then return to normal level just as quickly. In case of sudden rise (fall) of demands is called upward (downward) jumps in the paper.

For the most general form of a continuous time, Markov process for the demand can be written as (with unit increment of time, $dt = \Delta t = 1$).

$$dx(t) = \alpha(x(t), t) + (J_1 - x)\lambda_{go}(x) + (J_2 - x)\lambda_{back}(x) + \sigma P dz \quad (4)$$

$$\alpha(x(t), t) = a_1[a_2 \sin \omega_0 t + a_3 - P(t)] \quad (5)$$

where $\alpha(x(t), t)$ (including constants $a_1 \sim a_2$) denote the trend of time series , J_1, J_2 are amounts of jumps added to $x(t)$, and dz is a standard increment of Brownian motion. Two factors $\lambda_{go}(x), \lambda_{back}(x)$ describe the probability of occurrence of jumps. The probability $\lambda_{go}(x)$ means the occurrence of beginning of a jump, and $\lambda_{back}(x)$ means the probability of reverse jump to return previous level. We assume J_1, J_2 obey to normal distributions.

$J_1 \sim N(a_{11}, s_{11}), J_2 \sim N(a_{12}, s_{12})$

where $N(a, s)$ means the normal distribution with mean a and standard deviation b. Also for downward jumps we use

$J_1 \sim N(a_{21}, s_{21}), J_2 \sim N(a_{22}, s_{22})$

We assume relatively simple forms for the probabilities $\lambda_{go}(x), \lambda_{back}(x)$ with piece-wise linear functions. Fig.1 depicts the schematic forms for these probabilities, and is seen that expect for the linear transition areas , these probabilities take constant values such as θ_{11}. When prices are low they follow a time dependent, mean reverting, stochastic Brownian motion. As price rise, so does the probability of an beginning of upward spike. When a spike occurs, the time series instantly jumps into the high value regime, while in this regime the probability of a backward jumps is relatively high which will bring the values back into low value regime. Fig.2 depicts simulated time series including upward and downward jumps.

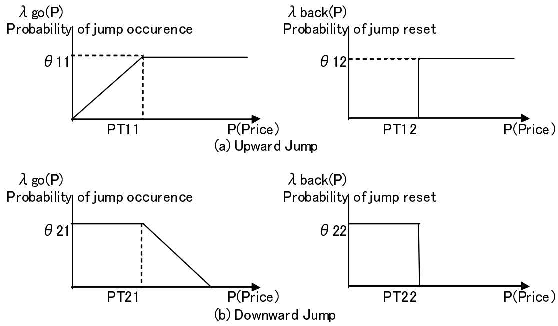

Fig.1-Probabilities $\lambda_{go}, \lambda_{back}$ for upward (upper) and downward (lower) jump.

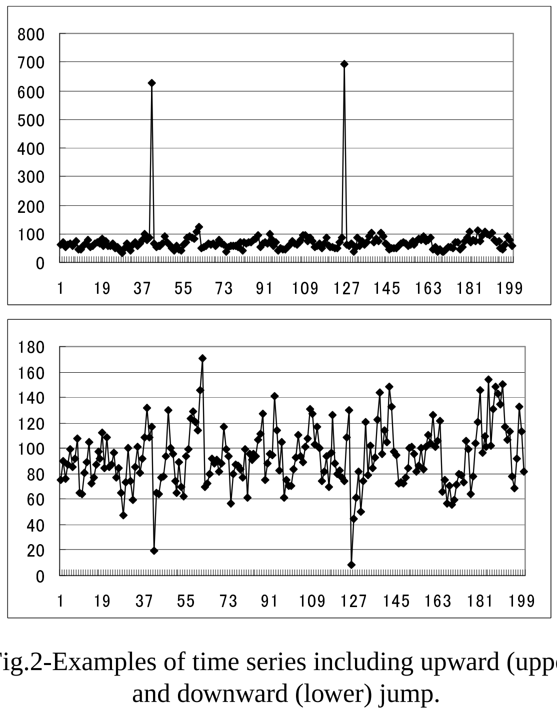

Fig.2-Examples of time series including upward (upper) and downward (lower) jump.

3.1. Optimization of production

We denote the expected cash flow up to time t as $V(t)$. The choice of $R(t)$ is expressed as the differential equation.

$$dR(t) = [c(t) - D(t)]dt \tag{6}$$

We seek the strategy $c(P, f, t, R)$ that maximize the expected cash flows

$$E[\int_t^T e^{-\rho(t-\tau)} F(P, f, c, t, R) d\tau] \tag{7}$$

$$\begin{aligned} F(.) &= P(t)D(t) - A(c(t), f(t)) - H(R) \\ &-B([D(t) - c(t) - R(t)]) \end{aligned} \tag{8}$$

where $A(.), H(.), B(.)$ denote the cost function for production, holding cost function and back order cost function, respectively. ρ is the discount factor to adjust the time value of money. The function $B(.)$ means the cost when the production is not sufficient to the demand of market and supplied form other firm. By employing the multiple-dimensional version of Ito's lemma, we expand the above equation in a Taylor's series, and yields as.

$$\begin{aligned} &\max_c [L(V) + F(x, c, t, R) + (c - D)V_R \\ &+ \sum_{k=1}^{2} \epsilon_k E[V_k^{(+,x)} - V] = 0 \end{aligned} \tag{9}$$

$$V_k^{(+,P)} = V(P + \gamma_k(P, t, J_k), f, t, R) \tag{10}$$

where $V_k^{(+,x)}$ denotes the value of $V(.)$, given that jump processes in x occurred.

Only two terms in the above equation involves c, so the optimal value for c maximize.

$$\max_c [F(P, f, x, c, t, R) + (c - D)V_R] \tag{11}$$

This result implies that when c is chose to maximize above equation, then,

$$\begin{aligned} 0 \;=\; & L(V) + F(x, c, t, R) + (c - D)V_R \\ & + \sum_{k=1}^{N_x} \epsilon_k E[V_k^{(+,x)} - V] \end{aligned} \tag{12}$$

The terminal condition at time T is evident from equation (1), and is given by

$$V(x, T; c) = 0 \tag{13}$$

3.2. Terms for information sharing

It is very hard to include many parameters in the formulation of jump processes, then we restrict ourselves to cases where our objects of information sharing are following four terms.

(1) ω_0
(2) σ
(3) $PT_{11}(PT_{21})$
(4) mean value s_{11} of normal distribution in $J_1 \sim N(a_{11}, s_{11})$ and a_{21} in $J_1 \sim N(a_{21}, s_{21})$

Other parameters such as $\theta_{11}\theta_{12}$ $(\theta_{21}, \theta_{22})$ are assumed to be constant. We also assume that parameters PT_{12}, PT_{22} are proportional to PT_{11}, PT_{21}, and $s_{11}(s_{21})$ is proportional to $a_{11}(a_{21})$. The estimation process for these parameters are realized as in Fig.3 based on the multi-stage fuzzy inferences and the GP procedure (however, details are omitted here due to the restriction of pages)[10].

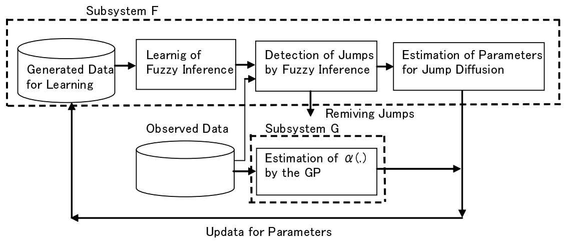

Fig.3-Estimation scheme of parameters such as ω_0[10]

4. Applications

Now we show simulation studies to evaluate the information sharing in the production under demand including jumps. Following are assumed to be given as initial values.

Upward jumps:
$\alpha(x, t) = 0.4[(15 \sin 0.26t + 27 - x)]$
$\sigma = 0.2, PT_{11} = 100, PT12 = 100$
$a_{11} = 700, s_{11} = 100, a_{12} = 100, s_{12} = 10$
$\theta_{11} = 0.01, \theta_{12} = 0.85$

Downward jumps:
$\alpha(x, t) = 0.4[(15 \sin 0.26t + 27 - x)]$
$\sigma = 0.2, PT_{21} = 100, PT22 = 50$
$a_{21} = 50, s_{21} = 20, a_{22} = 50, s_{22} = 20$
$\theta_{21} = 0.01, \theta_{22} = 0.85$

As we already mentioned, four terms ω_0, σ ,$PT_{11}(PT_{21})$ and mean value a_{11}, a_{21} are assumed to be objects for information sharing between manufacturer and retailer. The valuation scheme of information sharing of the paper is based on the estimation of expected cash flows of the manufacturer throughout the period $t = 1 \sim T$, namely $V(0)$. If the transmission of information about four terms between manufacturer and retailers are complete (correct), then the value of $V(0)$ (attained cash flow) is the same as optimal value of production (denoted as $V(0)^P$. However, the transmitted data for parameters are not correct, then the value of $V(0)$ (denoted as $V(0)^S$) deviate from $V(0)^P$, and is smaller than $V(0)^P$, while the manufacturer use the wrong (deteriorated) value of parameters for the production. Therefore, the value of information sharing is evaluated by calculation relative deviation between $V(0)^P$ and $V(0)^S$. For simplicity, we assume that transmission error occurs independently in four terms, and consider cases where terms are multiplied by a certain constant as the distortion (deterioration) of transmission. Then, we define following four cases for the wrong (deteriorated) transmission of parameters (terms) as Case I〜Case IV. In Case I〜Case IV, terms are multiplied by 1.2, 1.5, 1.8, and 0.5 respectively. For example, for term ω_0, in Case I the value $1.2 \times \omega_0$ is transmitted in place of ω_0.

Table 1 and 2 show the result of evaluation of information sharing based on simulation studies for cases by using the decrease of cash flow defined as the indicator of deterioration $Y = (V(0)^P - V(0)^S)/V(0)^P$. As is seen from the results, term ω_0 affects always to the efficiency of information sharing for all cases, and term σ affect increasingly along the deviation of term from correct data. On the other hand, shapes of probabilities of occurrence of jumps are not serious for the decrease of efficiency of information sharing. The results implies that we could allocate transmission capability mostly to ω_0 and σ.

Table 1. Deterioration of information denoted by Y (upward jump).

temrs	Case I	Case II	Case III	Case IV
ω_0	0.12	0.11	0.11	0.12
σ	0.04	0.12	0.19	0.14
PT_{11}	0.01	0.01	0.01	0.01
a_{11}	0.01	0.02	0.03	0.05

Table 2. Deterioration of information denoted by Y (downward jump).

forward	Case I	Case II	Case III	Case IV
ω_0	0.13	0.12	0.12	0.13
σ	0.03	0.10	0.16	0.14
PT_{21}	0.004	0.008	0.008	0.005
a_{21}	0.0004	0.002	0.004	0.009

5. Conclusion

In this paper we showed the valuation of information sharing in a two-level supply chain facing demands with jump diffusion processes. We reduced the problem to cases where only 4 parameters are to be shared. As applications, we showed simulation results for valuing the information sharing under deteriorated transmissions. For future works, we intend to apply the method of the paper to real world data, and further works will be done.

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