

# Non-Overshoot Position Control of Pump-controlled Asymmetric Cylinder Systems with Reference Management

Jian-feng Tao<sup>†</sup>, Yuzo Ohta<sup>‡</sup>, Hao Guo<sup>‡</sup>, and Cheng-liang Liu<sup>†</sup>

<sup>†</sup> School of Mechanical Engineering, Shanghai Jiao-tong University,  
 Shanghai 200240, China

<sup>‡</sup> Graduate School of System Informatics, Kobe University  
 Kobe 657-8501, Japan  
 Email: jftao@sjtu.edu.cn

**Abstract**—Pump-controlled asymmetric cylinder systems are widely used on industry machines, especially on hydraulic presses, but analysis and synthesis of such systems is very hard due to the nonlinearity caused by the different working areas of cylinder piston. The requirement of non-overshoot position precision control and the constraint of control input make the problem more difficult. In this paper, a simplified mathematic model is proposed. A control algorithm based on reference management is adopted to transform the nonlinear system into a constrained linear system, so that classical linear analysis and synthesis method can be used to approach the non-overshoot requirement and satisfy the input constraints. The numerical simulation is carried out and the results show that the proposed method is valid.

## 1. Introduction

Due to the maturity of the concept and the technology of electrically controlled pump<sup>[1-2]</sup>, pump-controlled asymmetric cylinder systems are getting more and more application from industry. After its success in plastic injection modeling machines<sup>[3,4]</sup>, hydraulics researchers and engineers are trying their best in applying this promoting technology on metal forming press machines<sup>[5-7]</sup>. Most of the aforementioned applications adopted close-loop circuits powered by bidirectional electrically controlled pump for space-saving and use auxiliary pump to keep the cylinder chamber with certain pressure for high response performance and to compensate the unbalanced flow between the input flow and output flow of the asymmetric cylinder. Therefore, the costs, including the expensive components, the complex circuits and the complex control algorithm, are high. In fact, for some stationary industry applications, such as for metal forming presses, people do not care the space occupied by oil tank, and because the load is constant or does not vary rapidly, the fast response performance comparable to valve controlled system is unnecessary. An open loop circuit controlled unidirectional electrically controlled pump, direction valve and some other auxiliary components is enough for the press position control<sup>[2]</sup>. Unfortunately, there are few researches reporting the design, modeling and control of such systems.

In this paper, a pump controlled asymmetric cylinder system, which is similar to the system in Ref.[2], is researched. In Section.2, the schematic and the special requirement of the system is described. In Section.3, the mathematic model is given and in Section.4 a control algorithm based on reference management are proposed. In Section.5 mathematic simulation is carried out to validate the proposed method. Short conclusions are given in Section.6.

## 2. System Descriptions

### 2.1. Schematic

Fig.1 shows the schematic of the presented pump controlled asymmetric cylinder system. The system consists of unidirectional speed variable electrically controlled pump(1), safety valve (2), direction valve (3), hydraulic controlled check valve (4-1,2), asymmetric cylinder (5), pressure sensor (6-1,2), force sensor (7), displacement sensor (8), spring (9), damper (10), and motion controller (11).

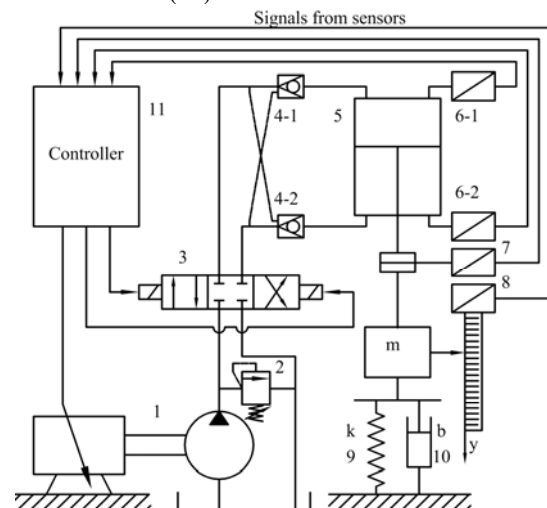


Figure.1 Schematic of pump controlled asymmetric cylinder system

When direction valve is on its left position, pump's output flow passes through the and check valve (4-1) and

enters cylinder's chamber without rod; oil in the chamber with rod flows through the another check valve(4-2) and direction valve, and returns to the oil tank; then the rod of the cylinder overcomes the spring force and damping force, and moves down to the desired position. Similar actions happen to the system to make the rod return to its original position, while the valve is set to the right position. The displacement sensor is set to monitor the rod's position. The controller collects pressure and displacement signals to generate the control input of the d pump, so that the desired dynamic performance and steady state performance are achieved.

## 2.2. Requirements

For precision positioning function, the requirement of no overshoot step response is important, especially for forming machines. In the pressing process, the deformation of products has plastic deformation ingredient. Therefore, if the step response has an overshoot, the precision of the product size maybe has lost, although there is no error between the desired position and the cylinder's final position. On the other hand, since we use an unidirectional pump to control the system's position, the way to pull the system from the overshoot position to the desired position is to change the direction valve to its another non-mid position. A slow valve may lead to more vibrations around the desired position. Then, a fast valve is required, and this increases component costs. The switch of the valve also lead to that we have to describe the system's dynamic behavior with a nonlinear mathematic model as shown in Section. 3. As known to all, the analysis and synthesis of nonlinear control system are difficult problems. Considering the reasons mentioned above, we take no overshoot as the most important requirement. Since there is usually no special requirement for the system's return position, we only consider the control of cylinder moving down to a desired position in this paper.

## 3. System modeling

### 3.1. Assumptions

Referring to Fig.1, we assume that: 1) the oil temperature and the system effective bulk modulus are constant; 2) electrically controlled pump is a speed variable pump, and the dynamics of electric motor is negligible; 3) there are no line losses and dynamics; 4) pressure drops of the direction valve and hydraulic controlled check valve are small and negligible; 5) the dynamics of the direction valve and hydraulic controlled check valve have no effect on the systems dynamics; and the safety valve is close in system's normal operation; 6) Friction force and gravity of the rod are small and negligible; 7) the variation of the cylinder chambers introduced by the piston movement is small and negligible.

### 3.2. Mathematic model

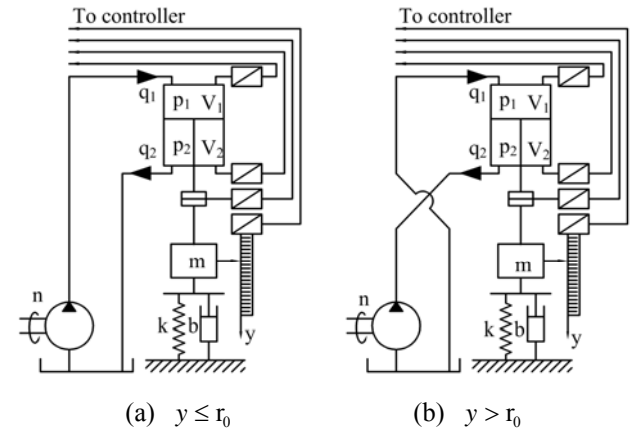


Figure.2 Simplified hydraulic circuit for pump controlled asymmetric cylinder

Under the assumptions in Section 3.1, we can simplify the hydraulic circuit as shown in Fig.2. When the current position  $y \leq r_0$  (the desired position), the lower chamber pressure  $p_2 = 0$ , and the continuity equation for the upper chamber can be written as

$$q_1 = \frac{V_1}{E_y} \dot{p}_1 + A_1 \dot{y} + K_{ci} p_1 \quad (1)$$

where,  $V_1$  is the volume of upper chamber,  $E_y$  is the system effective bulk modulus,  $p_1$  is the upper chamber pressure,  $A_1$  is the area of piston side without rod,  $K_{ci}$  is the inner leakage coefficient,  $q_1$  is equal to the pump's output flow, that is,

$$q_1 = D_m n, 0 \leq n \leq \omega_{max} \quad (2)$$

where,  $n$  is the rotary speed, and  $D_m$  is the pump's displacement.

When  $y > r_0$ ,  $p_1 = 0$ , and the continuity equation for the down chamber can be written as

$$A_2 \dot{y} - K_{ci} p_2 = \frac{V_2}{E_y} \dot{p}_2 + q_2 \quad (3)$$

where, the value of  $q_2$  is equal to that of the pump's output flow, but the direction is reverse, that is,

$$q_2 = -D_m n, 0 \leq n \leq \omega_{max} \quad (4)$$

Applying Newton's second law to the forces on the piston, we obtain the following equation

$$p_1 A_1 - p_2 A_2 = m \ddot{y} + b \dot{y} + ky \quad (5)$$

where,  $m$  is the mass of the rod,  $b$  is the coefficient of the damp, and  $k$  is the stiffness of the spring.

It is obvious that the system is a piece-wise linear system. Considering the requirement in Section 2.2 and supposing that we can guarantee  $y \leq r_0$  when the system move to the desired position, the system can be described by the follow state equations

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}u \\ y &= \mathbf{C}x^T \end{aligned} \quad (6)$$

where  $x = \begin{bmatrix} y \\ \dot{y} \\ p_1 \end{bmatrix}$ ,  $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{A_1}{m} \\ 0 & -\frac{E_y A_1}{V_1} & -\frac{E_y K_{ci}}{V_1} \end{bmatrix}$ ,

$\mathbf{B} = \begin{bmatrix} 0 & 0 & \frac{E_y D_m}{V_1} \end{bmatrix}^T$ ,  $\mathbf{C} = [1 \ 0 \ 0]$ ,  $u = n$ . Both the input and the output are subject to constraints, that is,

$$0 \leq u \leq \omega_{\max}, y \leq r_0 \quad (7)$$

#### 4. Controller Design

The position control problem of pump controlled asymmetric cylinder in this paper can be treated as typical constrained control problem<sup>[8]</sup>. We divide the design of control algorithm into two steps:

**Step 1:** assuming that the constraints in (7) is satisfied, use linear state feedback theory to find a state feed back matrix  $\mathbf{K}$  and a feedforward gain  $\mathbf{P}$ , and let

$$u = \mathbf{P} \cdot r_0 - \mathbf{K} \cdot x \quad (8)$$

which is subject to

$$\lim_{t \rightarrow \infty} (r_0 - y(t)) = 0 \quad (9)$$

and assigns the poles to the desired value<sup>[9]</sup>.

**Step 2:** discretize the continuous system ( $\mathbf{A} - \mathbf{BK}$ ,  $\mathbf{BP}$ ,  $\mathbf{C}$ ) to a discrete system  $\Sigma$

$$\begin{aligned} x(k+1) &= \mathbf{A}_d x(k) + \mathbf{B}_d r(k) \\ \Sigma: \quad y(k) &= \mathbf{C}_d x(k) \\ z(k) &= \mathbf{L}_d x(k) + \mathbf{D}_d r(k) \end{aligned} \quad (10)$$

and use reference management concept<sup>[10]</sup> to reconstruct a new reference signal  $r(k)$ , which minimizes the following objective function

$$J = \sum_{k=0}^N \|r_0 - y(k)\|^2 + w \sum_{k=0}^N \|r_0 - r(k)\|^2 \quad (11)$$

subject to

$$\begin{aligned} z(k) &\in \{(z_1(k), z_2(k))^T \mid 0 \leq z_1 \leq \omega_{\max}, z_2 \leq r_0\}, \\ k &= 0, 1, \dots, N \end{aligned} \quad (12)$$

here,  $\mathbf{L}_d = \begin{bmatrix} -\mathbf{K} \\ 1 & 0 & 0 \end{bmatrix}$ ,  $\mathbf{D}_d = \begin{bmatrix} \mathbf{P} \\ 0 \end{bmatrix}$ .

##### 4.1. Calculations of $\mathbf{K}$ and $\mathbf{P}$

Calculate the controllability matrix  $\mathbf{Q}$  of the system in (6), we obtained that

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & \frac{A_1 E_y D_m}{m V_1} \\ 0 & \frac{A_1 E_y D_m}{m V_1} & -\frac{b E_y D_m}{m V_1} + \frac{b E_y^2 D_m K_{ci}}{m V_1} \\ \frac{A_1 E_y D_m}{m V_1} & -\frac{E_y^2 D_m K_{ci}}{V_1^2} & -\frac{E_y^2 D_m A_1^2}{m V_1^2} + \frac{E_y^3 D_m K_{ci}^2}{V_1^2} \end{bmatrix} \quad (13)$$

It is obviously that  $\mathbf{Q}$  is full rank. Therefore, the poles of system can arbitrarily be assigned provided that complex conjugate eigenvalues are assigned in pairs. The feedforward gain  $\mathbf{P}$  can be calculated by

$$\mathbf{P} = \frac{1}{\mathbf{C}(\mathbf{I}_3 - \mathbf{A} + \mathbf{BK})^{-1} \mathbf{B}} \quad (14)$$

where  $\mathbf{I}_3$  is 3-by-3 identity matrix.

##### 4.2. Calculation of $r(k)$

Define vectors  $\mathbf{r}$ ,  $\mathbf{r}_0$ ,  $\mathbf{s} \in R^{N+1}$ , whose  $i$ th element are defined as

$$[\mathbf{r}]_i = r(i), i = 0, 1, 2, \dots, N \quad (15)$$

$$[\mathbf{r}_0]_i = r_0, i = 0, 1, 2, \dots, N \quad (16)$$

$$[\mathbf{s}]_i = r_0 - \mathbf{C}_d \mathbf{A}_d^i \mathbf{x}(0), i = 0, 1, 2, \dots, N \quad (17)$$

and matrixes  $\mathbf{M} \in R^{(N+1) \times (N+1)}$ , a lower triangular Toeplitz matrix, whose  $(i, j)$ th element is defined as

$$[\mathbf{M}]_{i,j} = \begin{cases} 0, & i = j = 1, 2, \dots, N+1 \\ \mathbf{C}_d \mathbf{A}_d^{i-j-1} \mathbf{B}_d, & 1 \leq j < i \leq N+1 \end{cases} \quad (18)$$

Define  $\mathbf{h} \in R^3$  and  $\mathbf{H} \in R^{3 \times 2}$  as

$$\mathbf{H} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T, \mathbf{h} = [\omega_{\max}, 0, r_0]^T \quad (19)$$

Minimizing (11) subject to (12) is equivalent to minimizing the following objective function

$$\tilde{J} = \mathbf{r}^T \mathbf{G} \mathbf{r} + \mathbf{f} \mathbf{r} \quad (20)$$

subject to

$$\mathbf{A}_l \cdot \mathbf{r} \leq \mathbf{b}_l \quad (21)$$

where  $\mathbf{G} = \mathbf{M}^T \mathbf{M} + w \mathbf{I}_{N+1}$ ,  $\mathbf{f} = -2(\mathbf{s}^T \mathbf{M} + w \mathbf{r}_0^T)$ ,  $\mathbf{I}_{N+1}$  is a  $(N+1)$ -by- $(N+1)$  identity matrix;  $\mathbf{A}_l \in R^{3(N+1) \times (N+1)}$  is a block lower triangular Toeplitz matrix whose  $(i, j)$  blocks are given by

$$[\mathbf{A}_l]_{i,j} = \begin{cases} \mathbf{H} \mathbf{D}_d, & i = j = 1, 2, \dots, N+1 \\ \mathbf{H} \mathbf{L}_d \mathbf{A}_d^{i-j-1} \mathbf{B}_d, & 1 \leq j < i \leq N+1 \end{cases} \quad (22)$$

and  $\mathbf{b}_l \in R^{3(N+1)}$ , whose  $i$ th block is defined by

$$[\mathbf{b}_l]_i = (h - \mathbf{H} \mathbf{L}_d \mathbf{A}_d^i \mathbf{x}_0), i = 1, 2, \dots, N+1 \quad (23)$$

#### 5. Numerical simulation

Consider a pump controller asymmetric cylinder system with  $m=500\text{Kg}$ ,  $b=0.02\text{N.s/m}$ ,  $k=1 \times 10^7 \text{N/m}$ ,

$E_y=8.8 \times 10^8 \text{ Pa}$ ,  $A_1=0.0491 \text{ m}^2$ ,  $V_1=0.0098 \text{ m}^3$ ,  $V_2=0.0023 \text{ m}^3$ ,  $K_{ci}=1 \times 10^{-10} \text{ m}^3/(\text{s.Pa})$ ,  $D_m=6.4 \times 10^{-6} \text{ m}^3/\text{rad}$ ,  $\omega_{\max}=209.44 \text{ rad/s}$ ,  $r_0=0.01 \text{ m}$ . Assign the system poles to  $\lambda_1 = -49.54$ , and  $\lambda_{1,2} = -8.26 \pm 15.25i$  with Matlab Function PLACE, we obtained  $\mathbf{K} = [-2.32 \times 10^4 \quad -8.01 \times 10^3 \quad 9.93 \times 10^{-5}]$ , and  $\mathbf{P} = 264.12$ . With Matlab Function C2D, we discretize  $(\mathbf{A}-\mathbf{BK}, \mathbf{B}\mathbf{P}, \mathbf{C})$  to  $(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d)$ . Solving problem described by (20) and (21) with Matlab Function QUADPROG, we obtained the new reference signal  $r(k)$  as shown in Fig.3.

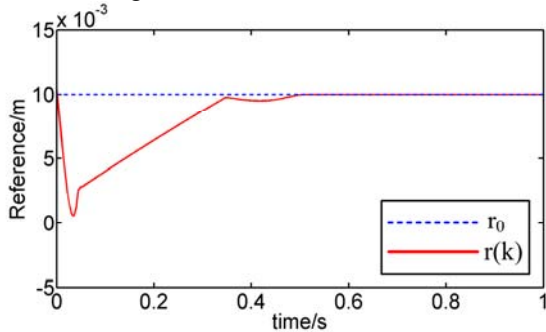


Fig.3 reference and shaped reference ( $N=500$ ,  $w=5$ )

In Fig.4, the dash-dot line is the control input of the system  $\Sigma$  with input constraints, when the reference is  $r_0$ . The solid line is for system  $\Sigma$  with constraints in (12), and with  $r(k)$  as reference input. It is obviously that the reference management plays its role in guaranteeing the constraints being satisfied. The corresponding system outputs are shown Fig.5, in which we can find that our method approaches the non-overshoot control goal perfectly. Large  $w$  is helpful to reduce system's response time.

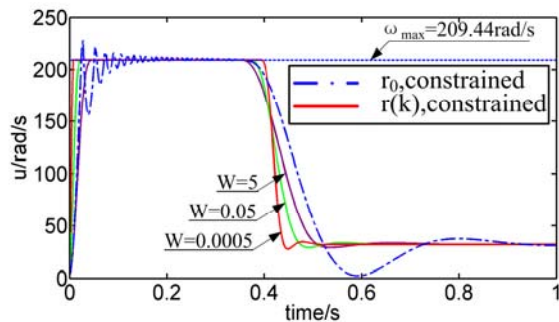


Fig.4 control inputs ( $N=400$ )

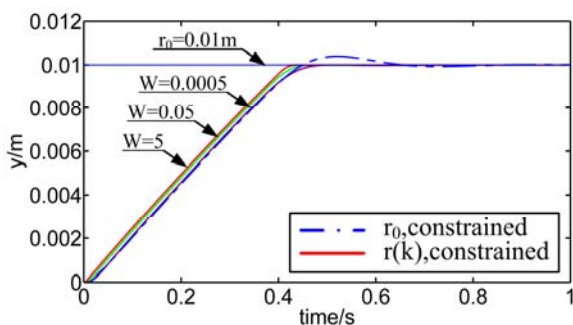


Fig.5 system outputs ( $N=400$ )

## 6. Conclusion

The reference management is used to transfer pump controlled asymmetric cylinder system from a constrained piece-wise linear system to a constrained linear system, and at the same time, the non-overshoot position control is approached.

Since the proposed method requires that the system model and parameters are known, it would be necessary to consider how to deal with the model uncertainty and/or disturbances. Combining the proposed method with robust control will be an interesting problem.

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